Performance Degradation for the GLR Test-Statistics for Spatial Signal Detection

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Abstract—Antenna arrays are widely used in modern radio systems in sonar and communications. The solving of the detection problems of a useful signal on the background of noise is based on the GLRT method. There is a large number of problem which depends on the known a priori information. In this work, in contrast to the majority of already solved problems, it is used only difference spatial properties of the signal and noise for detection. We are analyzing the influence of the degree of non-coherence of signal and noise unhomogeneity on the performance characteristics of different GLRT statistics. The description of the signal and noise is carried out by means of the spatial covariance matrices $\mathbf{C}$ in the cases of different number of known information. The partially coherent signalis is simulated as a plane wave with a random angle of incidence of the wave concerning a normal. Background noise is simulated as random process with uniform distribution function in each element.

The results of investigation of degradation of performance characteristics for different cases are represented in this work.

Keywords—GLRT, Neumann-Pearson's criterion, test-statistics, degradation, spatial processing, multielement antenna array

I. INTRODUCTION

MULTICHANNEL adaptive signal detection is interesting with a practical point. It is more and more widely using the multielement antenna array in the modern radio systems, hydrolocations and communications. The solving of this problem is based on GLRT method application [1] - [5]. The GLRT replaces the unknown parameters by their maximum likelihood estimates. In different applications is a large variety of tasks depending on the known a priori information. In the most cases of solved tasks the temporary structure of the useful signal, the direction of its arrival a form of a wavefront set (for example pane wave) is supposed known [3] - [15]. In contrary of these works in the present paper on the basis of previous works [16] - [20] used only difference spatial properties of the signal and noise for detection. In the present work the detailed comparison of test-statistics is completes a series of studies, starting from completely coherent signal on the homogeneous noise background to essential noncoherent signal on the unhomogeneous noise background.

II. MODELS AND METHODS

Consider a p-element receiving antenna array with arbitrary locations of sensors. The p-dimensional input signal $\vec{z}$ is

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assumed to be complex random Gaussian vector and N samples of the signal $\vec{z}[1], \vec{z}[2], ..., \vec{z}[N]$ are statistically independent and identically distributed zero-mean random vectors with spatial covariance matrix $\mathbf{C}$. Consider enough general case when the useful signal is spatially correlated, and gaussian noise is spatially and temporarily white. Then the detection problem formulation is consolidated to a classical statistical two-alternative:

Null hypothesis (noise only),
$H_0 : \mathbf{C} = \mathbf{C}_0,$

Alternative hypothesis (signal plus noise), $\qquad$ (1)

$H_1 : \mathbf{C} \neq \mathbf{C}_0,$

where H_0 is the null hypothesis about only noise presence, and H_1 is the alternative hypothesis about additive mix of noise and useful signal presence. The introduction of noise characteristics occurs with the help of definition of covariance matrix $\mathbf{C}$ [16]-[17]. In the first case (hypothesis H_{01}), we have no a priori information about noise, in addition to the condition of the independence of its samples in various elements of the antenna. In the second case (hypothesis H_{021}), in addition to the independence of noise in the elements of antenna, a priori information on its homogeneity (the same power in the different elements of the antenna). In the third case (hypothesis H_{03}), in addition to the independence of noise in the known and the noise power (covariance matrix noise $\mathbf{C}$ is a unit). In the fourth case (hypothesis H_{04}), the characteristics of the noise are the same as in the third, but added information on the spatial coherence of the received signal. The decision about signal presence or absence is accepted by a way of comparison of the generalized likelihood ratio V with some threshold value V_{th}. For four of the cases the test statistics of the generalized likelihood ratio equal to, respectively

$$V_1 = \frac{|A|^N}{\prod\limits_{i=1}^{p} (a_{ii})^N}, \qquad (2)$$

$$V_2 = \frac{|A|^N}{\left(\frac{spA}{p}\right)^{pN}} \qquad (3)$$

$$V_3 = \left(\frac{e}{N}\right)^{pN} |\mathbf{A}|^{\mathbf{N}} \mathbf{e}^{-\mathbf{spA}} \qquad (4)$$

$$V_4 = \frac{|A|^N}{\left(\frac{spA}{p}\right)^{pN}} \qquad (5)$$

Here

$$\mathbf{A} = \sum_{\alpha=1}^{N} \vec{z}_\alpha \vec{z}_\alpha^\dagger \qquad (6)$$

- the scaled correlation matrix. According to Neumann-Pearson's criterion threshold value of a statistic V_{th} is defined from a condition of the probability of the right detection at the given false alarm probability.

III. SIMULATION RESULTS

The above considered detection algorithms were simulated numerically. According to suggested null hypotheses of H_{01} and H_{02} noise was simulated as unhomogeneous with covariance matrix

$$\mathbf{C}_{01} = \begin{array}{cccc} \sigma_{11}^2 & 0 & \dots & 0 \\ 0 & \sigma_{22}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{pp}^2 \end{array} \qquad (7)$$

and as homogeneous with covariance matrix

$$\mathbf{C}_{02} = \begin{array}{cccc} \sigma^2 & 0 & \dots & 0 \\ 0 & \sigma^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^2 \end{array} \qquad (8)$$

correspondingly.

In the homogeneous noise case the powers in the antenna array elements were identical, in the unhomogeneous noise case this powers were different and the power averaged over all antenna elements coincided with that of homogeneous noise in the one element (Fig.1).
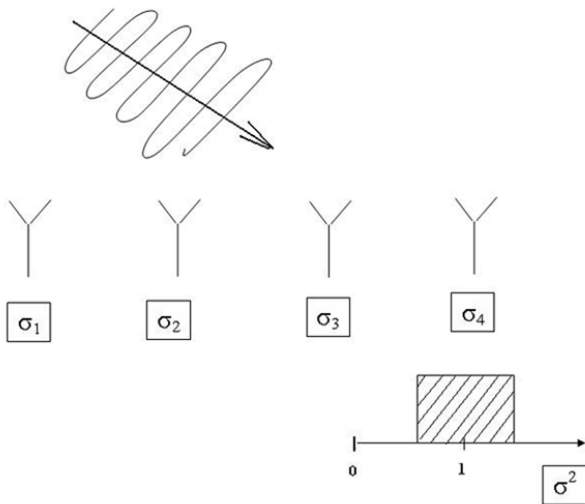

Fig. 1. Noise in the various elements of the antenna array

In both considered cases, in homogeneous and unhomogeneous, the trace of a covariance matrix of noise remained equal to number of elements of an antenna arrray. Detail dependence of the detection characteristics of the degree ofnoise heterogeneity was investigated in [18] and [19]. Here is studied performance degradation depending on shake angle of arrival for different test-statistics.

The useful signal were simulated as a patially coherent plane wave. In the patially coherent model a plane wave with a fluctuating arrival angle used. Obviously, the useful signal in this case can be written in the following form:

$$\mathbf{S(t)} = \mathbf{a(t)S} = (e^{\mathbf{i}\theta_0}, \dots, e^{\mathbf{i}(2\pi\delta(\mathbf{p}-1)\sin((\theta(\mathbf{t})+\theta_0)))}). \quad (9)$$

Here $a(t)$ is the complex Gaussian amplitude of a signal, $\mathbf{S}$ is the vector fazor of the useful signal wave front , θ_0 is an initial phase of a signal in the first antenna element, δ is the ratio of the distance between elements of the antenna array to a wavelength of a source signal, $\theta(t)$ is a variable angle of incidence of the wave concerning a normal. It was supposed that the law of an incidence angle change variation of a wave represents the normal process with independent increments:

$$\theta[n+1] = \theta[n](1-b) + q\xi[n], \qquad (10)$$

there $\xi[n]$- white Gauss noise with zero mean and unit dispersion, q - factor that determines the full capacity generating noise, b - factor that determines the speed of decay of correlation. The exact expression for the time correlation function of the angle of incidence of a plane wave $\theta(t)$ written in the form

$$K_\theta[\tau] = (1-b)^{|\tau|}\sigma_\theta^2. \qquad (11)$$

The standard deviation σ_θ of the arrival angle of the plane wave determine the coefficients q and b as expression

$$\sigma_\theta = \frac{q\sigma_\xi^2}{1-(1-b)^2}. \qquad (12)$$

The correlation time τ_θ of a random angle of arrival $\theta[n]$ determined as the time in which correlation function (11) decays e times. From the analysis of the equation (10) for the correlation time τ_θ it is easy to get the following formula:

$$\tau_\theta = ln(\frac{1}{1-b}) \qquad (13)$$

The dependence of the correlation time of the parameter b is shown in Fig. 2.

It is obvious that by varying the the coefficients b and q, we can get various models of fluctuations of the useful signal wave front.

In this experiment the linear equidistant 4-element antenna array was simulated. The distance between elements of array was equal to half wavelength ($\delta = 0.5$). On Fig. 3 the experimentally obtained dependence the correlation coefficient between the useful signals in the elements of antenna array depending on the distance between them for different values of standard deviation angle of arrival useful signal (10) is presented. The diagrams show the correlation coefficient of useful signals in extreme elements of the antenna array becomes equal to 0.5 already at mean-square deviation angle of arrival $\sigma_\theta = 4°$.
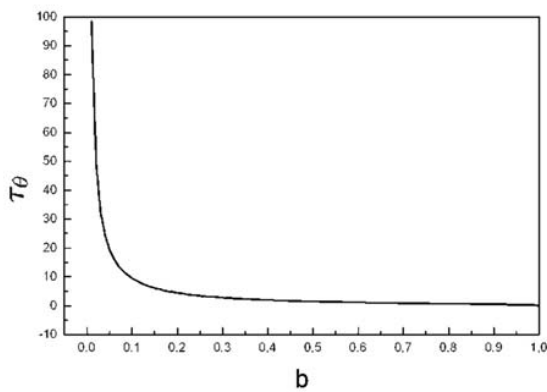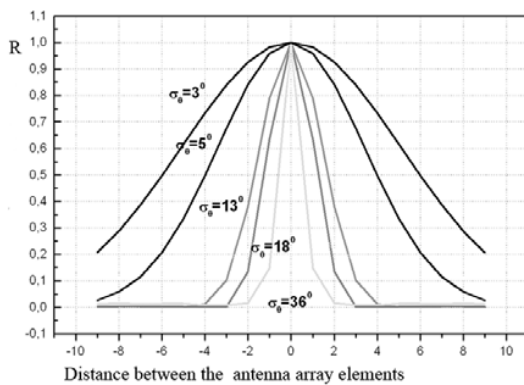
Fig. 2.   The correlation time for the parameter b

Fig. 3.   The correlation coefficient for the distance between the antenna array elements

Fig. 4.   Receiver operating characteristics for V1-V4 for plane wave with a different variance of fluctuating arrival angle on the homogeneous noise background

Fig. 5.   Receiver operating characteristics for V2 separately for plane wave with a different variance of fluctuating arrival angle on the homogeneous noise background

Fig. 6.   Receiver operating characteristics for V1-V4 for plane wave with a different variance of fluctuating arrival angle on the unhomogeneous noise background

The correlation time of the angle of arrival τ_θ was equal to $\tau_\theta \simeq 0.1T$, where T is the time of sampling. The entire sample of size N=8. Such choise of parameters for partially coherent useful signal provide substantial spatial non-coherency in the aperture of the receiving antenna array (the correlation coefficient in the extreme elements of the antenna was practically equal to zero, see Fig. 2).

Comparing of the application efficiency for all four statistics V_1, V_2, V_3, V_4 for the useful signal detection was carried out. The threshold values for used GLR test-statistics were calculated using the specified P_{FA} and receiver operating characteristics (probability of the right detection as functions of the signal-to-noise ratio SNR in one antenna element) were constructed.

To explore the dependence of the detection characteristics from the fluctuations of the angle of arrival for each statistics were considered 4 cases: variance of the angle of arrival varied from 0.01 to 0.5 square radians (which corresponds the standard deviation changing from 5.7 to 40.5 degrees). On Fig. 4 and Fig. 6 the receiver operating characteristics for all four statistics at different dispersions on homogeneous and unhomogeneous background noise respectively shows. On
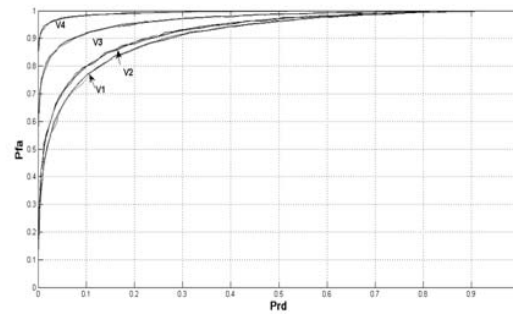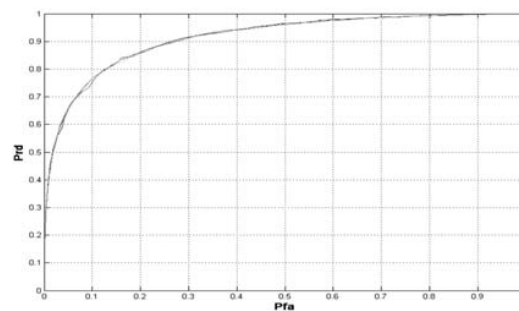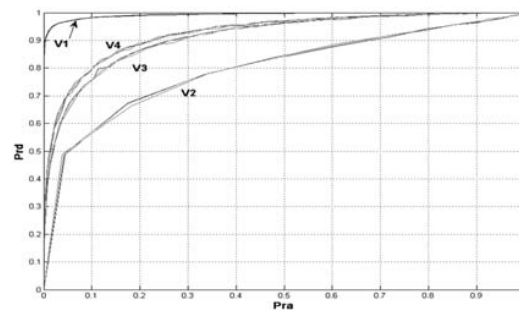
Fig. 5 and Fig.7 these plots can examine in detail for statistics V_2 taken for the example (for other statistics dependence on variance amendments is similar). It is obvious that the degrade of the characteristics of detection for each test-statistic occurs only with the change of the degree of heterogeneity of noise. This fact confirms earlier studies, see [19]. But the fluctuations of the angle of arrival have weak influence on the receiver operating characteristics for of GLRT-statistics. This is quite an unexpected result.
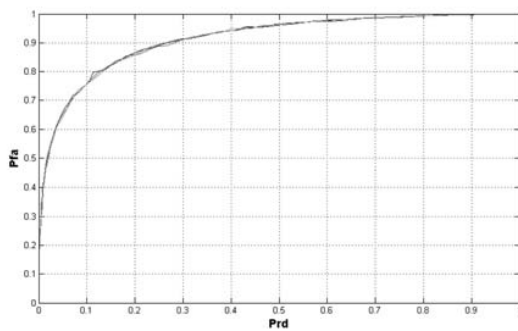
Fig. 7. Receiver operating characteristics for V2 separately for plane wave with a different variance of fluctuating arrival angle on the unhomogeneous noise background

IV. CONCLUSION

In the present work performance degradation for the GLR test-statistics for the patially coherent spatial signal detection was discussed. The detailed comparison of test-statistics is completes a series of studies, starting from a basic case of completely coherent to essential non-coherent signal. The model of the patially coherent spatial signal was a plane wave with a fluctuating arrival angle. Two types of the noise background, homogeneous and unhomogeneous, considered. Studies have shown a weak dependence of the detection characteristics (receiver operating characteristics) for of detecting the degree of signal non-coherency (degree of coherence is caused by phase changes).

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