Application of Transportation Linear Programming Algorithms to Cost Reduction in Nigeria Soft Drinks Industry

A. O. Salami

Abstract—The transportation problems are primarily concerned with the optimal way in which products produced at different plants (supply origins) are transported to a number of warehouses or customers (demand destinations). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. The objective of this study is to determine ways of minimizing transportation cost in order to maximum profit. Data were sourced from the records of the Distribution Department of 7-Up Bottling Company Plc., Ilorin, Kwara State, Nigeria. The data were computed and analyzed using the three methods of solving transportation problem. The result shows that the three methods produced the same total transportation costs amounting to N1, 358, 019, implying that any of the method can be adopted by the company in transporting its final products to the wholesale dealers in order to minimize total production cost.

Keywords—Allocation problem, Cost Minimization, Distribution system, Resources utilization.

I. INTRODUCTION

A. Background Information

WHENEVER there are physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales. A lot of situation do arise which would make the marketing managers or sales managers in an organization to determine the total number of goods that goes to a particular depot/sales point at a particular period. Basing this type of decision(s) on chance and not on scientific approach could result to sub-optimization; this type of situation has made many firms to be running at a minimum profit or sometimes at a loss [4].

Operations research (OR) are the application of scientific methods to problems arising from operations involving integrated systems of men, machines and materials. It normally utilizes the knowledge and skill of an interdisciplinary research team to provide the managers of such systems with optimum operating solutions [3]. The basic objective of these scopes of OR in management is to minimize the total cost [5]. The transportation problem is a special class of linear programming problem, which deals with shipping

Akeem Salami is with the Department of Business Administration, College of Management Sciences, Federal University of Agriculture, Abeokuta P.M. B 2240, Ogun State 110001, Nigeria (e-mail: Akeemsalami20022002@yahoo.com).

commodities from source to destinations. The objective of the transportation problem is to determine the shipping schedule that minimize that total shipping cost while satisfying supply and demand limits.

The classical transportation problem is one of the many well-structured problems in operations research that has been extensively studied in the literature. Other examples are the travelling salesman and shortest-route problems. As noted, the transportation problem is one of the subclasses of the linear programming problems for which simple and practical computational procedures have been developed which serves as major advantage of the special structure of the problem.

The origin of transportation was first presented by Hitchcock in 1941, who also presented a study entitled "The Distribution of a Product from Several sources to numerous Localities". This presentation is considered to be the first important contribution to the solution of transportation problems [6]. Author [7] also presented an independent study, not related to Hitchcock's, and called it "Optimum Utilization of the Transportation System". These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem, received this name because many of its applications involve determining how to optimally transport goods. However it cannot be used to solve optimization problems in complex business situations until only in 1951, when George B. Dantzig applied the concept of Linear Programming in solving the Transportation models.

B. Objective of the Study

The objective of this study is to determine the best methods of minimizing transportation cost using the following three models of transportation algorithms:
1) North-West Corner method
2) Minimum / least cost method
3) Vogel's Approximation Method (VAM)

C. Methodology

The data used for analysis in this paper were mainly from secondary source. These data were obtained from the distribution department of 7-up Bottling Company, PLC, Ilorin Depot. It was extracted from the distribution file of the company. Five (5) depots located in the western zone of Nigeria were selected and they represents the supply capacities while sixteen (16) demand requirements are to be met by the five depots selected.

The depots that were selected based on proximity to the wholesale consumers are: Oshogbo, Ibadan, Ilorin, Mokwa and Ogbomoso. These five depots supplied the following customers with their 7-up demand requirements in these locations: Okuku, Ede, Ejigbo, Oyo, Awe, Ilora, Offa, Erin-Ile, Oro Ilofa, Jebba, Patigi, Ogbomoso, Oko, Odo-Oba, and Inisha. The days that fall within the Christmas and New Year celebration periods (December 15, 2012 to January 15, 2013], during which there were always higher demand for the company's products was understudied in this paper.

Transportation problem is a special type of linear programming problems that involves the following steps:
1) Finding an initial feasible solution
2) Testing the solution for optimality
3) Improves the solution when it is not optimal
4) Repeating steps (ii) and (iii) until the optimal solution is obtained.

Three methods of solving transportation problems viz North-West corner, Vogel's approximation and least-cost methods were used in analyzing data in this paper.

II. CONCEPTUAL CLARIFICATIONS

A. Assumptions of Transportation Model

Transportation methods can only be applied after a series of assumptions must have been made. However, the assumption made in the transportation model must be clearly stated as follows:
1) Only homogenous product is considered and this must be a commodity such that all supplies are identical and customers will accept these items from any of the source.
2) The quantities available and demanded are assumed known and equal in total. Though the latter assumption can be removed by adding to the problem a dummy depot as application which absorbs surplus at no cost. In [2], the model assumes that all transport routes can be waived by replacing the unusable routes by ones having a very high transport cost. This ensures that they could not feature in any optimum solution.
3) The model assumes that all transport costs are known, but this may not be so in practice since many routes have not been used in the past. It is also assumed that the costs should not only be known but they are directly proportional to the quantity.

B. Nature of Transportation Analysis

Transportation model can only be applied after the assumption must have been made. However, the assumption made in transportation model must be unambiguous. What is meant by transportation analysis is that they are involved in every extra truckload that is located to a route. Nonetheless, there are many situations for which the above model provides a satisfactory description and the transportation method can be used to give the solution.

There are other business situations which can be modeled in exactly the same form, as the transportation model that is described above. The transportation method handles any

model which can be summarized as presented in Table I below [1].

TABLE I
TRANSPORTATION MATRIX FORMAT

Destination /origin	D1		D2		D3		D4		Capacity per time period
O1	X_{11}	C_{11}	X_{12}	C_{12}	X_{13}	C_{13}	X_{14}	C_{14}	B1
O2	X_{21}	C_{21}	X_{22}	C_{22}	X_{23}	C_{23}	X_{24}	C_{24}	B2
O3	X_{31}	C_{31}	X_{32}	C_{32}	X_{33}	C_{33}	X_{34}	C_{34}	B3
O4	X_{41}	C_{41}	X_{42}	C_{42}	X_{43}	C_{43}	X_{44}	C_{44}	B4
Destination requirement per time period	D1		D2		D3		D4		TOTAL

In Table I above, O1, O2, O3 and O4 represent plants/origin of the goods. B1, B2, B3 and B4 on the right hand side represent the capacity of plants respectively D1, D2, D3 and D4 represent depots/destinations.

The Cij is the cost coefficient, where i = 1, 2, 3 and 4 and j = 1, 2, 3 and 4. They represent the cost of transporting goods from source to destination. All j stands for column while I stands for row.

The total cost will be given as:

$$\sum_{i-1}^{4} \quad \sum_{j-1}^{4} Cij \, Xij \qquad (1)$$

where Xij represents the size of item from origin i to destination j. Cij represent the respective costs of moving goods from source to destination.

Consequently, an optimal solution occurs when the basic feasible solution achieved cannot be improved upon.

1. The Transportation Tableau

The special structure of a transportation model allows for the presentation of data in a concise format known as transportation tableau. Author [10], opined that, the following rules apply for formulating transportation tableau:
1) The transportation tableau has one row for each supply sources and column for each demand points
2) The supply availability of each point is given in "Availability Column" to the right of the main body of the tableau.
3) The "Box-in" values in the upper right hand corner in each tableau cell is the objective function co-efficient (or cost) of the relevant decision variable.

2. Transportation Model

The main objective of transportation model according to [1] is to have effective distribution of goods and services and more so at the minimum cost possible. There are balanced and unbalanced transportation problems. The situation where total demand is equal to the total supply, it is a balanced transportation problem. But the situation whereby the total requirement by the customer is not up to the source capacity is called an unbalanced transportation problem. When the

model is unbalanced, a dummy row or column is added to the table with zero transportation cost. Using Table I for example, a model for its equivalent would look like the following:

$$\text{Min } Z = C_{11}X_{11} + C_{12}X_{12} + \ldots + C_{14}X_{14} \qquad (2)$$

Subject to:

$$X_{11} + X_{12} + \ldots + X_{14} = b1$$
$$X_{21} + X_{22} + \ldots + X_{24} = b2$$
$$X_{31} + X_{32} + \ldots + X_{34} = b3 \qquad \text{Plant Constraint}$$
$$X_{41} + X_{42} + \ldots + X_{44} = b4$$

$$X_{11} + X_{21} + \ldots + X_{31} + X_{41} = d1$$
$$X_{12} + X_{22} + \ldots + X_{32} + X_{42} = d2$$
$$X_{13} + X_{23} + \ldots + X_{33} + X_{43} = d3 \qquad \text{Depot Constraint}$$
$$X_{14} + X_{24} + \ldots + X_{34} + X_{44} = d4$$

Reference [2] formulated the transportation problem as a special linear programming problem and then developed a special form of the simple technique for solving such problems, now known as the transportation technique. Though the above transportation model presented in form of linear programming problem could be solved by using simplex method algorithm, but because of the special structure of the constraint, Danzig's solution procedure has advantage of the special nature (structure) of the co-efficient matrix that makes it better.

This is because it can used to solve hundreds of constraints subject to a single objective. The standard transportation problem is to optimize an objective function subject to many constraints as presented in (3).

$$\text{Minimize } Z = \sum_{i=1}^{4} \sum_{j=1}^{4} C_{ij}X_{ij} \qquad i = 1,2,\ldots 4 \qquad (3)$$

$j = 1,2\ldots4$
Subject to:
$\Sigma X_{ij} \leq b_i \; \forall \; i = 1,2,\ldots 4$
$\Sigma X_{ij} \leq d_j \; \forall \; j = 1,2,\ldots 4$
and $X_{ij} \geq O$ for all i and j.

An additional requirement is that the sum of the demand must be equal to the sum of the supply, otherwise dummy supply or demand should be introduced with zero transportation cost
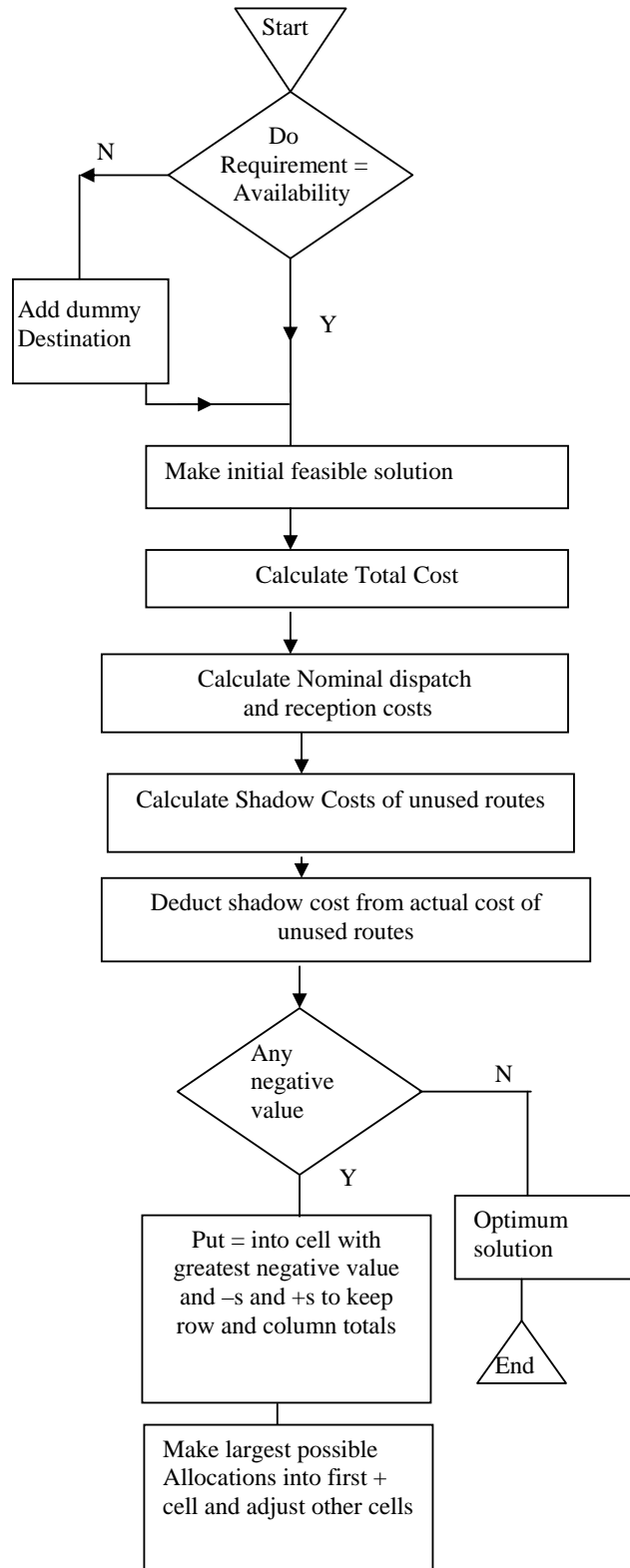i.e. $\Sigma b_i = \Sigma d_j$

Fig. 1 Outline of transportation methods flowchart

The methodology developed and summarized in Fig. 1, provides a detailed description of the principles involved in

the transportation technique. It was summarized by [8].

C. Methods of Solving Transportation Problems

Transportation problem is solved using any of the transportation methods listed below to obtain the initial solution according to [9].
1) Vogel's approximation method
2) North-West corner rules method
3) Least-Cost Method

1. Vogel's Approximation Method (VAM)

This is an advance method and as such can be complicated. VAM according to [9] has the advantage of arriving at optimal solution at first attempt i.e. it both initiates and simultaneously solves optimally.

Steps:
1) Determine the lowest cost in each column and subtract from the next lowest cost in that same column. The result is the opportunity cost of that column.
2) Determine the lowest cost in each row and subtract from the next lowest cost in that row. The result is the opportunity cost in that row
3) The row or column with the highest opportunity cost is selected. Allocation is then made to the cheapest cell in that row or column selected. If a tie (equality) occurs, select any one. Block necessary cells during allocations.
4) After step iii, repeat the process form steps i to iii, (ignoring blocked cells) and stop when the last allocation is made.

2. North-West Corner Rule Method

Here, the distribution pastern starts from the north-west corner of the table. Distribution continues until the last sources is distributed by allocating as many units as possible to the first route on the top left corner of the table while moving one route to the right to satisfy row requirements or one route down to satisfy column requirements. No attention is paid to relative cost of the different routes while making the first assignments. This method does not guarantee optimal solution.

3. Least Cost Method

Here, the method first guesses what the optimal solution might be with the initial table. It then uses this guess to distribute along the routes. Allocations are made on the basis of economic desirability i.e. we begin with the cell that has the lowest cost and to the next until all the distribution are exhausted. Efforts should be made to block cells which rows and columns have been satisfied.

D. Obtaining Improved Solutions

After obtaining the initial solutions, improved solutions can be obtained until optimal solution is determined according to [8]. In finding the optimal solution, the following procedure must be observed.
1) Evaluating the necessary contribution of unused routes or cells
2) Test for optimality
3) Select the route to start utilizing

4) Determine the amount to be moved over the selected routes
5) Develop the new solution

E. Calculating the Next Contribution of Unused Routes

Two common alternatives are used to evaluate the next contribution of unused routes of a given transportation tableau. These are:
1) Stepping stone method
2) Modified distribution method (MODI)

1. Steps in Stepping Stone Method

1) Start at the unused route to be evaluated and mark with a plus (+) sign.
2) Examine the row that contains the chosen routes and mark the other routes with a minus (-) sign. The used routes are referred to as steeping-stones. If there is more than one stepping stone, mark the one which contains another used routes within its column.
3) Scan the column that contains the routes and mark with a plus (+) sign. The unused route that contains another used route within its row.
4) Continue the process until each row containing a plus (+) sign also contains a minus (-) sign and until the same condition applies to the columns.
5) Determine the net contribution of these chains of adjustments by adding the per unit cost of all routes containing a plus (+) sign and subtracting the per unit cost of all the routes containing the minus (-) signs.

2. Steps Involved in Modified Distribution Methods (MODI)

1) Obtain the initial table using any of the initial method
2) For each used route, break the cost into two components, i.e. Dispatch and Reception Costs $C_{ij} = D_i + R_j$.
3) Solve the system of equation obtained in step 2 above by assuming the first dispatch Cost to be Zero.
4) Break each of the unused routes into Dispatch and Reception Costs
5) With the aid of the solution of the system of equation in step (iv), for each of the unused route, evaluate $C_{ij} - (D_i + R_j)$ i.e. Actual Cost – Shadow Costs
6) Check for the negative values in iv above, if there is none, the table is said to be optimal.

F. Testing the Optimality

In maximization problem, the optimal solution is obtained when the next contribution of unused cells have positive (+) or zeros. In minimization problems, the optimal level is reached when the net contribution of the unused cells are negative or zero. In minimization problem, select the route with the largest negative net contribution value. In maximization problem, select the route with the largest positive (+) net contribution value.

The largest amount that can be moved into the chosen route is found by examining those route containing negative signs (-). A new transportation tableau is then drawn and the value of the objectives function is determined. This procedure is

repeated until the optimal solution is obtained.

G. Degeneracy in Transportation Technique

Degeneracy according to [8] is a situation whereby there are allocations less than $(M + N - 1)$ cells in transportation problem. 'M' is number of rows and 'N' number of column. Feasible solution is only attained in any transportation problem satisfying allocation in $M + N - 1$. When degeneracy occurs, the solution is made basic by putting unusable zero in one of the unused cells. Improvement can be made for degenerate problem in the same way as when the initial solution is basic.

H. Unbalanced Transportation Problem

The transportation is unbalanced when the total quantity demanded by the depots is not equal to the total supply or capacity of the plants. To solve an unbalanced transportation problem, a dummy depot must be created to absorb the excess first. After which degeneracy is checked to allow for efficient allocation to routes.

I. Profit Maximization in Transportation Problem

Transportation problem according to [1] may be designed so that the objective (which is to make the allocation from sources to destination such that contributions (or profit) may be maximized) is allowed.

The steps to follow in order to achieve this are listed below:

1) Make initial feasible allocation with maximum profit and so on according to their sizes.
2) The optimal level is reached when all contribution of cell in the unused routes are all negatives or zeros. If not, the next cell to use is the one with the largest positive net contribution.

III. PRESENTATION AND ANALYSIS OF DATA

A. Presentation of Data

The data on transportation problem for this research study was collected from the Distribution Department of 7-Up Bottling Company Plc., Ilorin. The presentation of the data will assist in analyzing and deducing good model for allocation of the final products of the company to the wholesales dealers.

TABLE II
THE SUPPLY CAPACITIES OF THE FIVE DEPOTS (IN CRATES)

DEPOTS	CAPACITIES
Osogbo	1,728
Ibadan	2,304
Ilorin	2,880
Mokwa	1,440
Ogbomoso	2,592

TABLE III
THE DEMAND REQUIREMENTS (CRATES) OF THE WHOLESALE DEALERS

WHOLESALE	REQUIREMENTS
Okuku	500
Ede	728
Ejigbo	500
Oyo	1,000
Awe	920
Ilora	760
Offa	1,200
Erin-Ile	920
Oro	760
Ilofa	340
Jebba	500
Patigi	600
Ogbomoso	1,500
Oko	330
Odo-Oba	381
Inisha	381

TABLE IV
COST OF TRANSPORTING THE PRODUCTS FROM THEIR RESPECTIVE DEPOTS TO REQUIRED WHOLESALE DEALERS

DEPOT	EDE ₦	EJIGBO ₦	OYO ₦	AWE ₦	ILORA ₦	OFFA ₦	ERINLE ₦	ORO ₦	ILOFA ₦	JEBBA ₦	PATIGI ₦	OGBOM OSO ₦	OKO ₦	ODO-OBA ₦	INISHA ₦
Osogbo	122	122	126	126	126	124	125	122.5	129	130	132	127	125	124	123
Ibadan	125	124.5	122.	122.5	123	126	122.5	126.5	127	129	130	125	126	127	127
Ilorin	126	127	128	127	127	124	123.5	124.5	127	128	129	124	125	125	127
Mokwa	132	131	132	131	131	128	129	131	122	123	124	130	131	130	132
Ogbomoso	131	129	129	129	128	131	130	132	133	134	134.5	122	122.5	125	124

B. Evaluation

Data in Tables II and III shows that the demand is equal to supply which implies a balanced transportation problem. Table IV depicts the cost per unit of moving the goods from their various sources to destinations. In this research work, all the three methods mentioned in the study methodology were computed in determining the optimal solution.

TABLE V
THE SHIPPING LIST

FROM	TO	SHIPMENT	Cost per unit ₦	Shipment cost ₦
Osogbo	Okuku	500	122.5	61,250
Osogbo	Ede	728	122	88,816
Osogbo	Ejigbo	500	122	61,000
Osogbo	Offa	--	124	--
Osogbo	Inisha	--	123	--
Oyo	Oyo	1,000	122.5	122,500
Oyo	Awe	652	122.5	79,870
Oyo	Ilora	652	123	80,196
Oyo	Ilofa	--	127	--
Ilorin	Offa	1,200	124	148,800
Ilorin	Erin-Ile	920	123.5	113,155
Ilorin	Oro	760	124.5	94,620
Ilorin	Ilofa	--	127	--
Mokwa	Ilofa	340	122	41,480
Mokwa	Jebba	600	124	74,400
Mokwa	Patigi	600	124	74,400
Ogbomoso	Ogbomoso	1,500	122	183,000
Ogbomoso	Oko	330	122.5	40,425
Ogbomoso	Odo-Oba	381	123	46,863
Ogbomoso	Inisha	381	124	47,244
			Total Cost	**1,358,019**

IV. CONCLUSION

From the result of the analysis, it was found out that the transportation cost using the three methods were the same i.e. N1, 358, 019.00. Therefore, any of the methods can be adopted since they all produced the same optimal solution at the minimum transportation cost of N1, 358, 019.00. By implication, this result provides a practical analysis of transportation model solution methodology.

Based on the study findings as presented in Table V, the study recommends that optimum solution can be attained through the following routes:

Osogbo Depot should supply: Okuku., Ede, Ejigbo, Offa, and Inisha wholesale dealers.

Oyo Depot should supply: Oyo, Awe, Ilora and Ilofa wholesale dealers.

Ilorin Depot should supply: Offa, Erin-Ile, Oro and Ilofa wholesale dealers.

Mokwa Depot should supply: Ilofa, Jebba and Patigi wholesale dealers.

Ogbomoso Depot should supply: Ogbomoso, Oko, Odo-Oba and Inisha wholesale dealers.

REFERENCES

[1] T.O. Asaolu and M. L. Nassar, Application of Quantitative Technique to Management Accounting. Ile-Ife, Nigeria, Cedar Productions 2002, chapter 3 and 4.
[2] G.B. Dantzig, Application of the Simplex Method to the Transportation Problem, in T.C. Koopmans (ed.), Activity Analysis of Production and Allocation, John Wiley & Sons, New York, 1951 pp. 359–373.

[3]   W. J. Fabryoky, P.M. Glare and, P.E. Torgerson,   Industrial Operations Research Prentice-Hall, Inc., Englewood SHHSA, New Jersey, 1992 chapter 8.

[4]   P.K. Gupta and D.S. Hira, Problems in Operations Research: Principles and Solutions. S. Chend   and Co. Ltd., New Delhi, 1998, chapter 3.

[5]   P. K. Gupta and D. S. Hira, Operations Research: S.   Chand   and   Co. Ltd., New Delhi,   2005, chapter 4.

[6]   F. L. Hitchcock, "The distribution of a product from   several sources to numerous localities", Journal of Mathematics and Physics, 1941 20 pp. 224-230.

[7]   T. C. Koopman, "Optimum utilization of transportation   system", Proc. Intern. Statics. Conf.   1947, Washington D.C.

[8]   T. Lucey, Quantitative Techniques. 7th   edition, London: Ashford Colour Press, 2002.

[9]   O. O. Oyesiku, A. J. Abolade, S.O. Kajola and S. G. Napoleon, Basic of Operations   Research, 1st edition Ago-Iwoye: Ogun State University Press, 1999.

[10] S. D. Sharma, Operations Research. India: MC Kinsey and Company Inc, 1996.