

The Ability of Forecasting the Term Structure of Interest Rates Based On Nelson-Siegel and Svensson Model

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Abstract—Due to the importance of yield curve and its estimation it is inevitable to have valid methods for yield curve forecasting in cases when there are scarce issues of securities and/or weak trade on a secondary market. Therefore in this paper, after the estimation of weekly yield curves on Croatian financial market from October 2011 to August 2012 using Nelson-Siegel and Svensson models, yield curves are forecasted using Vector autoregressive model and Neural networks. In general, it can be concluded that both forecasting methods have good prediction abilities where forecasting of yield curves based on Nelson Siegel estimation model give better results in sense of lower Mean Squared Error than forecasting based on Svensson model. Also, in this case Neural networks provide slightly better results. Finally, it can be concluded that most appropriate way of yield curve prediction is Neural networks using Nelson-Siegel estimation of yield curves.

Keywords—Nelson-Siegel model, Neural networks, Svensson model, Vector autoregressive model, Yield curve

I. INTRODUCTION

THE yield curve is a representation of the relationship between market remuneration rates and the remaining time to maturity of debt securities, also known as the term structure of interest rates [1]. A full range of activities in the financial markets is actually determined by the relationship between the interest rate and maturity.

To estimate the yield curve central banks and financial managers use different models. In most cases dominate Svensson and Nelson-Siegel models [2]. Both models have their advantages and disadvantages. Nelson Siegel model is extremely popular in the practice; both individual investors and the central banks use this model. This model is simple and stable for the evaluation, it is quite flexible and very well suited for assessing yields for more bonds or one bond and for the time series of returns, for a large number of countries and time periods and for different classes of bonds. It also has good prediction ability [3]. Svensson model is an extension of the Nelson-Siegel model, which provides sufficient precision adjustment and smooth curve shape of periodic rents. It has become very popular in the mid 90's. Since then, it has been used by the substantial number of central banks worldwide to

assess the bond structure with a single coupon and future interest rates (forward rates). However, the model has some weaknesses, such as limited ability to adapt irregular shapes of the yield curve, the tendency of taking extreme values at the bottom of the curve and a relatively strong dependence on the estimations in different or even nonadjacent segments of the yield curve [3].

In Croatia still does not exist an official yield curve due to a scarce issue of Croatian bonds denominated in Kuna and weak trade on a secondary market. Due to the fact that a proper yield curve is not estimated for Croatian financial market, but also because of its obvious importance, it is necessary to provide a tool for yield curve forecasting purposes. In this paper two forecasting methodologies are contrasted: a vector autoregressive model and a neural network approach.

In the first part of the paper theoretical overview of the yield curve is provided, followed by an explanation of two most commonly used models for yield curve estimation: Nelson-Siegel and Svensson model. In the third part of the paper two forecasting methodologies are presented, and used as a forecasting tool in fourth part of the paper. Finally, the main conclusions of the paper are given in the last section.

II. YIELD CURVE ESTIMATION METHODS

A. Nelson-Siegel Model

Often used model for developing yield curve in the practice is the Nelson-Siegel model [4]. Nelson and Siegel introduced a simple, parsimonious model, which can adapt to the range of shapes of yield curves: monotonic, humped and S shape.

A class of functions that readily generates the typical yield curve shapes is that associated with solutions to differential or difference equations [5]. If the instantaneous forward rate at maturity T , $f(t, T)$, is given by the solution to a second-order differential equation with real and unequal roots, it is of the form:

$$f(t, T) = \beta_0 + \beta_1 e^{-\frac{T-t}{\tau_1}} + \beta_2 e^{-\frac{T-t}{\tau_2}} \quad (1)$$

where τ_1 and τ_2 are time constants associated with the equation, and β_0 , β_1 and β_2 are determined by initial conditions.

Now, zero-coupon rates $R(t)$ can be calculated by averaging the corresponding instantaneous forward rates:

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$$R(t, T) = \frac{1}{T-t} \int_t^T f(x, T) dx \quad (2)$$

A more parsimonious model that can generate the same range of shapes is given by the equation solution for the case of equal roots:

$$f(t, T) = \beta_0 + \beta_1 e^{-\frac{T-t}{\tau}} + \beta_2 \frac{T-t}{\tau} e^{-\frac{T-t}{\tau}} \quad (3)$$

By substituting (3) into (2) and integrating, and after a simple rearrangement of this expression, the yield to maturity is given by:

$$R(t, T) = \beta_0 + (\beta_1 + \beta_2) \frac{1 - e^{-\frac{T-t}{\tau}}}{\frac{T-t}{\tau}} - \beta_2 e^{-\frac{T-t}{\tau}} \quad (4)$$

It is obvious that the forward and zero-coupon yield curves are functions of four parameters: β_0 , β_1 , β_2 and τ .

It can be seen that

$$\lim_{T \rightarrow \infty} R(t, T) = \beta_0 \quad (5)$$

and β_0 corresponds to zero-coupon rates for very long maturities.

At the short end of the curve it is:

$$\lim_{T \rightarrow t} R(t, T) = \beta_0 + \beta_1 \quad (6)$$

which implies that the sum of parameter values β_0 and β_1 should be equal to the level of the shortest interest rates.

If β_1 is negative, the forward curve will have a positive slope and other way round. The parameter β_2 , indicates the magnitude and the direction of the hump and if it is positive, a hump will occur at τ whereas, in case it is negative, a U-shaped value will occur at τ . It can be concluded that parameter τ which is positive, specifies the position of the hump or U-shape on the entire curve. Consequently, Nelson and Siegel propose that with appropriate choices of weights for these three components, it is possible to generate a variety of yield curves based on forward rate curves with monotonic and humped shapes [5].

B. Svensson Model

Svensson [6] extended Nelson-Siegel model by introducing additional parameters that allow yield curve to have an additional hump. Thus this model is considered to be computably more demanding. Svensson suggested forward curve to be estimated as:

$$f(t, T) = \beta_0 + \beta_1 e^{-\frac{T-t}{\tau_1}} + \beta_2 \frac{T-t}{\tau_1} e^{-\frac{T-t}{\tau_1}} + \beta_3 \frac{T-t}{\tau_2} e^{-\frac{T-t}{\tau_2}} \quad (7)$$

The corresponding yield to maturity is of the form:

$$R(t, T) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{T-t}{\tau_1}}}{\frac{T-t}{\tau_1}} + \beta_2 \left(\frac{1 - e^{-\frac{T-t}{\tau_1}}}{\frac{T-t}{\tau_1}} - e^{-\frac{T-t}{\tau_1}} \right) + \beta_3 \left(\frac{1 - e^{-\frac{T-t}{\tau_2}}}{\frac{T-t}{\tau_2}} - e^{-\frac{T-t}{\tau_2}} \right) \quad (8)$$

From (7), it can be noticed that the β_2 is identical to the β_2 term from the Nelson-Siegel model with τ replaced by τ_1 . The two additional parameters β_3 and τ_2 explain the extended flexibility of the Svensson approach. The parameter β_3 defines the form (convex or concave) of the second hump of the spot interest rate curve, and the parameter τ_2 , like τ_1 in the Nelson-Siegel model, defines its position [7].

III. FORECASTING METHODS

A. Vector Autoregressive Model

Diebold and Li [8] chose, among the others, Vector Autoregressive (VAR) model to forecast parameters (factors) of the Nelson-Siegel model. VAR model groups all the factors to take into account the interaction between all the states variables [9].

VAR is a multivariate time series model that consists of multiple equations [10]. VAR model defined with n endogenous variables and k lags can be written as:

$$Z_t = a_0 + A_1 Z_{t-1} + \dots + A_k Z_{t-k} + B D_t + \varepsilon_t \quad (9)$$

where Z_t is n -dimensional vector of potentially endogenous variables, A_1, \dots, A_k are $n \times n$ coefficient matrices, D_t is a vector of other exogenous variables with coefficient matrix B . Vector a_0 is a vector of constants (intercept) and ε_t is vector of error terms, i.e. n -dimensional white noise process. The parameters of VAR model can be estimated using ordinary least squares method, where the optimal order, i.e. number of lags k can be found using information criteria: final prediction error (FPE), Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC). The advantages of VAR models are: simplicity of the model (it is not necessary to classify endogenous and exogenous variables in the model), ease of estimation (each equation can be estimated with ordinary least square method), quality of forecasted estimates.

B. Neural Networks

Neural network (NN) is an artificial intelligence method, which has recently received a great deal of attention in many fields of study. NN can be seen as a non-parametric statistical procedure that uses the observed data to estimate the unknown function [11]. A wide range of statistical and econometric

models can be specified modifying activation functions or the structure of the network (number of hidden layers, number of neurons etc.) [12]: multiple regression, vector autoregression, logistic regression, time series models, etc. NN often give better results than statistical methods because of their possibility of analyzing the missing data, data with noise and learning from the previous data. Empirical researches show that NN are successful in forecasting extremely volatile financial variables that are hard to predict with standard statistical methods such as: exchange rates [13], interest rates [14] and stocks. More recently they have been used to forecast estimated parameters of the yield curve ([8], [9] and [15]).

NN usually have input, hidden and output layer. Input neurons receive data from the external world and send it to one or more hidden neurons. In the hidden layer information from neurons are processed and sent to output neurons. Information then backpropagate through network and the values of weights between neurons are adjusted to the target output. The process in the network is repeated as much iterations (epochs) as needed to reach the output that is the closest to the targeted output.

For the given inputs with known outputs the goal is to train the network in order to estimate the functional form between inputs and outputs. To accomplish the learning some form of an objective function is required, in order to optimize the weights. Most commonly used goal function is the sum of squared errors defined as:

$$E = \frac{1}{2} \sum_{p=1}^n \sum_{k=1}^O (y_{pk} - \hat{y}_{pk})^2 \quad (10)$$

where the subscript p refers to observation, with a total of n observations, the subscript k to output unit with a total of O outputs, y is the observed response, and \hat{y} is the predicted response. This is a sum of squared difference between the observed and predicted response averaged over all inputs and observations.

To understand backpropagation learning, firstly the way information is passed forward through the network is presented. The process starts with the input units being presented to the input layer. Input layer simply transfers data to the hidden layer. The input into the j -th hidden neuron is:

$$h_{pj} = \sum_{i=1}^N w_{ji} x_{pi} \quad (11)$$

here N is total number of input nodes, w_{ji} is the weight from input unit i to output unit j , and x_{pi} is the value of the i -th input for pattern p . The j -th hidden unit applies an activation function to its net inputs and outputs:

$$v_{pj} = g(h_{pj}) = \frac{1}{1 + e^{-h_{pj}}} \quad (12)$$

assuming that $g(\cdot)$ is a sigmoid (logistic) function. Using nonlinear activation function allows a neural network to capture nonlinearity in data.

Similarly, output unit k receives a net input of:

$$f_{pk} = \sum_{j=1}^M W_{kj} v_{pj} \quad (13)$$

where M is the number of hidden units, and W_{kj} represents the weight from hidden unit j to output k . The unit then outputs the quantity:

$$\hat{y}_{pk} = g(f_{pk}) \quad (14)$$

assuming that $g(\cdot)$ is a identity (linear) function.

The goal is to find the set of weights w_{ji} , the weights connecting the input and hidden layer, and W_{kj} , the weights connecting hidden and output layer, which minimize the sum of squared errors given in (10).

In this case logistic activation function is used in the hidden layer and identity (linear) in the output layer.

IV. DATA AND METHODOLOGY

In order to calculate yield curve on a Croatian financial market data from Zagreb money market, where data for treasury bills can be found, and Reuters data base, where data for government bonds can be found, is collected. Yield curves are calculated on a weekly basis from 7th October 2011 to 24th August 2012 using both Nelson-Siegel and Svensson model. Even though on these dates there was pour trade on treasury bills and bonds (on observed dates the number of securities traded was mostly 10), yield curves are successfully estimated using above mentioned formulas.

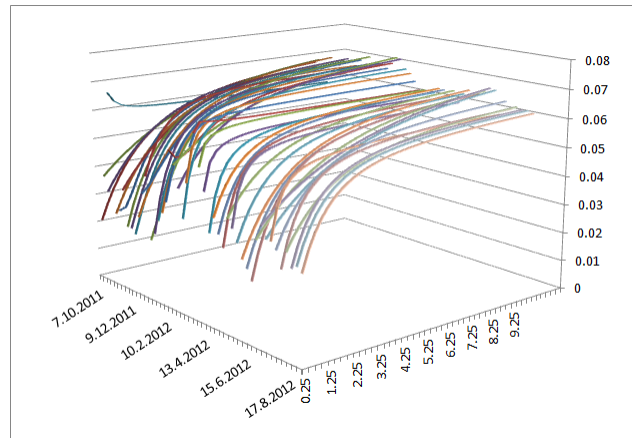


Fig. 1 Yield curves on Croatian financial market using Nelson-Siegel model

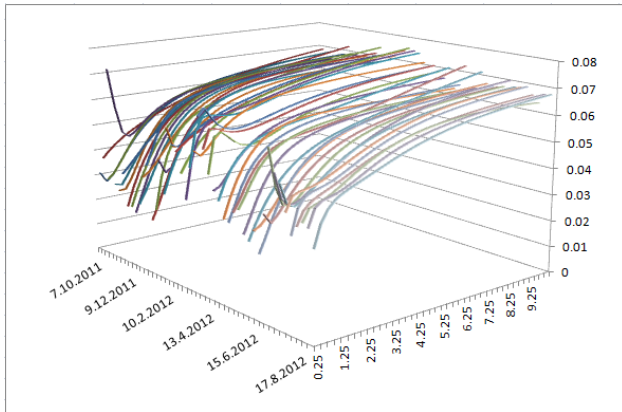


Fig. 2 Yield curves on Croatian financial market using Svensson model

Parameters β_0 , β_1 , β_2 and τ are estimated for Nelson-Siegel model, and β_0 , β_1 , β_2 , β_3 and τ_1 and τ_2 for Svensson model in MS Office Excel using least square method with quasi-Newton. In the case where it was particularly difficult to estimate parameters, using Simplex method in Statistica starting points for an estimation of parameters are generated. These appropriate start values are then used in subsequent quasi-Newton iterations¹. Figs. 1 and 2 present calculated yield curves using Nelson-Siegel and Svensson models respectively.

Tables I and II show means, standard deviations, and extreme low and high values of estimated parameters on Croatian financial market through the whole sample period for Nelson-Siegel and Svensson model respectively. For Nelson-Siegel model parameter β_0 corresponds to zero-coupon rates for very long maturities and equals 6.95% on average. The sum of parameters β_0 and β_1 represents the level of the shortest interest rates and equals 2.13% on average. Since β_1 is negative, the curve has a positive slope. The parameter β_2 , indicates the magnitude and the direction of the hump at time τ and since it is nearly equal to zero (0.0039 on average) the curves have a monotonic shape. In Svensson model the long term interest rate equals 7.45% on average, the short term interest rate is 1.72% on average, the parameter β_2 which is negative indicates that an U-shape occurs at time $\tau_1=0.891$ and β_3 which is positive indicates the position of a hump at time $\tau_2=0.6559$.

Table I shows high volatility of parameter τ from the Nelson-Siegel model with standard deviation of 0.95 and extreme values (minimum and maximum values) ranging from 0.04 to 3.94. The same situation is with the parameters β_2 , β_3 and τ_1 and τ_2 from the Svensson model, given in Table II.

¹ Simplex method is generally less sensitive to local minima and is usually used in combination with the quasi-Newton method [16]

TABLE I
DESCRIPTIVE STATISTICS OF ESTIMATED PARAMETERS USING NELSON-SIEGEL MODEL

Parameters	Mean	Std. Dev.	Min	Max
β_0	0.0695	0.0062	0.0460	0.0808
β_1	-0.0482	0.0119	-0.0675	-0.0123
τ	0.9001	0.9454	0.0478	3.9361
β_2	0.0039	0.0344	-0.0894	0.0970

Source: Author

TABLE II
DESCRIPTIVE STATISTICS OF ESTIMATED PARAMETERS USING SVENSSON MODEL

Parameters	Mean	Std. Dev.	Min	Max
β_0	0.0745	0.0103	0.0473	0.1051
β_1	-0.0573	0.0155	-0.0919	-0.0171
τ_1	0.8910	0.8558	0.0095	4.4552
β_2	-0.1912	0.9062	-5.0478	1.3427
β_3	0.2183	0.8999	-1.2840	5.0358
τ_2	0.6559	0.6665	0.0359	2.7959

Source: Author

After the estimation of the parameters using Nelson-Siegel model and Svensson model, yield curves are forecasted using Vector autoregressive and Neural network models, by predicting parameters β_0 , β_1 , β_2 and τ for Nelson-Siegel model, and β_0 , β_1 , β_2 , β_3 and τ_1 and τ_2 for Svensson model.

The parameters are predicted using Vector autoregressive (VAR) model in Stata11 by dividing the sample on two sets: first, the test set from 7th October 2011 until 15th June 2012 and second, the validation set from 22nd June 2012 until 24th August 2012. Based on final prediction error (FPE) and Akaike's information criterion (AIC) VAR (1) model is chosen, which can be defined as:

$$Z_t = a_0 + A_1 Z_{t-1} + \varepsilon_t \quad (15)$$

VAR (1) model is tested on test set and based out-of-sample forecast of parameters mean square error (MSE) is calculated both for Nelson-Siegel and Svensson model. The results are presented in Tables III and V respectively.

For prediction of Nelson-Siegel and Svensson parameters using Neural networks (NN), training sample from 7th October 2011 until 11th May 2012, testing sample from 18th May 2012 until 15th June 2012 and validation sample from 22nd June 2012 until 24th August 2012 is used. MLP model with one hidden layer is used with the process of trial and errors for defining the right number of units in a hidden layer and the activation function. The best neural networks are: MLP 4-14-4 for Nelson-Siegel model and MLP 6-18-6 for Svensson model; logistic activation function in hidden layer and identity activation function in output layer are set in advance. Based on validation sample (the same as out-of-sample forecast in VAR (1)) mean square error (MSE) is calculated and given in

Tables IV and VI for Nelson-Siegel and Svensson model respectively.

Neural network model gives marginally smaller MSE than VAR based method for Nelson-Siegel model (Tables III and IV). Both models predict the values of parameters β_0 , β_1 and β_2 extremely well, ending with small mean square errors. However, due to the fact that estimated parameter τ is varying extremely through forecasting period, both models' forecasting abilities are weak, ending with much larger mean square errors. Fig. 3 shows the yield curve estimated with

Nelson-Siegel model on 22nd June 2012 and yield curves predicted with neural network and vector autoregressive model. It shows good short term forecast abilities of both models and marginally better results of NN model. Fig. 4 shows the yield curve estimated with Nelson-Siegel model on 24th August 2012 and yield curves predicted with neural network and vector autoregressive model. It shows long term forecast abilities of both models and it can be concluded that both models perform slightly worse in longer term forecast horizons.

TABLE III
ESTIMATED AND PREDICTED PARAMETERS OF NELSON-SIEGEL MODEL USING VAR (1) WITH MSE

Date	β_0	$\hat{\beta}_0$	β_1	$\hat{\beta}_1$	τ	$\hat{\tau}$	β_2	$\hat{\beta}_2$
22.6.2012	0.0713	0.0684	-0.0402	-0.0507	2.7827	0.7737	0.0313	0.0032
29.6.2012	0.0792	0.0685	-0.0422	-0.0493	2.3586	0.8435	0.0000	0.0082
6.7.2012	0.0769	0.0693	-0.0444	-0.0495	1.7828	0.7983	0.0000	0.0054
13.7.2012	0.0619	0.0695	-0.0490	-0.0493	0.1801	0.8057	0.0000	0.0042
20.7.2012	0.0694	0.0695	-0.0419	-0.0491	0.8197	0.8234	0.0000	0.0043
27.7.2012	0.0659	0.0695	-0.0497	-0.0490	0.4400	0.8286	0.0000	0.0044
3.8.2012	0.0688	0.0695	-0.0394	-0.0490	1.0312	0.8290	0.0000	0.0044
10.8.2012	0.0667	0.0695	-0.0462	-0.0490	0.5670	0.8294	0.0000	0.0044
17.8.2012	0.0669	0.0695	-0.0443	-0.0490	0.6059	0.8297	0.0000	0.0044
24.8.2012	0.0663	0.0695	-0.0458	-0.0490	0.5763	0.8298	0.0000	0.0044
MSE	0.0166		0.0193		2.8403		0.0319	

Source: Author

TABLE IV
ESTIMATED AND PREDICTED PARAMETERS OF NELSON-SIEGEL MODEL USING NN (4-14-4) WITH MSE

Date	β_0	$\hat{\beta}_0$	β_1	$\hat{\beta}_1$	τ	$\hat{\tau}$	β_2	$\hat{\beta}_2$
22.6.2012	0.0713	0.0696	-0.0402	-0.0482	2.7827	0.8623	0.0313	0.0051
29.6.2012	0.0792	0.0695	-0.0422	-0.0482	2.3586	0.8654	0.0000	0.0049
6.7.2012	0.0769	0.0696	-0.0444	-0.0482	1.7828	0.8651	0.0000	0.0049
13.7.2012	0.0619	0.0696	-0.0490	-0.0482	0.1801	0.8647	0.0000	0.0050
20.7.2012	0.0694	0.0696	-0.0419	-0.0482	0.8197	0.8628	0.0000	0.0051
27.7.2012	0.0659	0.0696	-0.0497	-0.0482	0.4400	0.8643	0.0000	0.0050
3.8.2012	0.0688	0.0696	-0.0394	-0.0482	1.0312	0.8631	0.0000	0.0051
10.8.2012	0.0667	0.0696	-0.0462	-0.0482	0.5670	0.8645	0.0000	0.0050
17.8.2012	0.0669	0.0696	-0.0443	-0.0482	0.6059	0.8636	0.0000	0.0050
24.8.2012	0.0663	0.0696	-0.0458	-0.0482	0.5763	0.8638	0.0000	0.0050
MSE	0.0158		0.0161		2.7706		0.0302	

Source: Author

TABLE V
ESTIMATED AND PREDICTED PARAMETERS OF SVENSSON MODEL USING VAR (1) WITH MSE

Date	β_0	$\hat{\beta}_0$	β_1	$\hat{\beta}_1$	τ_1	$\hat{\tau}_1$	β_2	$\hat{\beta}_2$	β_3	$\hat{\beta}_3$	τ_2	$\hat{\tau}_2$
22.6.2012	0.0716	0.0758	-0.0666	-0.1056	0.8059	0.5322	0.0000	-0.1640	0.1245	0.6318	0.0489	0.7047
29.6.2012	0.0726	0.0771	-0.0584	-0.0827	0.4558	0.7231	-0.0972	-0.3082	0.1797	0.5484	0.1069	0.7544
6.7.2012	0.0709	0.0765	-0.0560	-0.0792	2.9877	0.7017	0.0565	-0.2498	0.1287	0.4570	0.0797	0.7513
13.7.2012	0.0736	0.0766	-0.0689	-0.0814	0.9757	0.6975	0.0000	-0.2813	0.1384	0.5041	0.0842	0.7581
20.7.2012	0.0473	0.0766	-0.0260	-0.0811	4.4552	0.6947	0.1025	-0.2833	0.0603	0.5006	0.2402	0.7482
27.7.2012	0.0756	0.0765	-0.0633	-0.0816	0.6421	0.6890	-0.1437	-0.2802	0.1585	0.5019	0.4186	0.7430
3.8.2012	0.0718	0.0765	-0.0445	-0.0817	0.6645	0.6882	-0.1029	-0.2804	0.0931	0.5037	0.4577	0.7421
10.8.2012	0.0495	0.0765	-0.0324	-0.0817	3.0436	0.6884	0.0882	-0.2796	0.0613	0.5029	0.1989	0.7421
17.8.2012	0.0728	0.0765	-0.0533	-0.0817	0.3243	0.6886	0.0783	-0.2795	-0.1013	0.5026	0.6126	0.7425
24.8.2012	0.0742	0.0765	-0.0562	-0.0817	0.5909	0.6888	0.2284	-0.2795	-0.2358	0.5025	0.7093	0.7427
MSE	0.0409		0.0476		4.9696		1.0500		0.8470		1.7108	

Source: Author

TABLE VI
ESTIMATED AND PREDICTED PARAMETERS OF SVENSSON MODEL USING NN (6-18-6) WITH MSE

Date	β_0	$\hat{\beta}_0$	β_1	$\hat{\beta}_1$	τ_1	$\hat{\tau}_1$	β_2	$\hat{\beta}_2$	β_3	$\hat{\beta}_3$	τ_2	$\hat{\tau}_2$
22.6.2012	0.0716	0.0776	-0.0666	-0.0577	0.8059	0.6628	0.0000	-0.0779	0.1245	0.1945	0.0489	0.7709
29.6.2012	0.0726	0.0777	-0.0584	-0.0570	0.4558	0.6778	-0.0972	-0.0353	0.1797	0.1959	0.1069	0.7976
6.7.2012	0.0709	0.0776	-0.0560	-0.0573	2.9877	0.6749	0.0565	-0.0678	0.1287	0.2129	0.0797	0.7947
13.7.2012	0.0736	0.0797	-0.0689	-0.0516	0.9757	0.7988	0.0000	0.3840	0.1384	0.1969	0.0842	1.0330
20.7.2012	0.0473	0.0777	-0.0260	-0.0568	4.4552	0.6843	0.1025	-0.0244	0.0603	0.1929	0.2402	0.8031
27.7.2012	0.0756	0.0820	-0.0633	-0.0480	0.6421	0.8744	-0.1437	0.7416	0.1585	0.2016	0.4186	1.2255
3.8.2012	0.0718	0.0772	-0.0445	-0.0578	0.6645	0.6738	-0.1029	-0.1127	0.0931	0.2049	0.4577	0.7759
10.8.2012	0.0495	0.0778	-0.0324	-0.0564	3.0436	0.7022	0.0882	-0.0203	0.0613	0.2306	0.1989	0.8495
17.8.2012	0.0728	0.0808	-0.0533	-0.0507	0.3243	0.8186	0.0783	0.5080	-0.1013	0.2068	0.6126	1.1159
24.8.2012	0.0742	0.0772	-0.0562	-0.0584	0.5909	0.6612	0.2284	-0.1546	-0.2358	0.2175	0.7093	0.7639
MSE	0.0449		0.0482		5.0453		1.1471		0.6138		2.0393	

Source: Author

Vector autoregressive model gives marginally smaller MSE than NN based method for Svensson model (Tables V and VI) for most of the parameters. Both models predict the values of parameters β_0 and β_1 well, ending with small mean square errors. However, due to the fact that estimated parameters β_2 , β_3 , τ_1 and τ_2 are varying extremely through forecasting period, both models' forecasting abilities are weak, ending with much larger mean square errors. Fig. 5 shows the yield curve estimated with Svensson model on 22nd June 2012 and yield curves predicted with neural network and vector autoregressive model. It shows rather poor short term forecast abilities of both models. Fig. 6 shows the yield curve estimated with Svensson model on 24th August 2012 and yield curves predicted with neural network and vector autoregressive model. It shows long term forecast abilities of both models and it can be concluded that both models perform somewhat better than in the short term, but still not good enough.

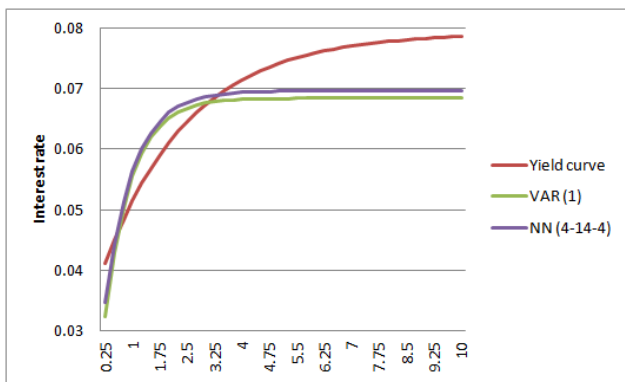


Fig. 3 Nelson-Siegel yield curve and predicted yield curves using VAR and NN models on 22.06.2012

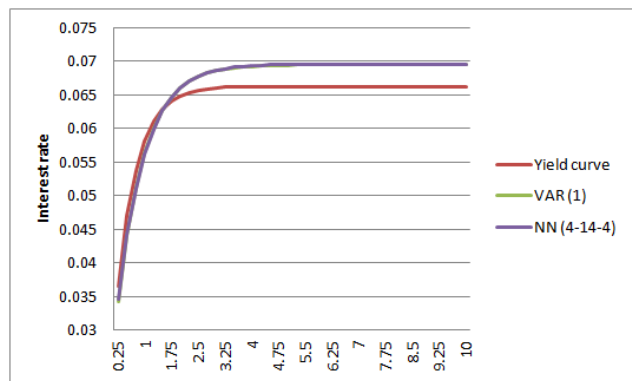


Fig. 4 Nelson-Siegel yield curve and predicted yield curves using VAR and NN models on 24.08.2012

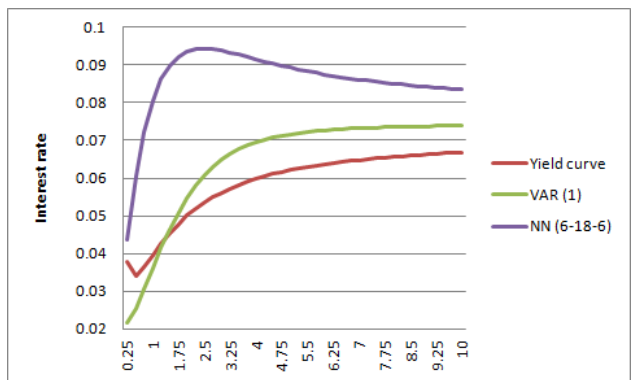


Fig. 5 Svensson yield curve and predicted yield curves using VAR and NN models on 22.06.2012

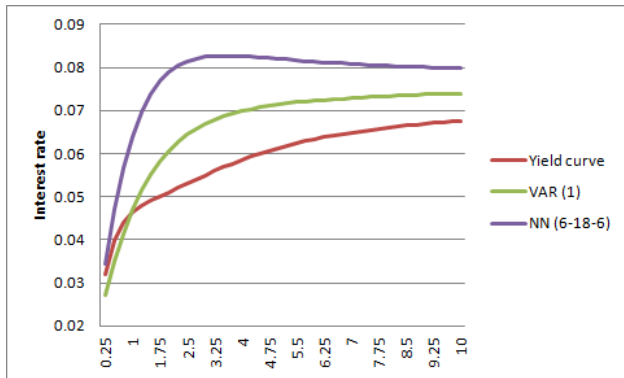


Fig. 6 Svensson yield curve and predicted yield curves using VAR and NN models on 24.08.2012

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