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Monotone Rational Trigonometric Interpolation

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Abstract—This study is concerned with the visualization of monotone data using a piecewise C^1 rational trigonometric interpolating scheme. Four positive shape parameters are incorporated in the structure of rational trigonometric spline. Conditions on two of these parameters are derived to attain the monotonicity of monotone data and other two are left free. Figures are used widely to exhibit that the proposed scheme produces graphically smooth monotone curves.

Keywords—Trigonometric splines, Monotone data, Shape preserving, C^1 monotone interpolant.

I. Introduction

SHAPE preserving is of great practical importance in modeling free form curves. The need for shape preserving interpolating scheme came into existence mainly due to visualization of scientific data. The aim is to present huge set of data in the form of a single image that is easy to apprehend. Scientific data visualization, in the form of smooth curves is a substantial method to grasp numerous physical phenomena. It has been applied in various fields like, Engineering, Industrial design, Computer aided geometric design and computer graphics, Computational Geometry, Biology, Chemistry, Medical and social sciences [1]. Data arising from some physical phenomenon carries certain shape features, one fundamental of which is monotonicity. A monotone data can be observed in E.S.R level in cancer patients and the level of blood urea in renal diseases just to exemplify. The primitive requirement while fitting a curve through a given monotone data set is that the curve should not only be smooth but also monotone. It is generally accepted that polynomial splines do not retain monotonicity of the given monotone data. Thus to get necessary condition, rational splines with some shape parameters in their structure were introduced [2], [3]. Lately, new spline methods in CAGD are introduced to have a grip on model complexity and to meet the demands for more effective tools in designing. One such attempt is introduction of trigonometric splines with shape parameters. The great advantage of trigonometric functions is that they can represent straight line segment, circular arc, ellipse and some transcendental curves like circular helix that are the crucial geometrical entity in almost every modelling system. Thus they proved to be a best substitute of rational Bézier curves and NURBS. In addition, these splines are more flexible than the traditional Bézier and B-spline techniques due mainly to the presence of shape parameters [4]–[7]. The study in this paper includes the use of trigonometric functions in shape preserving data visualization environment. A local shape preserving interpolation scheme, employing a piecewise C^1 rational trigonometric interpolant with four free shape parameters is presented to preserve the monotonicity of monotone data. Since trigonometric functions are continuous, they help in dealing with singularities. The addition of shape parameters in their structure controls their oscillatory behaviour thus insuring a smooth graphical representation of data. The worth of the developed scheme and the role of shape parameters is manifested in numerical examples.

The outline of the paper is: Section II includes a C^1 rational trigonometric interpolant with four shape parameters developed to preserve the monotonicity through monotone data. A graphical demonstration of the scheme is presented in Section III depicting that the scheme is worthful for both equally and unequally spaced data. Finally, the work is concluded in Section IV.

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II. C^1 Rational Trigonometric Interpolant

Consider a data set $\{(x_i, y_i, d_i) : i = 0\,(1)\,n\}$, where $x_0 < x_1 <, ..., < x_n$ is a partition of an arbitrary interval $I = [a, b]$, $y_0, y_1, ..., y_n$ are the monotone function values at these data points; that is, either $y_i \leq y_{i+1}$ or $y_i \geq y_{i+1}$ for $i = 0\,(1)\,n-1$ and d_i are derivatives at partition points which are either given or estimated by some numerical method [8] . We aim to construct a C^1 rational trigonometric interpolant $P(x)$ such that

$$P(x_i) = y_i \tag{1}$$

and $P(x)$ is monotone as well. On each subinterval $[x_i, x_{i+1}]$, we define a piecewise C^1 rational trigonometric interpolant with four positive shape parameters λ_i, μ_i, ν_i and δ_i as follows:

$$P(x) = \frac{A_0 b_0 + A_1 b_1 + A_2 b_2 + A_3 b_3}{\lambda_i g_0 + \mu_i g_1 + \nu_i g_2 + \delta_i g_3} \tag{2}$$

where

$$\begin{cases} b_0 = (1 - \sin u)\cos^2 u \\ b_1 = \sin u \cos^2 u \\ b_2 = \cos u \sin^2 u \\ b_3 = (1 - \cos u)\sin^2 u \end{cases}$$

and

$$\begin{cases} A_0 = \lambda_i f_i \\ A_1 = \mu_i f_i + \dfrac{2h_i d_i \lambda_i}{\pi} \\ A_2 = \nu_i f_{i+1} - \dfrac{2h_i d_{i+1} \delta_i}{\pi} \\ A_3 = \delta_i f_{i+1} \end{cases}$$

with $P^{'}(x_k) = d_k, k = i, i+1$, $h_i = x_{i+1} - x_i$ and $u = \frac{\pi}{2}\frac{x - x_i}{h_i}$.

Theorem 1: Let $\{(x_i, y_i)\}_{i=0}^{n}$ be a given monotone data set. The rational trigonometric interpolant defined in (2)

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preserves the monotonicity if the free shape parameters μ_i and ν_i, on each subinterval $[x_i, x_{i+1}]$ satisfy the following conditions:

$$\mu i > max\{0, \frac{2\lambda_i d_i}{\pi \Delta_i}\}, \quad \nu i > max\{0, \frac{2\delta_i d_{i+1}}{\pi \Delta_i}\} \tag{3}$$

Proof: Suppose that the data set $\{(x_i, y_i) : i = 0(1)n\}$ is monotonically increasing, that is:

$$y_i \leq y_{i+1}$$

Equivalently, $\Delta_i = \dfrac{y_{i+1} - y_i}{h_i} \geq 0$
For monotonicity, the necessary conditions on derivatives are:

$$d_i \geq 0, \quad i = 0(1)n$$

There exist following two cases for rational trigonometric interpolant (2) to carry the monotonic behaviour of data.

1) $\Delta_i = 0$. In this case, the interpolant reduces to

$$P_i(x) = y_i, \quad \forall x \in [x_i, x_{i+1}]$$

2) $\Delta_i \neq 0$, then $P_i(x)$ is monotonically increasing if and only if

$$P_i^{'}(x) \geq 0, \quad \forall x \in [x_i, x_{i+1}]$$

For $x \in [x_i, x_{i+1}]$, $P_i^{'}(x)$ is given in (4).
with

$$\begin{cases} K_1 = \frac{2\lambda_i^2 d_i}{\pi} \\ K_2 = \lambda_i(\nu_i \Delta_i - \frac{2\delta_i d_{i+1}}{\pi}) \\ K_3 = \lambda_i \delta_i \Delta_i \\ K_4 = \mu_i \nu_i \Delta_i - \frac{2\lambda i \nu_i d_i}{\pi} - \frac{2\mu i \delta i d_{i+1}}{\pi} \\ K_5 = \delta_i(\mu_i \Delta_i - \frac{2\lambda_i d_i}{\pi}) \\ K_6 = \frac{2\delta_i^2 d_i}{\pi} \end{cases} \tag{5}$$

Since the denominator of (4) is always positive, thus the sufficient conditions for the curve to preserve the monotonicity of monotone data are:

$$K_j \geq 0, \quad j = 1, ..., 6$$

It is clear that $K_j \geq 0$ for j=1,3,6
and for j=2,4,5, $K_j \geq 0$, if:

$$\mu_i \geq \frac{2\lambda_i d_i}{\pi \Delta_i} \tag{6}$$

$$\nu_i \geq \frac{2\delta_i d_{i+1}}{\pi \Delta_i} \tag{7}$$

Since $\mu_i, \nu_i \geq 0$, thus equations (6) and (7) can be written as:

$$\mu i > max\{0, \frac{2\lambda_i d_i}{\pi \Delta_i}\}, \quad \nu i > max\{0, \frac{2\delta_i d_{i+1}}{\pi \Delta_i}\} \tag{8}$$

which is the desired result.

■

TABLE I: Monotone Data Set

i	1	2	3	4	5
x_i	0	6.0	10.0	29.5	30.0
y_i	0.01	15.0	15.0	25.0	30.0

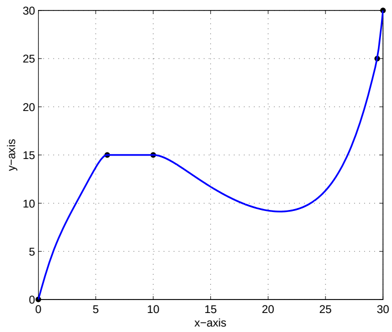

Fig. 1 Non-monotone curve for monotone data

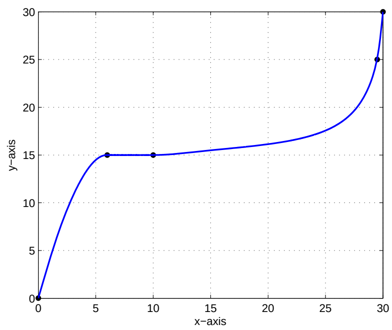

Fig. 2 Monotone curve for monotone data

TABLE II: Monotone Data Set

i	1	2	3	4	5	6	7	8	9	10	11
x_i	1	2	3	4	5	6	7	8	9	10	11
y_i	0.0001	0.0006	0.0027	0.0123	0.0551	0.2402	0.7427	0.9804	0.9990	0.9999	1.0000

III. Demonstration

Example 1: Consider a monotone data set given in Table I. Figure 1 shows the curve drawn by rational trigonometric spline with the values of shape parameters set as $\lambda_i = 0.2$, $\mu_i = \nu_i = 0.8$ and $\delta_i = 0.3$. It can be observed that although the data set is monotone, but the resulting curve is not. This deficiency is removed in figure 2 when the curve is constructed using the monotonicity preserving rational trigonometric interpolant discussed in section II with the values of free shape parameters $\lambda i = \delta_i = 1$.

Example 2: Figure 3 shows a non-monotone curve produced by using a monotone data set given in table II for arbitrary values of shape parameters $\lambda_i = \mu_i = \nu_i = \delta_i = 1.0$. On the other hand a monotone curve in figure 4 for the same data set is drawn by applying the proposed scheme by taking $\lambda i = \delta_i = 0.5$.

$$P_i^{'}(x) = \frac{\pi}{2(q_i(u))^2}\{K_1\cos^5 u + K_2\cos^2 u\sin u(\cos^2 u\sin u + \sin u(1-\sin u)(2-\sin^2 u))$$
$$+K_3\cos u\sin u(\cos u\sin u(-1+\cos u+\sin u)+2(1-\sin u)(1-\cos u))$$
$$+K_4\cos^2 u\sin^2 u + K_5\cos u\sin^2 u(\cos u\sin^2 u + \cos u(1-\cos u)(2-\cos^2 u)) + K_6\sin^5 u\} \quad (4)$$

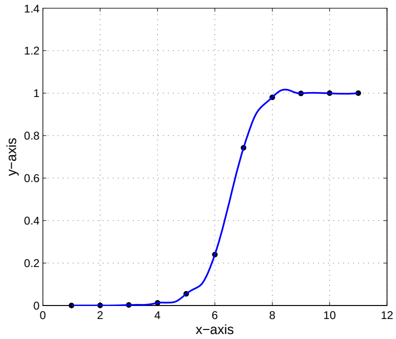

Fig. 3 Non-monotone curve for monotone data

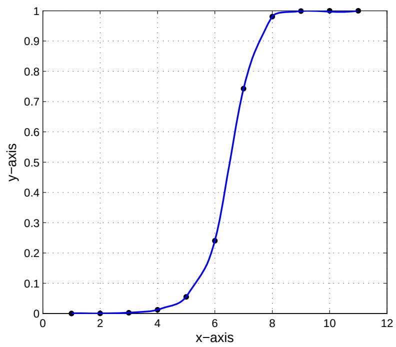

Fig. 4 Monotone curve for monotone data

IV. Conclusion

In this study a piecewise C^1 monotone rational trigonometric spline with four shape parameters is constructed. Monotonicity of data is assured by imposing restrictions on two shape parameters while leaving the other two free for the user. The scheme is tested for monotone data sets producing graphically smooth and geometrically monotone curves.

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References

[1] M. Grave, Y. L. Lous and W. T. Hewittm, *Visualization in Scientific Computing*, Springer-Verlag, Vol. 199, 1994.
[2] M. Z. Hussain and M. Hussain, *Visualization of data preserving monotonicity*, Applied Mathematics and Computation, 2007, pp. 1353–1364.
[3] M. Z. Hussain and M. Sarfraz, *Monotone piecewise rational cubic interpolation*, International Journal of Computer Mathematics, Vol. 86, 2009, pp. 423–430.
[4] X. Han, *Quadratic trigonometric polynomial curves with a shape parameter*, Computer Aided Geometric Design, vol. 19, 2002, pp. 503–512.
[5] X. Han, *Cubic trigonometric polynomial curves with a shape parameter*, Computer Aided Geometric Design, vol. 21, 2004, pp. 535–548.
[6] X. Wu, X. Han and S. Luo, *Quadratic trigonometric spline curves with multiple shape parameters*, 10th IEEE International Conference on Computer-Aided Design and Computer Graphics, 2007 , pp. 413–416.
[7] N. Choubey and A. Ojha, *Trigonometric splines with variable shape parameter*, Journal of Mathematics, vol. 38, 2008. pp. 91-105.
[8] M. Sarfraz, S. Butt and M. Z. Hussain, *Visualization of shaped data by a rational cubic spline interpolation*, Computers & Graphics, Vol. 25, 2001, pp. 833–845.