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Multi-Objective Multi-Mode Resource-Constrained Project Scheduling Problem by Preemptive Fuzzy Goal Programming

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***Abstract*—**This research proposes a preemptive fuzzy goal programming model for multi-objective multi-mode resource constrained project scheduling problem. The objectives of the problem are minimization of the total time and the total cost of the project. Objective in a multi-mode resource-constrained project scheduling problem is often a minimization of makespan. However, both time and cost should be considered at the same time with different level of important priorities. Moreover, all elements of cost functions in a project are not included in the conventional cost objective function. Incomplete total project cost causes an error in finding the project scheduling time. In this research, preemptive fuzzy goal programming is presented to solve the multi-objective multi-mode resource constrained project scheduling problem. It can find the compromise solution of the problem. Moreover, it is also flexible in adjusting to find a variety of alternative solutions.

***Keywords*—**Multi-mode resource constrained project scheduling problem, Fuzzy set, Goal programming, Preemptive fuzzy goal programming.

I. Introduction

PROJECT management (PM) issues have attracted interest for both practitioners and academics for a long time. Since the program evaluation and review technique (PERT) and the critical path method (CPM) were developed in the 1950s, many models including mathematical programming techniques, heuristics and meta-heuristics have been used to solve PM problems [1]. CPM is widely used for project planning and scheduling in many projects. It concerns about the time and determines critical activities to minimize project makespan, but the resource availability is not considered [2].

Resource-constrained project scheduling problem (RCPSP) is concerned with scheduling project activities over time and resource simultaneously [3]. The main focus on project makespan minimization has led to the development of various exact and (meta-) heuristic procedures for scheduling project with renewable resource constraints [4]. The extension of RCPSP is the multi-mode resource-constrained project scheduling problem (MRCPSP), which is a generalization of the RCPSP which each activity can be performed in one of several modes [5]. The objective of MRCPSP is to find a mode and a start time for each activity such that the makespan is minimized and the schedule is feasible with respect to the precedence and resource constraints [4]. Several exact and (meta-) heuristic approaches to solve the MRCPSP have been proposed in these recent years. For exact procedures, mixed integer programming formulations, linear programming, branch and bound algorithm, branch and cut algorithm and enumeration scheme-based procedure are presented [6]-[8]. Many of (meta-) heuristic approaches such as heuristics and combined heuristics, local search methods, genetic algorithm, Boolean satisfiability problem algorithm, frog-leaping algorithm, simulated annealing approach and tabu search [4], [6], [9], [10].

Most of objective in MRCPSP is makespan or project duration. However, the cost of the project is also important. Minimization of both project time and cost is a critical matter in today's competitive environment [2]. So, among these problems, the discrete time/cost tradeoff problem (DTCTP) is a well-known problem where the duration of each activity is a discrete of the amount of a single resource committed to it [10]. Many models have been proposed and can be categorized into two types: deterministic case and uncertainty case. In the deterministic case, dynamic programming enumeration algorithm or branch and bound algorithm are applied [10]. However, in fact, there are many cases that the project's parameters may not be presented in a precise manner [11]. Several stochastic models have been developed with uncertain activity durations [4], [10], [11]. Another type of dealing with uncertainty, fuzziness can be used. Fuzzy sets rather than crisp numbers are employed in goals, duration of project activities or parameters. Fuzzy minimum total crash cost is constructed by [11]. Fuzzy multi-objective two-phase fuzzy goal programming is developed and proposed by [1]. This method determines all objective in the same level. In real-world PM decisions, the satisfying goal values should normally be imprecise/fuzzy due to incomplete and unobtainable information over the project planning horizon. Moreover, some objectives may highly important than the others.

In this research another type of fuzzy methods is developed, called a preemptive fuzzy goal programming (PFGP) model. Two objectives are minimizations of both project time and total project cost is determined in the proposed model. The project time has higher priority than the project cost, so in the proposed model preemptive is used. Moreover, the total project costs of the current used models are reevaluated and shown the problem of lacking cost components.

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II. Total Project Cost

The total project cost is all costs to achieve the target of the project. It composes of direct and indirect costs. Most of research papers use some of the total project cost components in their objective, but there is no research paper considers the total project cost components. From the survey of existing researches, it can be concluded as shown in Table I.

TABLE I
Summary of Cost Functions in MRCPSP

	[1], [2]	[10]-[13]	[11],[14]-[16]	Proposed method
Direct cost	√			√
Crash cost	√	√	√	√
Fixed indirect cost	√			√
Variable indirect cost		√		√
Penalty cost	√			√
Reduced interest cost				√

III. Mathematical Programming Model

A. Index sets

i index for activities (i =1,…,N).
m index for modes (m =1,…,M).
k index for resources (k = 1,…,K).

B. Variables

x_{im} equals to 1 if activity i is performed in mode m.
equals to 0 otherwise

p_{im}, n_{im} positive and negative deviation in normal time of activity i if it is executed in mode m

d^+, d^- positive and negative deviation of the total project time

C. Parameters

d_{im}, $d(\min)_{im}$, $d(\max)_{im}$, $d(\text{nor})_{im}$ duration of activity, minimum duration of activity, maximum duration of activity and normal duration of activity i if it is executed in mode m.

t_i the total time from starting to the ending of activity i.

f_{im} the fixed direct cost of activity i if it is executed in mode m.

v_{im} the crash cost of activity i if it is executed in mode m.

fi_{im} the fixed indirect cost of activity i if it is executed in mode m.

vi_{im} the variable indirect cost of activity i if it is executed in mode m.

t_{nc} due date of the project.

c penalty cost per period.

it interest per period that can be reduced if the project is finished early.

r_{imk} the total number of resource k for activity i executed in mode m.

R_k the number of resources for each type of resources.

re_{imk} cost of activity i if it is executed in mode m for resource k.

λ_1, λ_2 satisfaction levels of goals 1 and 2.

λ_1^* acceptable satisfactory level of goal 1.

ρ_1, ρ_2 positive deviations of goals 1 and 2.

η_1, η_2 negative deviations of goals 1 and 2.

A is the set of pairs of activities between, which a finish-start precedence relationship with time lag 0 exists. Activity i is a predecessor of activity j.

D. Mathematical Model for MRCPSP

In the proposed MRCPSP model, Two objectives are minimizations of both project time (z_1) and total project cost (z_2) are determined as shown in (1)-(2). The project time or makespan is the main objective and has higher priority than the total project cost. In the total project cost composed of direct cost, crash cost, indirect variable cost, penalty cost, reduced interest cost and resource cost.

$$\min\ z_1 = t_N \tag{1}$$

$$\begin{aligned}\min\ z_2 = {} & \sum_{m=1}^{M}\sum_{i=1}^{N}(f_{im}\cdot x_{im}) + \sum_{m=1}^{M}\sum_{i=1}^{N}(p_{im}\cdot x_{im}\cdot v_{im}) + \\ & \sum_{m=1}^{M}\sum_{i=1}^{N}(fi_{im}\cdot x_{im}) + \\ & \sum_{m=1}^{M}\sum_{i=1}^{N}(vi_{im}\cdot t_N) + (c\cdot d^-) - (it\cdot d^+) + \\ & \sum_{m=1}^{M}\sum_{i=1}^{N}(r_{imk}\cdot re_{imk}\cdot x_{im})\end{aligned} \tag{2}$$

Constraints

$$\sum_{m=1}^{M} x_{im} = 1 \quad \text{for } i = 1,\ldots,N \tag{3}$$

$$t_i \le \sum_{m=1}^{M} t_j - (d_{jm}\cdot x_{jm}) \text{ for all } (I,j)\in A \tag{4}$$

$$t_N = \sum_{m=1}^{M} t_j - (d_{jm}\cdot x_{jm}) \tag{5}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{N}(r_{imk}\cdot x_{im}) \le R_k \text{ for } k = 1,\ldots,K \tag{6}$$

$$d_{im} = d(\text{nor})_{im} + (n_{im} - p_{im}) \text{ for } I = 1,\ldots,N;\ m = 1,\ldots,M \tag{7}$$

$$t_N = t_{nc} + (d^- - d^+) \tag{8}$$

$$d(\min)_{im} \le d_{im} \le d(\max)_{im} \tag{9}$$

$$n_{im}\cdot p_{im} = 0 \tag{10}$$

$$d^+\cdot d^- = 0 \tag{11}$$

$$d^+, d^-, n_{im}, p_{im} \ge 0 \tag{12}$$

Equation (3) ensures each job completed exactly once. Equation (4) ensures that precedence relationships are maintained. A is a set of all pairs of immediate predecessor jobs. Equation (5) shows the total project time. Equation (6) is the resource limitation constraint. Equations (7) and (8) are deviations of activity durations and project durations. Range of duration for each activity is shown in (9). Equations (10), (11) show the relationships of positive and negative deviations of activity durations and project duration, and the remaining constraint shows nonnegativity constraint.

E. Preemptive Fuzzy Goal Programming Model for MRCPSP

Goal programming is one of the most popular multi-objective solution methods. However, it is too rigid for decision makers in the selection of the satisfactory solution among the efficient solution set. So, the fuzzy goal programming model has been proposed. The minimax fuzzy goal programming is applied to MRCPSP by [1]. However, the minimax method is not appropriate if one objective is high important than the other. In the MRCPSP, the project completion time should be minimized first and then the total project cost is determined by relaxing the first objective. So, the preemptive fuzzy goal programming is proposed in this research. Based on the idea of preemptive goal programming, deviations of both goals should be minimized orderly as shown in (13) and the goals of both objectives can be set as shown in (14), (15). Equations (16), (17) are non-negativity constraints.

$$\text{lex min}=[(\rho_1+\eta_1),(\rho_2+\eta_2)] \tag{13}$$

$$z_1-\rho_1+\eta_1=\tau_1 \tag{14}$$

$$z_2-\rho_2+\eta_2=\tau_2 \tag{15}$$

$$\rho_1,\eta_1\geq 0 \tag{16}$$

$$\rho_2,\eta_2\geq 0 \tag{17}$$

In order to set fuzzy goals, membership functions of the goal are constructed. Fuzzy set is applied to each goal of the objective function. Defining the membership function of each goal is based on the Positive-Ideal Solution (PIS) and the Negative-Ideal Solution (NIS) [17], [18]. The PIS is the best possible solution when each objective function is optimized. The NIS is the feasible worst value of each objective function. So, the PIS is used to set the most preferred value and has the satisfactory degree of 1. By the same way, the satisfactory degree of 0 is assigned to the NIS. Acceptable deviation from the goal can be calculated from the difference between PIS and NIS or it can be evaluated by DM. Then, the membership function of the goal based on the DM's preference can be shown as Fig. 1.

Membership functions of goals 1 and 2, $\mu(z_1)$ and $\mu(z_2)$ can be represented by

$$\mu(z_1)=1-\left(\frac{z_1-PIS}{NIS}\right) \tag{18}$$

$$\mu(z_2)=1-\left(\frac{z_2-PIS}{NIS}\right) \tag{19}$$

The, preemptive fuzzy goal programming model is described in the following steps

<u>*Step 1*</u>max $=\lambda_1$

Subject to

$$\lambda_1\leq\mu(z_1)\,. \tag{20}$$

and constraints (1)-(12), (14), (18),(19).

<u>Step 2</u> max $=\lambda_2$

Subject to

$$\lambda_1^*\leq\mu(z_1)\quad, \tag{21}$$

$$\lambda_2\leq\mu(z_2)\,, \tag{22}$$

and constraints (1)-(12), (15), (18), (19).

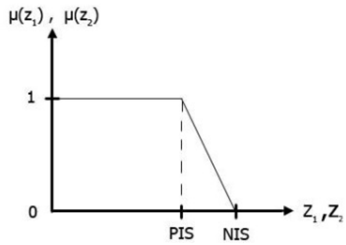

Fig. 1 Membership function of two goals

IV. Numerical Example

A case study of this research is adapted from [10]. Information about each activity, time, crash cost, and direct cost are shown in Table II. Activity relationships, modes and resources are shown in Table III. Interest per day is $2,000. Indirect variable cost per day is $7,000 and fixed indirect cost is $20,000. Penalty cost per day is $6,000. The due date of this project is 86 days. Limitation of resources R1 and R2 are 9 and 4, respectively. Resource costs for R1 and R2 are $2,000 and $4,000 respectively. From Tables I, II groups costs components are commonly used in constructing the objective for MRCPSP. One is the total cost of crash cost and indirect variable cost (Type 1). Another is the total cost of direct cost, crash cost, fixed indirect cost and penalty cost (Type 2). The entire project cost components are not included in both of these objectives. So, in the proposed model, all cost components as are determined (Type 3).

If we consider only one objective of the total project cost, then we will find that cost components effects project completion time. The determination of the completed total project cost causes to reduce project completion time because the minimum cost of Type 3 is shortest. Moreover, Type 1 and type 2 are also different.

TABLE II
Activity, Time, Crash Cost and Direct Cost for Each Activity

Activity ID	Shortest time (Day)	Crash cost$ (x1,000)	Direct cost$ (x1,000)
1	0	0	0
2	32	13	7
	13	19	12
3	23	14	8
	9	21	14
4	15	16	8
5	22	14	8
	11	19	13
6	18	15	8
	6	23	15
7	11	19	13
	20	14	8
8	18	12	7
9	14	14	8
10	8	21	14
	24	14	8
11	10	19	13

TABLE III
ACTIVITY RELATIONSHIPS, MODES AND RESOURCES

Activity ID	Successor	Mode	R1 (Persons)	R2 (Persons)	Normal time (Days)
1	2,3,4	1	0	0	0
2	5	1	5	0	45
		2	0	3	18
3	5	1	7	0	39
		2	0	4	15
4	5	1	6	0	24
5	6,7	1	2	0	36
		2	0	3	18
6	8	1	4	0	27
		2	0	2	9
7	8	1	0	4	18
		2	5	0	33
8	9	1	4	0	30
9	10	1	2	0	20
10	11	1	0	2	13
		2	4	0	39
11	12	1	0	2	18

Solving by the proposed method the compromise solution can be obtained. The decision maker can select the appropriate solution by adjusting the satisfaction level according to his/her preference. If the project completion time has been minimized the then, the project completion time will be 76 days but the total cost is $1.604 million. While the total cost is minimized, then the project completion time will be very high, 85 days but the total cost is $1.424 million. These are not compromised. This compromise solution selected by the decision maker is shown in Table IV, the project completion time is 84 days but the total costs are $1.445 million. The selection of activity mode is shown in Table V.

TABLE IV
RESULTS OF SINGLE OBJECTIVE OPTIMIZATIONS AND THE PROPOSED METHOD

Model	Minimize project duration	Minimize project cost	Preemptive fuzzy goal programming
Project duration (days)	76	105	84
Project cost (Million dollars)	1.604	1.345	1.445
λ_1	1	0	0.75
λ_2	0	1	0.61

TABLE V
SELECTION OF ACTIVITIES MODE BY THE PROPOSED MODEL

Activity ID	Mode	Time of each activity by preemptive fuzzy goal programming (days)
2	2	18
3	2	15
4	1	18
5	2	11
6	1	27
7	1	18
8	1	18
9	1	14
10	1	13
11	1	10

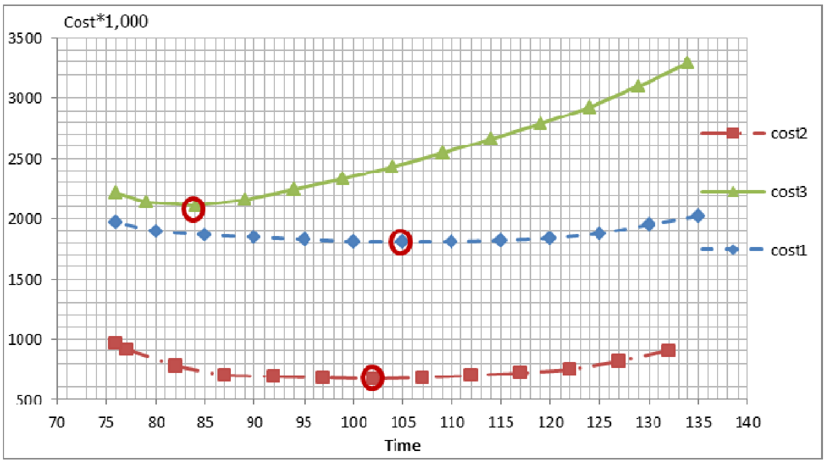

Fig. 2 Relationship of the total project cost and project completion time of three types of cost functions

V. CONCLUSION

This research presents a preemptive fuzzy goal programming model for multi-objective multi-mode resource constrained project scheduling problem. Two main objectives of the problem are minimization of the total time and the total cost of the project, which are considered in priority. Objective in a multi-mode resource-constrained project scheduling problem is often a minimization of makespan. However, both time and cost should be considered. From investigation of cost function, we found that lack of cost components effect to the total project time. Incomplete total project cost causes an error in finding the project scheduling time. So, we proposed the cost function with all cost components of the project. In this research, preemptive fuzzy goal programming is presented to solve the multi-objective multi-mode resource constrained project scheduling problem. It can find the compromise solution of the problem that the decision maker can select the appropriate one by adjusting the value of the acceptable satisfaction level.

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