International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:8, No:2, 2014

Supremacy of Differential Evolution Algorithm in Designing Multiplier-Less Low-Pass FIR Filter

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***Abstract*—**In this communication, we have made an attempt to design multiplier-less low-pass finite impulse response (FIR) filter with the aid of various mutation strategies of Differential Evolution (DE) algorithm. Impulse response coefficient of the designed FIR filter has been represented as sums or differences of powers of two. Performance of the proposed filter has been evaluated in terms of its frequency response and associated hardware cost. Supremacy of our approach has been substantiated by comparing our result with many of the existing multiplier-less filter design algorithms of recent interest. It has also been demonstrated that DE-optimized filter outperforms Genetic Algorithm (GA) based design by a large margin. Hardware efficiency of our algorithm has further been validated by implementing those filters on a Field Programmable Gate Array (FPGA) chip.

***Keywords*—**Convergence speed, Differential Evolution (DE), error histogram, finite impulse response (FIR) filter, total power of two (TPT), zero-valued filter coefficient (ZFC).

I. Introduction

PROCESSING of digital data is accomplished by a wide variety of digital filters which can broadly be classified into two main categories, namely finite impulse response (FIR) and infinite impulse response (IIR) filter [1]. FIR filters are normally suitable because of its number of attractive characteristics such as linear-phase response, guaranteed stability etc. [23]. However, for achieving equivalent magnitude response, FIR filters become computationally more expensive than its IIR counterparts [25].

A number of research articles have been published in recent times in order to get rid of the complicated structure of FIR filter. One such efficient method has been proposed [13] in which the coefficients of FIR filter have been represented by means of canonical signed digit (CSD) for reducing the hardware complexity and corresponding delay. Signed powers of two (SPT) representation of FIR coefficients has been demonstrated in [18] where, without increasing the number of adders, the number of SPT terms for each coefficient has been varied to yield a superior filter performance to those designed by means of mixed integer linear programming (MILP) and simulated annealing (SA) [2].

Several methods for designing discrete-coefficient-value linear phase FIR filters have been reported in recent past. Among them, the design with minimum normalized peak ripple magnitude by Lim [21] and one two-stage algorithm consisting of designing a prototype filter using fast time-domain approximation followed by a Trellis search by Chen and Willson [7] have drawn considerable attention.

Design examples have demonstrated the power of the algorithm in producing filters with a better frequency response than methods like [18], [20], [24] while using fewer SPT terms. Minimization of number of adders to meet the given amplitude criterion has been executed by using one systematic algorithm in [15] which has been compared with the works in [7] and [20] to prove its supremacy.

Yao illustrated an explicit representation for the distribution of the SPT terms of CSD numbers in [33]. Design examples have proved the superiority of the proposed method to produce filters possessing a small amount of SPT terms. The design flow for a multiplier-less linear-phase FIR filter has been discussed in [14] where the supremacy of the proposed scheme has been firmly established than techniques as proposed by Lim and Parker [20] and Samueli [24].

Realization of a number of constant multiplications by means of a minimum number of adders and subtractors has grown enough interest amongst the researchers for little more than a decade. An adder graph type algorithm for solving the multiple constant multiplication (MCM) problem with the aid of a novel heuristic inspired by difference method has been proposed in [11]. Authors have proved the supremacy of their approach over previous state-of-the-art approaches like region adder graph (RAG) and cumulative benefit heuristic (HCUB). One truncated MCM using pattern modification technique (PMT) for FIR filter implementation has been introduced in [12]. It has been demonstrated that PMT algorithm reduces the area cost by 35%, compared to non-truncated MCM algorithms, without increasing quantization error.

Efficient amalgamation of evolutionary computation mechanisms in the field of hardware efficient digital filter design has been studied extensively in the literature. Construction of FIR filter coefficients has been carried out by means of Genetic Algorithm (GA) in [3], [10], [19] where the coefficients of the filter have been constrained as sums of power of two. With the aid of the proposed genetic technique, promising results have been achieved which is comparable or better than other state-of-the art techniques. Speed of the optimization process in the same design problem has later been significantly enhanced by virtue of some recent variants of GA like Micro Genetic Algorithm (μGA) [4], modified Micro Genetic Algorithm [5] etc.

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Limitation of GA in locating the optimum solution is currently being overcome by some robust evolutionary optimization techniques among which Differential Evolution (DE) is the latest [6], [16]. A number of works have been reported in the literature which has employed DE for the design of FIR filters of various kinds [17], [22]. Motivated by this urge, an attempt has been made in this paper to design a multiplier-less low-pass FIR filter with the help of DE algorithm. Since DE is very much sensitive to the choice of its mutation strategy, this work focuses proper attention for selecting the most appropriate mutation scheme for finding out the best possible FIR filter coefficients.

In this article, we have judiciously employed DE algorithm for locating the optimum FIR coefficients encoded in SPT form. Five different mutation strategies of DE have been considered in this regard and their performances have been evaluated by means of different metrics such as computational complexity of optimization process, magnitude response behavior and the resulting hardware cost of the designed filter structure. Superiority of the optimum filter response has been substantiated by comparing its performance with many of the existing powers-of-two architectures developed by various conventional and intelligent algorithms. Robustness of the designed filter over other FIR models from literature has further been established by implementing it on a Field Programmable Gate Array (FPGA) chip through XILINX ISE Design Suite 12.3.

II. Modeling of FIR Filter

A. Design Principle of Conventional FIR Filter

Structure of traditional FIR filter simply consists of a number of delay elements, adders and multipliers where the number of such elements is determined by the length of the filter [1], [23], [25], [32]. This conventional structure becomes unsuitable in most of the practical systems because of the presence of power-hungry multipliers and hence the search for multiplier-free structure is on high rise. Impulse response coefficients of any FIR filter of length L may be represented by means of a row-vector as:

$$h(n) = \{h_0\ h_1\ h_2\ h_3\ h_4\ h_5\ \ldots\ldots h_{L-1}\} \tag{1}$$

Resulting output sequence $y\,(n)$ from this FIR filter for an input sequence $x\,(n)$ can have its mathematical illustrations as follows [23], [25]:

$$y(n) = \sum_{k=0}^{L-1} h_k . x(n-k) \tag{2}$$

As seen from (2), computation of the output sequence at i^{th} instant $y\,(i), i \in Z^{+}$ necessitates the use for 'L' multiplications along with 'L-1' additions. The process of multiplication is a precise mathematical task when performed on a digital hardware and thus demands for the replacement of its hardware by some simplified equivalent blocks like delay elements and adders.

B. Design Principle of Multiplier-less FIR Filter

Being motivated from the urge of designing hardware efficient filter, system designers have adopted several approaches to get rid of multipliers in the filter model. One such well recognized approach has been to replace the multipliers i.e. the filter coefficients by means of simple delay elements and adders without affecting the performance of the filter. Resulting impulse response coefficient of the designed filter thus may have the form like [15]:

$$h_i = \sum_{j=0}^{\lambda-1} c_{ij} . 2^{-j},\ i = 0,1,2, \ldots\ldots, L-1 \tag{3}$$

It can be easily observed from (3) that the individual coefficient h_i can be treated as sums or differences of power of two with the parameter 'λ' stands for the resolution of SPT terms. The variable c_{ij} in (3) holds the key in formulating the coefficients in the sense that they either allow or reject any powers-of-two terms 2^{-j} to be included into h_i. As the term c_{ij} takes the binary decision of either inclusion or exclusion, it can assume values only from the binary set $\mathbb{B} = \{0, 1\}\ or\ \{0, -1\}$. As a matter of fact, the proper assignment of these coefficients for different values of the subscript is a problem of optimization and has to be dealt with some powerful evolutionary optimization techniques of current interest.

III. Supremacy of Differential Evolution in the Field of Optimization

The theory of optimization technique has proved its supremacy in a wide variety of science and engineering research problems over a number of years. Inability of derivative based linear optimization techniques in locating the global minima through a non-linear rough surface has been overcome in the later part of the last century through the invention of a number of intelligent optimization methods guided by genetic and social behavior of animals [26], [28]. This has given a new dimension to the field of optimization and subsequently increased the application of such mechanisms in a number of engineering problems of current interest [27], [31]. Among a number of artificially intelligent optimization techniques, DE has opened a new door of evolutionary computation and has been successfully confirming its supremacy till date. Main body of the algorithm takes very few lines to code in any programming language making DE simple and uncomplicated. As pointed out by a number of studies, the performance of DE is significantly superior to its antecedents like G3 with PCX, MA-S2, ALEP, and CPSO-H in problems having unimodal, multimodal search spaces with separable and non-separable variables [9]. The second important observation about DE is the presence of a very few control parameters associated with it, namely weighting factor, recombination or cross-over probability and population size. Finally, the space complexity is another important issue which has helped in extending DE for handling large scale and expensive optimization problems.

DE is a multi-objective, robust parallel direct search mechanism employed for finding out the optimal D-dimensional solution from a search space consisting of NP number of agents or potential solutions. The set of vectors at any iteration 'G' can be represented mathematically as a set $\mathbb{S}_G$, where

$$\mathbb{S}_G = \{x_{i,G}^j\};\ \forall\ i \in \mathbb{P} = \{1,2,3,\ldots\ldots,NP\}\ and\ \ j = \{1,2,3,\ldots\ldots,D\} \quad (4)$$

The size of the population does not change during the optimization mechanism and it is nominal to assume a set size that obeys the following inequality [6]:

$$5D \leq NP \leq 10D \quad (5)$$

In order to generate new parameter vector for the next generation, DE uses a special type of differential operator where the weighted difference between two parameter vector of current generation is added to a third vector, yielding the mutant vector for the next iteration as [8], [9]:

$$v_{i,G+1}^j = x_{r_1,G}^j + F.\left(x_{r_2,G}^j - x_{r_3,G}^j\right)\ \ \forall\ i, r_1, r_2, r_3 \in \mathbb{P}\ and\ i \neq r_1 \neq r_2 \neq r_3\ with\ \ j = \{1,2,3,\ldots\ldots,D\} \quad (6)$$

Since the above process perturbs the current parameter vectors by using three other distinct parameter vectors, it is quite similar to the genetic mutation process where the genomes of chromosomes undergo changes and accordingly the process is called as 'mutation' mechanism. The parameter 'F', which acts as amplification factor in (6), has proved to play a significant role in convergence behavior of DE and rightly termed as 'weighting factor'. It is real and constant that can assume value from the range [0, 2]. However its value is chosen as 0.5 in most of the applications.

Diversity of the perturbed vector is enhanced in the next step of DE where, depending upon some random parameters, one or more genomes of the mutant vector of the present generation enter the target vector $U_{i,G+1} = \{u_{i,G+1}^j\}\ \forall\ j = \{1,2,3,\ldots\ldots,D\}$ for the next generation [28], [30]:

$$u_{i,G+1}^j = \begin{cases} v_{i,G+1}^j & if\ rand_{i,j} \leq CR\ \ or\ \ j = randn_i \\ x_{i,G}^j & if\ rand_{i,j} > CR\ \ and\ \ j \neq randn_i \end{cases}\ \ \forall\ i \in\ \mathbb{P}\ \ and\ j = \{1,2,3,\ldots\ldots,D\} \quad (7)$$

Equation (7) involves a control parameter CR, called 'crossover probability' which actually takes care of the number of genomes from the mutant vector that may be allowed to enter as element in the target vector. Proper care has to be taken during the selection of this parameter which is normally chosen from the set [0, 1].

Process of selection is carried out in the final step of DE where it is decided whether or not the target vector of the current generation may get entered as a parameter vector for the next generation $X_{i,G+1} = \{x_{i,G+1}^j\}\ \forall\ j = \{1,2,3,\ldots\ldots,D\}$. It involves a comparison between the target vector and the present parameter vector using one greedy criterion. The comparison is based on proper formulation of an objective or cost function $\varphi(.)$ as outlined in (8) [29], [30]:

$$X_{i,G+1} = \begin{cases} U_{i,G+1} & iff\ \ \varphi(X_{i,G}) \geq \varphi(U_{i,G+1}) \\ X_{i,G} & iff\ \ \varphi(X_{i,G}) < \varphi(U_{i,G+1}) \end{cases} \quad (8)$$

The process of mutation, cross-over or recombination and selection continues in an iterative way until the maximum number of iteration is exhausted or the optimum solution has been achieved.

IV. Mathematical Formulation of FIR Filter Design

Role of different mutation schemes of DE in designing multiplier-less low-pass FIR filter has been extensively studied in this work. Being an optimization algorithm, the efficiency of DE is always monitored by a mathematical expression called the cost function whose formulation is very much dependent on the specific problem of concern. In many situations, for a fixed structure of error function, the cost functional value and thus the fitness value fluctuates appreciably depending upon the selection of structural strategy of the optimization technique. Thus the fitness value and cost functional value may be regarded as two different functions of identical parameters, as outlined in the following equations:

$$fitness\ value = \psi(\phi, \mathbb{M}, \mathbb{R}, \zeta) \quad (9)$$

$$cost\ functional\ value = \sigma(\phi, \mathbb{M}, \mathbb{R}, \zeta) \quad (10)$$

The parameter $'\phi'$ in (9) and (10) reflects the formulation of the error function and takes the form as shown:

$$\varphi(k, p_i) = \begin{cases} \min_i[\max_k\{|1 - H_{p_i}(k)|\} & for\ \ k \leq \frac{\varpi N}{2\pi} \\ \min_i[\max_k\{|H_{p_i}(k)|\} & for\ k > \frac{\varpi N}{2\pi} \end{cases} \quad (11)$$

where $H(k) = \sum_{n=0}^{L-1} h(n) e^{\frac{-j2\pi kn}{L}}$ and $p_i \in \mathbb{P}$, where implies the set of population.

The other parameters within the parenthesis in (9) symbolize the set of mutation strategy, recombination strategy and control parameter and has been outlined as in (12), (13) and (14) respectively:

$$\mathbb{M} = \left\{\frac{rand}{1}, \frac{rand}{2}, rand\ to\frac{best}{1}, \frac{best}{1}, \frac{best}{2}\right\} \quad (12)$$

$$\mathbb{R} = \{binomial, exponential\} \quad (13)$$

$$\zeta = \{Weighting\ Factors, Recombination\ Probability\} \quad (14)$$

This paper investigates the most appropriate mutation rule for which the fitness value is the highest irrespective of the population size and iteration number of the evolutionary technique. Thus, the optimized mutation scheme results in a fitness value which satisfies the following inequality:

$$fitness\ value\|_{\phi,\mathbb{M}_{opt},\mathbb{R}_{nom},\zeta_n} > fitness\ value\|_{\phi,\mathbb{M}_i,\mathbb{R}_{nom},\zeta_n} \quad (15)$$

International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
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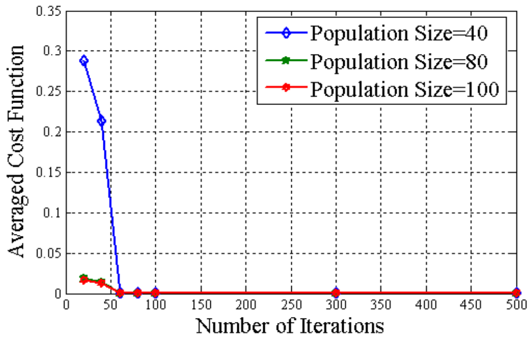
In this work, binomial recombination strategy has been chosen since it has been regarded as the conventional one for the most of the applications and it has been designated as $\mathbb{R}_{nom}$. Our main objective is to identify the suitable mutation rule $\mathbb{M}_{opt}$ for which the value of the fitness yields the highest score i.e. the lowest cost. However the control parameters of DE like weighting factor and recombination probability have been maintained at its nominal values ζ_n. Table I below summarizes the choice of several parameters for our design problem of interest.

TABLE I
CHOICE OF DIFFERENT PARAMETERS FOR DESIGNING MULTIPLIER-LESS LOW-PASS FIR FILTER USING DIFFERENTIAL EVOLUTION (DE) ALGORITHM

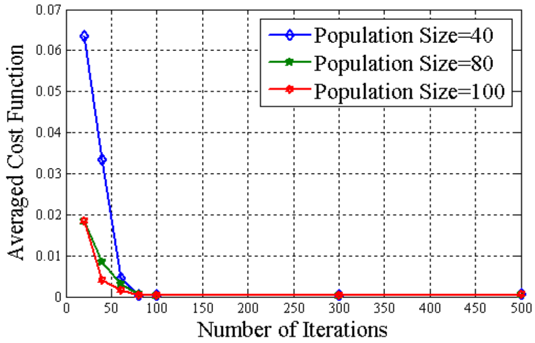
Parameter	Value
Mutation rule	rand/1, rand/2, best/1, best/2, rand to best/1
Weighting factor(s)	0.5
Cross-over or recombination rule	Binomial
Cross-over or recombination probability	0.5
Selection strategy	minimax
Error threshold	10^{-4}
Population size	40, 80, 100
Maximum iteration number	500

V. SIMULATION RESULTS

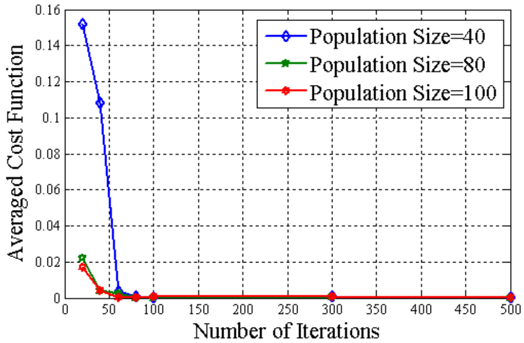
Computational efficiency of an optimization technique can be best evaluated in terms of its convergence curve, which shows the variation of averaged cost function with the number of iterations employed in the algorithm. Higher the computational efficiency, lesser the required number of iteration is. Convergence behavior of different mutation strategies of DE, as mentioned in Table I, has been depicted in Fig. 1.



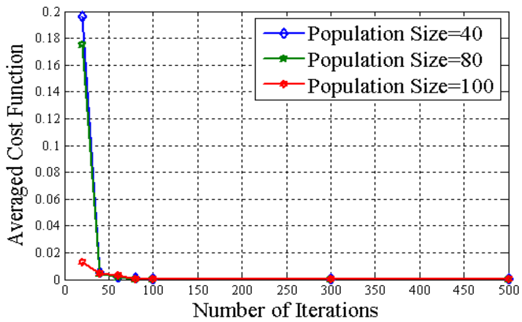
(a) rand/1



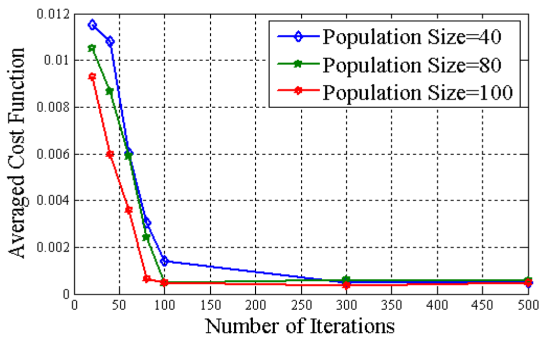
(b) rand/2



(c) best/1



(d) best/2



(e) rand to best/1

Fig. 1 Convergence behavior of DE for various mutation schemes

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ISSN: 2517-9438
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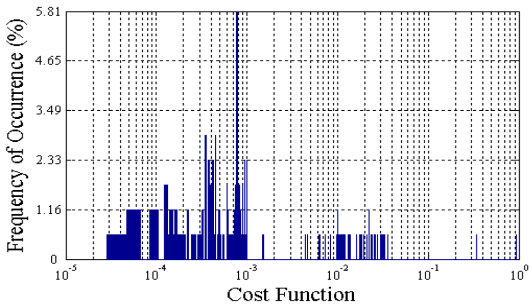
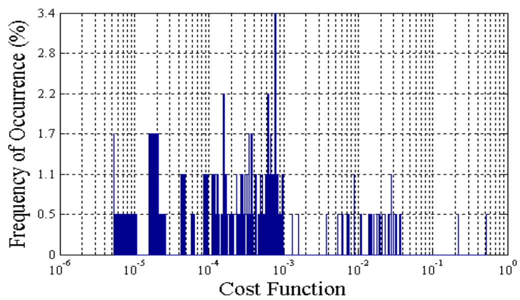
One useful observation can be made from Fig. 1 regarding the impact of mutation strategy on the size of the population of this evolutionary algorithm. As it can be seen from Fig. 1 (d), best/1 approach is the only mutation technique whose performance remains almost independent on the size of the population even at a lower number of iterations. For example, the cost function incurred with best/1 strategy is significantly less (1/5 to 1/10 times) than its counterparts for a population size of 40 with a number of iterations of 20. Moreover, for higher sizes of population, the best/1 technique also maintains its supremacy over its other competitors.

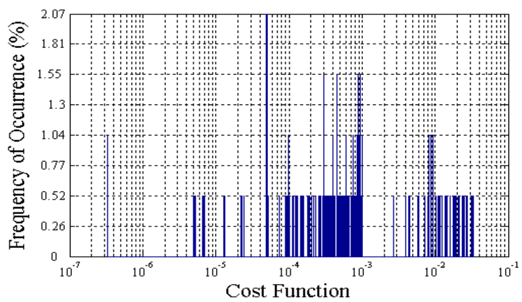
Fig. 1 only reflects the averaged value of the cost function with no information about the presence of individual cost function obtained through a number of trial runs. Occurrence of these individual values can be made presentable with the aid of a diagram called 'error histogram' which describes the variation of frequency of occurrence against each cost term. Such a diagram has been demonstrated in Fig. 2 for different mutation rules.



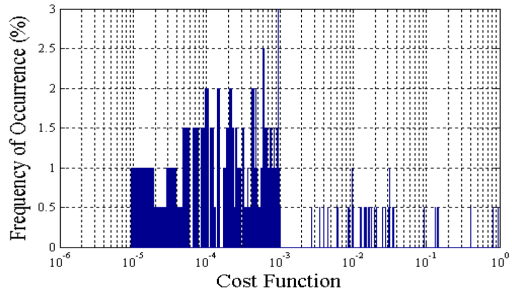
(a) rand/1



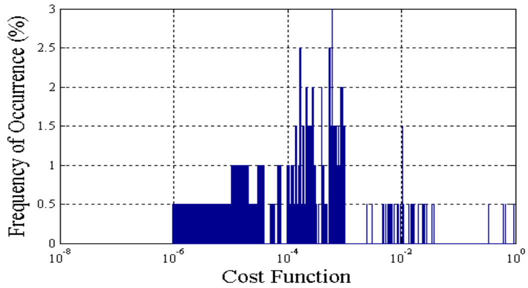
(b) rand/2



(c) best/1



(d) best/2



(e) rand to best/1

Fig. 2 Error histogram of DE for different mutation schemes

The effectiveness of best/1 strategy over the other existing techniques can well be understood from the plots of error histograms as depicted in Fig. 2. As for example, the maximum limit of cost functional value obtained with best/1 technique has been found to be 10^{-1} or 0.1; while the same limit is equal to 10^{0} or 1.0 for other mutation techniques. This fact has been substantiated by the presence of the discrete spikes between the range of cost functional values from 10^{0} (1.0) to 10^{-1} (0.1) for the other mutation strategies; whereas, best/1 strategy does not provide so.

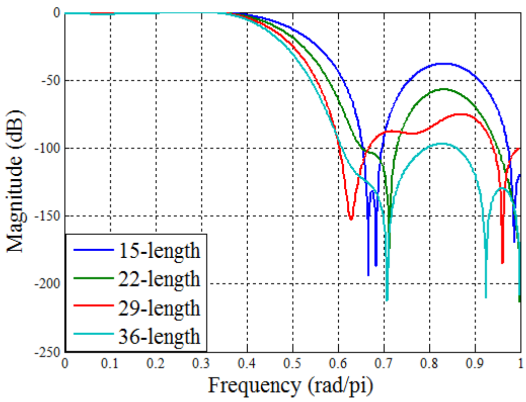
However, majority of the error values are concentrated near the centre of the plot (largely from 10^{-5} to 10^{-3}) irrespective of the mutation technique employed. This dense concentration of error values actually accounts for the saturation region which spans through a wider number of iterations in connection to the design problem of FIR filter. The interval from 10^{-5} to 10^{-3} may be subdivided into two adjacent intervals, namely I_1 (10^{-5} to 10^{-4}) and I_2 (10^{-4} to 10^{-3}) for the purpose of interpreting the experimental outcome from another angle. In I_1, frequency of occurrence of error samples for rand/1, rand/2, best/1, best/2 and rand to best/1 mutation schemes have been observed to be 4.68%, 6.25%, 8.29%, 9.04% and 4.52% respectively. Hence best/1 rule outperforms all except best/2 in I_1 since the efficiency of convergence is directly related to this frequency. As far as the frequencies in I_2 are concerned, best/1 yields a value of 66.32% followed by rand/1 (67.25%), rand/2 (69.89%), best/2 (72.36%) and rand to best/1 (74.37%). Hence best/1 strategy exhibits the best characteristics in I_2 which corresponds to the range of cost function higher than the specified error threshold as supplied in Table I. Numerical characterization of cost function resulting from different mutation techniques and the corresponding simulation time for each of these procedures has been listed in Table II. The entire simulation has been carried out using MATLAB 7.10 software with Intel Pentium 4 CPU with 2 GB of RAM.

TABLE II
NUMERICAL VALUES OF COST FUNCTION (POPULATION SIZE=80)

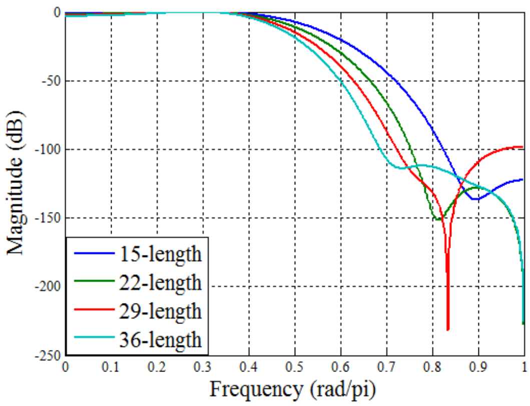
Rule	Cost functional value				Average simulation time
	Maximum	Minimum	Average	Standard deviation	
rand/1	0.0356	0.0000498	0.006115	0.009616	23.814 s
rand/2	0.0358	0.0000019	0.004959	0.009022	24.67 s
best/1	0.0332	0.00000515	0.00399	0.00737	15.503 s
best/2	0.148	0.000009	0.00463	0.0189	19.258 s
rand to best/1	0.6679	0.000021	0.028914	0.120897	20.897 s

From the numerical results presented in Table II, it can be inspected that the best/1 mutation technique requires the least average simulation time amongst all and thus proves to be the most efficient one as far as computational time is concerned. Furthermore, the error performance exhibited by this particular mutation scheme also looks very attractive since the average and standard deviation of cost function attains its minimum value with best/1 scheme only.

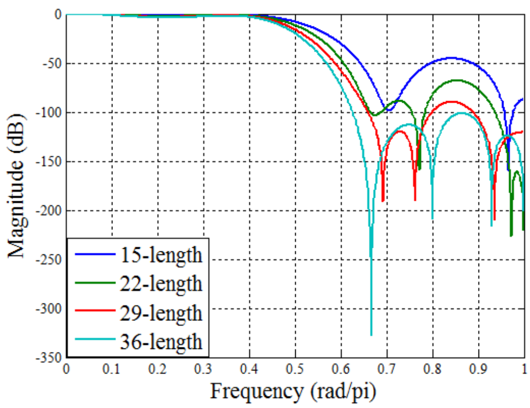
From the computational point of view, the supremacy of best/1 scheme has firmly been established but the effect of this superiority in terms of filter response has yet to be examined. For this purpose, the magnitude responses of the multiplier-free low-pass FIR filter for four distinct lengths, namely 15, 22, 29 and 36 have been plotted in Fig. 3 (a) through Fig. 3 (e) for five different mutation strategies considered in this work. In this context, specifications of the DE-optimized filter have been selected in accordance with those techniques with which its performance has been compared. As for example, filters designed in [7], [14], [15], [21] and [33] have their normalized pass-band and stop-band edge frequencies at 0.3 rad/pi and 0.5 rad/pi respectively. On the other hand, approaches described in [4] and [11] have chosen pass-band frequencies at 0.3 rad/pi & 0.45 rad/pi and stop-band frequencies at 0.54 rad/pi & 0.575 rad/pi respectively. As a matter of fact, this paper employs DE algorithm in designing multiplier-less low-pass FIR filter with pass-band and stop-band edge frequency at 0.3 rad/pi and 0.6 rad/pi respectively. This has made the comparison fair and meaningful from all sense.



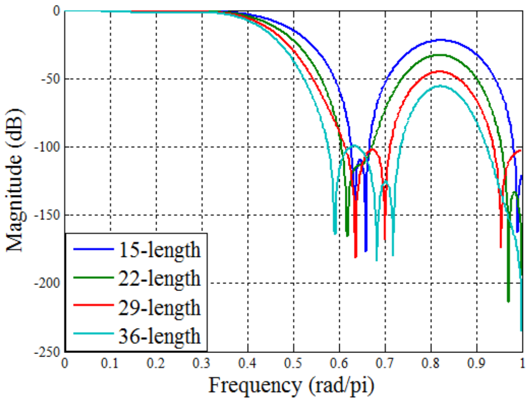
(a) rand/1



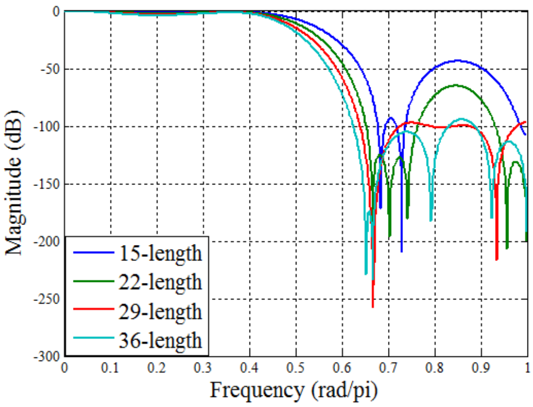
(b) rand/2



(c) best/1



(d) best/2



(e) rand to best/1

Fig. 3 Magnitude response of multiplier-less low-pass FIR filter designed with the aid of DE

Careful observation of the filter's magnitude response resulting from five different mutation strategies of DE has unambiguously suggested that selection of appropriate mutation rule becomes inevitably essential as far as the amount of attenuation in the stop-band region of characteristics is concerned. It can be well inspected that all the five rules exhibit fairly flat pass-band response irrespective of the order of the filter. As a comprehensive approach in comparing the stop-band characteristic, achievable attenuation has been computed at some discrete frequency points and accordingly tabulated in Table III below. Moreover, the maximum and minimum stop-band attenuation values have been listed in Table IV.

TABLE III
MAXIMUM AND MINIMUM STOP-BAND ATTENUATION RESULTING FROM FIVE MUTATION STRATEGIES OF DE

Mutation rule	Length	Frequency points			
		0.65 rad/pi	0.75 rad/pi	0.85 rad/pi	0.95 rad/pi
rand/1	15	88.8	52.76	38.58	71.4
	22	98.92	79.58	57.85	99.67
	29	117	89.24	76.27	112.6
	36	121.5	115	98.35	131.1
rand/2	15	30.23	61.32	117.3	126.6
	22	44.49	96.65	136.6	137.3
	29	59.18	114.2	139.3	100.6
	36	76.26	113.3	119.3	136.7
best/1	15	54.33	68.26	45.69	86.81
	22	86.06	95	68.03	115.7
	29	91.55	130.3	90.1	134.9
	36	141.7	113.1	102.8	127.2
best/2	15	110.4	31.37	23.29	56.63
	22	113	45.09	35.04	92.18
	29	115.3	61.87	47.84	133.5
	36	102.5	82.24	59.01	129.5
rand to best/1	15	56.59	77.92	43.63	69.57
	22	93.66	126.6	65.09	135.3
	29	119.5	97.21	99.96	117.6
	36	166.8	105.8	94.9	115.2

TABLE IV
MAXIMUM AND MINIMUM STOP-BAND ATTENUATION RESULTING FROM FIVE MUTATION STRATEGIES OF DE

Length	Stop-band attenuation (dB)	rand/1	rand/2	best/1	best/2	rand to best/1
15	Max	193.5	136.7	160	176.9	209.3
	Min	43.33	20.27	30.19	57.45	29.19
22	Max	213.6	228.3	227.1	214.2	206.3
	Min	63.6	29.68	46.16	89.41	45.09
29	Max	185	231.6	209.5	180.9	257.8
	Min	94.85	39.53	57.6	88.05	57.66
36	Max	212.8	226.2	327.2	235.7	229.1
	Min	92.58	50.43	73	118.3	71.7

Looking at Table III, it can be well apprehended that the 36-length FIR filter designed with the aid of best/1 strategy, followed by rand/1 and rand to best/1, corresponds to appreciably high attenuation value (comparable to or higher than 100 dB) at all the discrete frequency points considered. On the other hand, in terms of the maximum and minimum stop-band attenuation, no single strategy can be found out which outperforms others in this context.

Supremacy of DE-optimized multiplier-less low-pass FIR filter may further be substantiated by comparing the achievable frequency response with that obtained by other recent algorithms for designing discrete coefficient FIR filter. These include conventional techniques like polynomial time algorithm [21], trellis search [7], linear programming [15], multiple constant multiplication (MCM) [11]-[12] approach as well as intelligent techniques like Conventional Genetic Algorithm (CGA) [10], Micro Genetic Algorithm (μGA) [4] and so on. Robustness of our designed FIR filter has therefore been established by comparing its performance with those filters in terms of maximum and minimum stop-band attenuation. Such a comparative analysis has been illustrated in Table V below.

TABLE V
COMPARISON OF STOP-BAND ATTENUATION AMONG VARIOUS HARDWARE EFFICIENT FILTER STRUCTURES

Algorithm	Length of the filter	Maximum stop-band attenuation (dB)	Minimum stop-band attenuation (dB)
Lim [21]	35	220.3	63.91
Chen [7]	28	218.7	108.2
Kaakinen [15]	29	55.17	30.28
Guo [11]	28	~100	~45
CGA [10]	53	~120	41.46
μGA [4]	21	~120	40.63
Proposed (best/1)	15	160	30.19
	22	227.1	46.16
	29	209.5	57.6
	36	327.2	73

Relevance of the proposed approach in the field of powers-of-two FIR filter design may clearly be anticipated by looking at Table V. It has previously been shown that best/1 technique outperforms other four mutation schemes of DE in terms of convergence, computational efficiency and quality of the solution obtained for designing multiplier-less FIR filter. As a matter of fact, comparative analysis in Table V has been carried out considering best/1 strategy only. It can be unambiguously observed from the value of stop-band attenuation that DE-optimized filter surpasses both the conventional and intelligent search mechanisms by a significant margin. Specifically, compared to 36-length proposed filter, the amount of improvement in terms of maximum stop-band attenuation is around 32.67%, 33.16%, 83.14%, 69.43% and 63.33% than the works described in [21], [7], [15], [11] and [10] respectively. As far as the minimum attenuation in the stop-band region is concerned, the corresponding enhancement is approximately 12.45%, 58.52%, 38.36%, 43.21% and 44.34% than [21], [15], [11], [10] and [4] respectively. However, only the filter designed by trellis search algorithm [7] exhibits 38.09% more minimum stop-band attenuation than our design.

As the design of hardware efficient FIR filter model is considered in this work, a number of performance parameter is therefore selected in order to compare the hardware cost of different architecture. These include total powers of two (TPT) terms, total adders (TA), total delay flip-flop (TDF), and zero-valued filter coefficient (ZFC) in the impulse response. Such a relative analysis has been carried in our work and the outcome has been elaborately illustrated in Table VI.

TABLE VI
FILTER PERFORMANCE IN TERMS OF REQUIREMENT OF HARDWARE BLOCKS

Mutation strategy	Length of the filter	Total powers of two (TPT)	Total adders (TA)	Total delay flip-flop (TDF)	Zero-valued filter coefficient (ZFC)
rand/1	15	30	15	131	0
	22	40	22	194	4
	29	39	20	190	10
	36	52	32	250	16
rand/2	15	35	22	164	2
	22	34	20	154	8
	29	32	17	147	14
	36	42	28	198	22
best/1	15	28	13	135	0
	22	42	22	218	2
	29	51	30	253	8
	36	48	28	222	14
best/2	15	49	34	228	0
	22	48	28	240	2
	29	40	21	191	12
	36	52	30	230	14
rand to best/1	15	36	21	174	0
	22	46	26	226	2
	29	42	25	206	12
	36	48	26	230	14

Careful observation of the numerical entries in Table VI adds some essential flavors to our analysis. It can be verified that the 29-length FIR filter designed with the aid of rand/2 mutation strategy emerges as a hardware efficient architecture compared to others. More specifically, it includes significantly fewer TPT, TA and TDF while incorporating more ZFC. In order to examine its proficiency to be employed as a low-pass filter, we need to refer to the entries as summarized in Tables III and IV. It follows from those entries that the 29-length FIR filter, as optimized by rand/2 technique, exhibits satisfactory performance since it provides a maximum and minimum stop-band attenuation of around 230 dB and 40 dB respectively.

In order to prove the robustness of DE in designing multiplier-less low-pass FIR filter, a number of such multiplier-free FIR models has been selected from literature and consequently their performances have been compared in terms of these parameters in Table VII. FIR filter of length 29, designed with the help of rand/2 mutation rule of DE, is used as an optimum filter in this respect whose performance is compared with other state-of-the-art powers of two FIR structures.

TABLE VII
COMPARATIVE ANALYSIS AMONGST VARIOUS STATE-OF-THE-ART MULTIPLIER-LESS FIR FILTERS IN TERMS OF HARDWARE BLOCKS

Algorithm	Length	Word length (WL)	Total powers of two (TPT)	Total adders (TA)	Total delay flip-flop (TDF)	Zero-valued filter coefficient (ZFC)
Lim [21]	35	10	58	23	378	0
Chen [7]	28	12	60	61	445	2
Kaakinen [15]	29	11	49	26	326	6
Yao [33]	28	13	54	30	386	4
Jheng [14]	29	12	51	29	386	4
Proposed (rand/2)	29	8	32	17	147	14

Table VII unambiguously suggests the proposed approach as a useful means to construct hardware friendly FIR model because of a number of reasons. As can be seen from the last row of the above table, proposed filter model is always in need of fewer TPT, TA and TDF than the design in [21], [7], [15], [33] and [14]. Amongst the existing structures, filter designed in [15] necessitates the use of minimum TPT and TDF. In this context, the percentage improvement resulting from our approach is 34.69% and 54.91% respectively as compared to [15]. The impulse response coefficient of proposed FIR filter involves the use of 26.09% less TA than Lim's algorithm [21], which requires the least TA amongst other models considered in our work for the purpose of comparison. Moreover, using rand/2 mutation scheme of DE, we became successful in obtaining more than 45% ZFC which will speed up the system response significantly.

As a second method of comparing the hardware costs between other state-of-the-art multiplier-less FIR filters and the proposed design, all the filters have been realized on an FPGA kit using XILINX ISE Design Suite 12.3. Our achievement in this regard has been serially tabulated in Table VIII below.

TABLE VIII
COMPARATIVE ANALYSIS AMONGST VARIOUS STATE-OF-THE-ART MULTIPLIER-LESS FIR FILTERS AFTER REALIZING ON FPGA KIT

Algorithm	Length	I/O Buffer	Full Adder	Subtractor	Delay
Lim [21]	35	3	18	1	43
Chen [7]	28	3	17	1	35
Kaakinen [15]	29	3	14	1	39
Yao [33]	28	3	15	1	38
Jheng [14]	29	3	17	1	39
Proposed (rand/2)	29	3	8	1	28

Results outlined in Table VIII reveal the fact that all of these architectures are in need of identical number of I/O buffers and subtractors. However, in terms of the number of full-adder and delay blocks, different algorithms behave quite differently which may help in selecting the hardware efficient architecture. It can be unambiguously observed that the FIR filter designed with the aid of rand/2 mutation scheme is in need of minimum number of full adders and delay elements amongst all the architectures considered for the purpose of comparison.

Accumulation of all the results, as described in this work, has provided us with a number of interesting dimensions in the field of multiplier-less FIR filter design. It has already been established that best/1 mutation rule is fastest among all and thus requires less number of iterations to converge. Additionally, the filter performance resulting from best/1 rule is very much promising as pointed out in Tables III, IV and V. On the other hand, rand/2 mutation scheme outperforms all in reducing the overall hardware cost of the DE-optimized multiplier-less FIR filter. Moreover, it also shows significant improvement over other state-of-the-art powers-of-two filter in the same respect.

VI. CONCLUSION

Design of powers-of-two FIR filter has drawn considerable attention among the researchers since long. This paper focuses on this design problem with the aid of a robust evolutionary optimization technique, called Differential Evolution algorithm. In order to select the most favorable mutation strategy of DE, performances of five such different strategies have been examined. In connection to this, best/1 and rand/2 scheme have respectively been chosen as optimum choice as far as the low-pass characteristics and the resultant hardware cost of the FIR filter are concerned. Comparative analysis with other state-of-the-art multiplier-less architectures has demonstrated the superiority of our approach by a large margin. It has also been established that DE-based design fairly outperforms other design processes influenced by different variants of genetic algorithms. Future research may be directed by including the information of resulting hardware cost of FIR filter in formulating the cost function of the optimization process. Application of the designed filter in present and future generation wireless systems may also be studied.

REFERENCES

[1] A. Antoniou, *Digital Filters: Analysis, Design and Applications*, McGraw Hill, 2001.
[2] N. Benvenuto, M. Marchesi and A. Uncini, "Applications of Simulated Annealing for the Design of Special Digital Filters," *IEEE Trans. Signal Processing*, vol. 40, pp. 323-332, 1992.
[3] R. Cemes and D. Ait Boudaoud, "Genetic Approach to Design of Multiplier-less FIR Filters," *Electronics Letters*, vol. 29, no. 24, pp. 2090-2091, 1993.
[4] L. Cen and Y. Lian, "Complexity Reduction of High-speed FIR Filters using Micro-Genetic Algorithm," *Proceedings of the First International Symp. on Control, Communications and Signal Processing*, pp. 419-422, 2004.
[5] L. Cen, Y. Lian, "A Modified Micro-Genetic Algorithm for the Design of Multiplierless Digital FIR Filters,"*Proceedings of IEEE International Conference on Digital Techniques in Electrical Engineering*, pp.52-55, 2004.
[6] S. Chattopadhyay, S. K. Sanyal and A. Chandra, "Design of FIR Pulse-Shaping Filter: Superiority of Differential Evolution Optimization over Convex Optimization," *Proceedings of the 5th European Conference on Circuits and Systems for Communications (ECCSC'10)*, pp.189-192, 2010.
[7] C. Chen and A. N. Willson, "A Trellis Search Algorithm for the Design of FIR Filters with Signed-Powers-of-Two Coefficients," *IEEE Trans.*

on Circuits and Systems-II: Analog and Digital Signal Processing, vol. 46, no.1, pp.29-39, 1999.
[8] S. Das, A. Abraham and A. Konar, "Particle Swarm Optimization and Differential Evolution Algorithms: Technical Analysis," *Applications and Hybridization Perspectives. Studies in Computational Intelligence (SCI)*, vol. 116, pp. 1-38, 2008.
[9] S. Das and P. N. Suganthan, "Differential Evolution: A Survey of the State-of-the-art," *IEEE Trans. on Evolutionary Computation*, vol. 15, no. 1, pp. 4-31, 2011.
[10] P. Genetili, F. Piazza and A. Uncini, "Efficient Genetic Algorithm Design for Power-of-two FIR Filters," *Proceedings of the International Conference on Acoustics, Speech and Signal Processing*, vol. 2, pp. 1268-1271, 1995.
[11] R. Guo, L. S. DeBrunner and K. Johansson, "Truncated MCM using Pattern Modification for FIR Filter Implementation," *Proceedings of the 2010 IEEE International Symp. on Circuits and Systems (ISCAS)*, pp. 3881-3884, 2010.
[12] O. Gustafsson, "A Difference Based Adder Graph Heuristic for Multiple Constant Multiplication Problems," *Proceedings of the 2007 IEEE International Symp. on Circuits and Systems (ISCAS),* pp. 1097-1100, 2007.
[13] R. M. Hewlitt and E. S. Swartzlander, "Canonical Signed Digit Representation for FIR Digital Filters," *Proceedings of the 2000 IEEE Workshop on Signal Processing Systems (Sips 2000),* pp. 416-426, 2000.
[14] K. Jheng, S. Jou and A. Wu, "A Design Flow for Multiplierless Linear-Phase FIR Filters: From System Specification to Verilog Code," *Proceedings of the. 2004 IEEE International Symp. on Circuits and Systems*, vol. 5, pp. 293-296, 2004.
[15] J. Y. Kaakinen and T. Saramaki, "A Systematic Algorithm for the Design of Multiplierless FIR Filters," *Proceedings of the 2001 IEEE International Symp. on Circuits and Systems*, vol. 2, pp. 185-188, 2001.
[16] N. Karaboga and B. Cetinkaya, "Design of Digital FIR Filters using Differential Evolution Algorithm," *Circuits Systems and Signal Processing*, vol. 25, no. 5, pp. 649-660, 2006.
[17] N. Karaboga, "Digital Filter Design using Differential Evolution Algorithm," *EURASIP Journal of Applied Signal Processing*, vol. 8, pp. 1269-1276, 2005.
[18] D. Li, J. Song and Y. C. Lim, "A Polynomial-Time Algorithm for Designing Digital Filters with Power-of-Two Coefficients," *Proceedings of the IEEE International Symp. on Circuits and Systems*, vol. 1, pp. 84-87, 1993.
[19] Y. Lian and L. Cen, "A Genetic Algorithm for the Design of Low Power High-speed FIR Filters," *Proceedings of the Seventh International Symp. on Signal Processing and Its Applications*, vol. 1, pp. 181-184, 2003.
[20] Y. C. Lim and S. R. Parker, "FIR Filter Design over a Discrete Powers-of-Two Coefficient Space," *IEEE Trans. on Acoustic, Speech, Signal Processing (ASSP)*, vol. 31, pp. 583-591, 1983.
[21] Y. C. Lim, "Design of Discrete-Coefficient-Value Linear Phase FIR Filters with Optimum Normalized Peak Ripple Magnitude," *IEEE Trans. on Circuits and Systems*, vol. 37, pp. 1480-1486, 1990.
[22] B. Luitel and A. P. Engelbrecht, "Differential Evolution Particle Swarm Optimization for Digital Filter Design," *Proceedings of the IEEE World Congress on Computational Intelligence,* pp. 3954-3961, 2008.
[23] S. K. Mitra, *Digital Signal Processing: A Computer-based Approach*, 2nd Edition, McGraw Hill, 2001.
[24] H. Samueli, "An Improved Search Algorithm for the Design of Multiplier-less FIR Filters with Power-of-Two Coefficients," *IEEE Trans. on Circuits and Systems*, vol. 36, pp. 1044-1047, 1989.
[25] B. Somanathan Nair, *Digital Signal Processing: Theory, Analysis and Digital Filter Design*, Prentice-Hall, 2004.
[26] R. Storn and K. Price: Differential Evolution—A Simple and Efficient Adaptive Scheme for Global Optimization over Continuous Spaces. International Computer Science Institute, Berkeley. TR-95-012 (1995).
[27] R. Storn, "Differential Evolution Design of an IIR Filter," *Proceedings of the IEEE International Conference on Evolutionary Computation*, pp. 268-273, 1996.
[28] R. Storn and K. Price, "Differential Evolution—A Simple and Efficient Heuristic for Global Optimization Over Continuous Spaces," J*ournal of Global Optimization* vol. 11, no. 4, pp. 341-359, 1997.
[29] R. Storn and K. Price, "Differential Evolution: A Simple Evolution Strategy for Fast Optimization," *Dr. Dobb's J. Software Tools*, vol. 22, no. 4, pp.18–24, 1997.
[30] R. Storn, K. Price and J. Lampinen, "Differential Evolution- A Practical Approach to Global Optimization," *Springer*, 2005.
[31] R. Storn, "Designing Nonstandard Filters with Differential Evolution," *IEEE Signal Processing Magazine*, vol. 22, no. 1, pp. 103-106, 2005.
[32] L. Tan, *Digital Signal Processing: Fundamentals and Applications*, 1st Edition, Academic Press, 2008.
[33] C. Yao, "A Study of SPT Term Distribution of CSD Numbers and Its Application for Designing Fixed-point Linear Phase FIR Filters," *Proceedings of the 2001 IEEE International Symp. on Circuits and Systems*, vol. 2, pp. 301-304, 2001.