Multi-Linear Regression Based Prediction of Mass Transfer by Multiple Plunging Jets

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Abstract—The paper aims to compare the performance of vertical and inclined multiple plunging jets and to model and predict their mass transfer capacity by multi-linear regression based approach. The multiple vertical plunging jets have jet impact angle of $\theta = 90^O$; whereas, multiple inclined plunging jets have jet impact angle of $\theta = 60^O$. The results of the study suggests that mass transfer is higher for multiple jets, and inclined multiple plunging jets have up to 1.6 times higher mass transfer than vertical multiple plunging jets under similar conditions. The derived relationship, based on multi-linear regression approach, has successfully predicted the volumetric mass transfer coefficient (K_La) from operational parameters of multiple plunging jets with a correlation coefficient of 0.973, root mean square error of 0.002 and coefficient of determination of 0.946. The results suggests that predicted overall mass transfer coefficient is in good agreement with actual experimental values; thereby, suggesting the utility of derived relationship based on multi-linear regression based approach and can be successfully employed in modeling mass transfer by multiple plunging jets.

Keywords—Mass transfer, multiple plunging jets, multi-linear regression.

I. INTRODUCTION

APPLICATIONS of plunging jets in environmental and chemical engineering are wide and numerous, including aeration and floatation in water and wastewater treatment, bubble floatation of minerals, oxygenation of mammalian-cell bio-reactors, biological aerated filter, fermentation, stirring of chemicals as well as increasing gas-liquid transfer, cooling system in power plants, plunging columns, breakers and waterfalls [1]-[7]. Plunging jet systems provide a simple and inexpensive mass transfer method and is an attractive way to effect mass/oxygen transfer than conventional mass/oxygenation systems for various reasons [1], [7]-[9]: it does not require compressor blower; it facilitates make-up of the "closed" system, which enhance complete utilization of oxygen and volatile reactants; it is simple in design, construction and operation; it does not require separate stirring devices because the water jet itself achieves aeration and mixing; it is energetically attractive as a means of straightforward contacting mechanism in fouling or hazardous environments; and it is free from operational difficulties such as clogging in air diffusers, limitations on the installation of mechanical aerators by the tank width, etc. Supported by these potential advantages, there has been a growing interest in the oxygenation by plunging water jets in the last few years.

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Numerous studies have been reported on air-water oxygen transfer by single plunging jets [10]-[18]. Some of these studies have also suggested empirical relationships between various jet parameters for estimating mass/oxygen transfer capacity. The simplest relationships for single water jets plunging vertically ($\theta = 90^O$) as proposed by [19], [15] and [14] respectively are:

$$K_L A_{(20)} = 3.1 \times 10^{-4} + 4.85 \times 10^{-2} v_j^3 d_j^2 \tag{1}$$

$$K_L A_{(20)} = 9 \times 10^{-5} P \tag{2}$$

$$K_L A_{(20)} = 0.029 \left(P/V \right)^{0.65} \tag{3}$$

where $K_L A_{(20)}$ is volumetric oxygen transfer factor at standard conditions (m^3/h); v_j is jet velocity at exit (m/s); d_j is jet diameter (m); P is jet power (W); $K_L A_{(20)}$ is volumetric oxygen transfer coefficient at standard conditions (1/s); and P/V is jet power per unit volume (kW/m^3).

Few studies have also been reported on air-water oxygen transfer by multiple plunging jets [20]-[22], and have concluded that multiple plunging jets system has higher mass/oxygen transfer than a single jet system at a given jet power and thus useful in practical situations where large volumes of wastewater at higher discharges are to be oxygenated. A couple of these studies have also suggested relationships between various jet parameters for estimating and predicting oxygen transfer capacity by multiple plunging jets. The relationships for vertical multiple plunging jets ($\theta = 90^O$) as proposed by [20] and for inclined multiple plunging jets ($\theta = 60^O$) as proposed by [21] respectively are:

$$K_L a_{(20)} = 0.113 \, n^{0.84} \, v_j^{2.14} \, d_j^{1.53} \tag{4}$$

$$K_L a_{(20)} = 0.103 \, n^{0.81} \, v_j^{2.11} \, d_j^{1.43} \tag{5}$$

where, $K_L a_{(20)}$ is the volumetric mass/oxygen transfer coefficient at standard conditions (per sec) and n is the number of jets in multiple plunging jets oxygenation system.

Within last few years, various predictive techniques have been used in civil and environmental engineering applications [23]-[30] and found to be working well. Taking a feather out of it, multi-linear regression has been used to model and predict mass transfer by multiple plunging jets. An attempt has

been made to suggest a single relationship for predicting/estimating mass transfer by vertical and inclined multiple plunging jets.

II. DATA SET AND METHODOLOGY

Data used in the present study are taken from two earlier studies carried out by Deswal and Verma [20] and Deswal [21] on mass transfer by multiple vertical (jet impact angle, θ = 90°) and multiple inclined (jet impact angle, θ = 60°) plunging jets respectively. The studies, as stated earlier, have also proposed empirical relationships to predict the oxygen transfer coefficient ($K_L a_{(20)}$) by multiple vertical plunging jets (4) and multiple inclined plunging jets (5) for a given configuration under required flow conditions. The dataset consists of eighty eight experimental observations, forty four each for vertical and inclined multiple plunging jets, on different configurations in terms of jet diameter and number of jets.

The parameters considered for multi-linear regression analysis include volumetric mass/oxygen transfer coefficient at standard conditions ($K_L a_{(20)}$, in per sec), jet impact angle (θ, in degree), jet velocity at exit (v_j, in m/s), jet diameter (d_j, in m) and number of jets (n). As is evident from (1) to (5) proposed by the researchers, the relationship between the jet parameters is non-linear; therefore, a following functional relationship is initially assumed:

$$K_L a_{(20)} = p_1\ \theta^a n^b\ v_j^c\ d_j^d \qquad (6)$$

where, p_1 is proportionality constant.

Taking log of both sides of (6) so as to reduce the equation to a linear form,

$$\log K_L a_{(20)} = \log p_1 + a \log \theta + b \log n + c \log v_j + d \log d_j \quad (7)$$

which is a multi-linear equation with four explanatory variables. Now to develop a multi-linear model, $\log K_L a_{(20)}$ is taken as the output parameter and the four explanatory variables namely $\log \theta$, $\log n$, $\log v_j$ and $\log d_j$ are taken as input parameters. A ten-fold cross-validation was used with data points. Cross-validation is a method of estimating the accuracy of a regression model in which the input data set is divided into several parts (number defined by the user), with each part in turn used to test a model fitted to the remaining parts. The output of the multi-linear regression provided the values of p_1, a, b, c & d and, in turn, the developed equation of the form (6) in which θ^a is replaced by 'Inclination Factor' (I_f) specific for a particular jet impact angle. The developed multi-linear regression equation is as under:

$$K_L a_{(20)} = 0.095\ I_f\ n^{0.82}\ v_j^{2.13}\ d_j^{1.48} \qquad (8)$$

For vertical jets (θ=90°), I_f = 1; and for inclined jets (θ=60°), I_f = 1.29.

It is important to note here that (8) could have been developed in the form of (3) and (4) i.e. in terms of kinetic jet power per unit volume (P/V); but in that case it would not have been possible to show the effect of configuration and/or geometry of multiple jets i.e. the number of jets and jet impact angle.

III. RESULTS

To assess the variation and effect of multiple jets under similar flow conditions, the ratio of mass transfer by multiple jets to a single jet is plotted against the kinetic jet power per unit volume (P/V) for vertical (MVJ) and inclined (MIJ) multiple jets in Figs. 1 and 2 respectively.

Both of the figures reveal that mass transfer increases with increase in number of multiple jets (n) as the kinetic jet power per unit volume increases. It suggests the utility of multiple plunging jets in practical situations, where large volumes of wastewaters are to be aerated at higher kinetic jet power per unit volume.
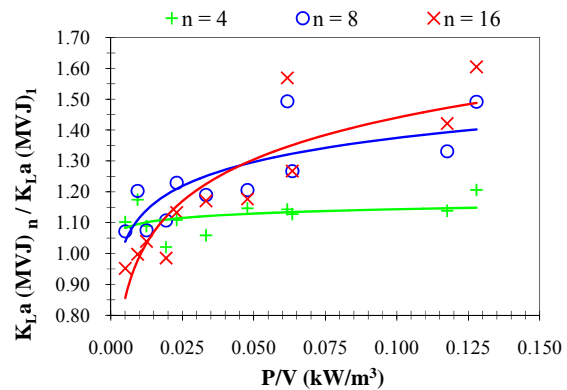

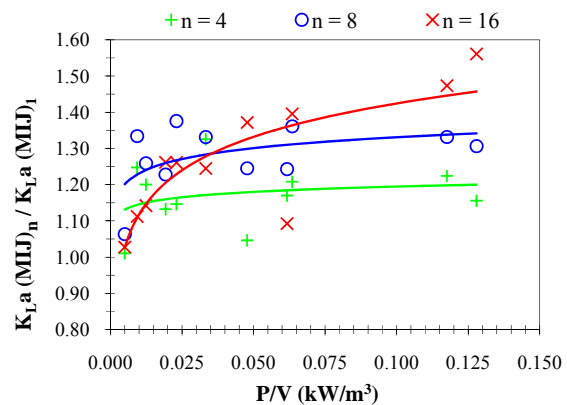Fig. 1 Variation of mass transfer with vertical multiple plunging jets (MVJ)

Fig. 2 Variation of mass transfer with inclined multiple plunging jets (MIJ)

The comparative performance of vertical (MVJ) and inclined (MIJ) multiple plunging jets under similar flow conditions is analyzed by plotting the ratio of mass transfer by MIJ to MVJ at given kinetic jet power per unit volume. It is observed that inclined multiple jets (MIJ) outperforms vertical multiple jets (MVJ). The inclined multiple jets have up to 1.6 times higher mass transfer than vertical multiple jets under similar conditions.
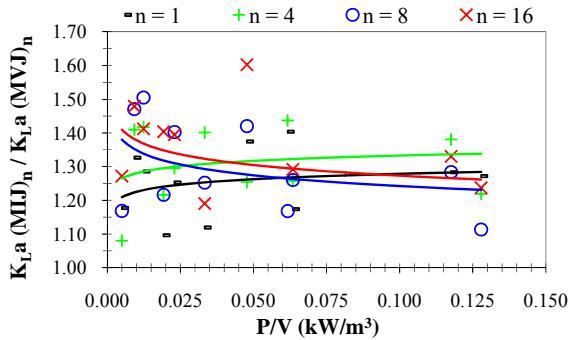

Fig. 3 Comparison of inclined and vertical multiple plunging jets
(MIJ v/s MVJ).

The usefulness of the developed multi-linear regression equation (8) in predicting overall mass/oxygen transfer coefficient by multiple plunging jets (both vertical and inclined, i.e. both MVJ and MIJ) has been assessed by comparing its statistical results with (4) for vertical multiple plunging jets (MVJ) only and (5) for inclined multiple plunging jets (MIJ) only. Table I provides values of correlation coefficient and RMSE obtained using (4), (5) and (8). The statistical results indicate that the developed multi-linear regression equation (8) has fairly comparable results.

TABLE I
CORRELATION COEFFICIENT AND RMSE VALUES

Approach	Correlation coefficient	Root mean square error
Equation (4), for vertical multiple jets [20].	0.961	0.0026
Equation (5), for inclined multiple jets [21].	0.981	0.0021
Multi-linear regression (8), for vertical and inclined multiple plunging jets.	0.973	0.0024

Fig. 4 provides the graph plotted between actual and predicted values of overall oxygen transfer coefficient by multiple plunging jets oxygenation system using (4) and (5) for vertical and inclined multiple plunging jets respectively and the developed multi-linear regression equation (8). The results of (8) shows a scattering within ±15%. Further, a higher value of R^2 (= 0.946) confirms that the developed (8) works well in predicting the overall oxygen transfer coefficient by both vertical and inclined multiple plunging jets.
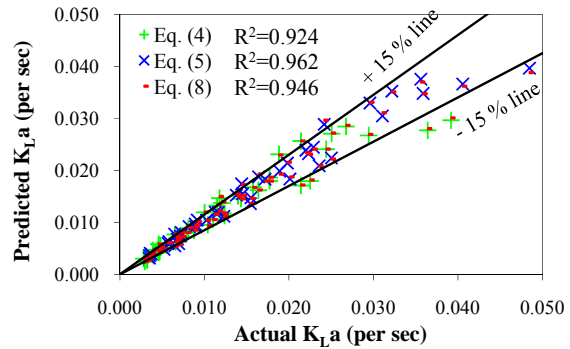

Fig. 4 Actual v/s predicted volumetric mass transfer coefficient

Fig. 5 represents the variation of actual and predicted overall oxygen transfer coefficient by multiple plunging jets with the number of test data. The first 44 test data numbers are for inclined multiple plunging jets (MIJ) and the rest 45 to 88 test data numbers are for vertical multiple plunging jets. It is evident from this plot that overall oxygen transfer coefficient predicted by developed multi-linear regression equation (8) is in good agreement with actual experimental values of both vertical and inclined multiple plunging jets. Thus, suggesting a better utility and performance of (8) in comparison to (4) and (5).
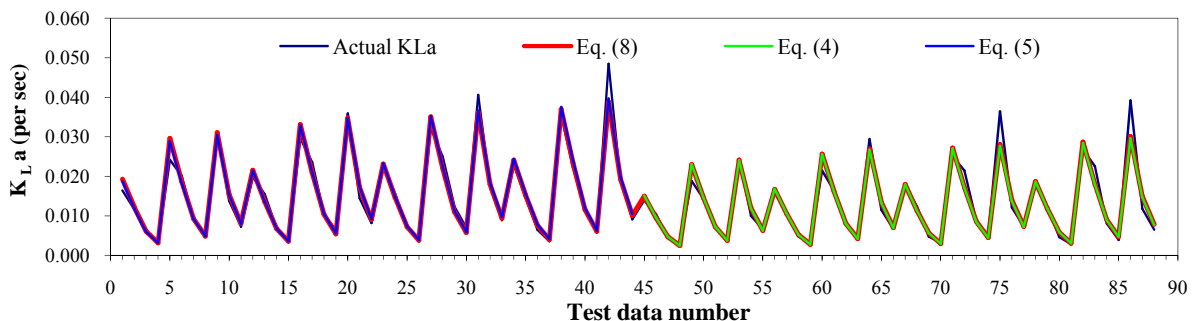

Fig. 5 Variation in predicted values by (8) in comparison to (4), (5) and the actual values of overall oxygen transfer coefficient at standard condition (K_La) by vertical and inclined multiple plunging jets

IV. CONCLUSION

The study has compared the performances of vertical and inclined multiple plunging jets and found that mass transfer is higher for multiple jets than a single jet, and inclined multiple plunging jets have up to 1.6 times higher mass transfer than vertical multiple plunging jets of same configuration under similar conditions. The study has also proposed a single multi-linear regression equation for predicting overall mass/oxygen transfer coefficient by vertical and inclined multiple plunging jets. The results presented are quite encouraging and the developed (8) can be utilized in comparing the performance of single and multiple plunging jets of different configurations/geometry and also in deciding the optimum configuration of multiple plunging jets for given flow conditions.

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