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Stress Analysis of Laminated Cylinders Subject to the Thermomechanical Loads

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Abstract**—**In this study, thermo elastic stress analysis is performed on a cylinder made of laminated isotropic materials under thermomechanical loads. Laminated cylinders have many applications such as aerospace, automotive and nuclear plant in the industry. These cylinders generally performed under thermomechanical loads. Stress and displacement distribution of the laminated cylinders are determined using by analytical method both thermal and mechanical loads. Based on the results, materials combination plays an important role on the stresses distribution along the radius. Variation of the stresses and displacements along the radius are presented as graphs. Calculations program are prepared using MATLAB® by authors.

Keywords**—**Isotropic materials, laminated cylinders, thermoelastic stress, thermomechanical load.

I. Introduction

DEVELOPMENTS in the modern technologies demand the invention of the new materials. These materials are assumed to work under the various loading types. The accurate control of the stress, temperature and strain distribution of materials is significant for an effective design.

Cylinders are widely used as structural components in the mechanical parts such as aircraft engines and chemical plants [1], [2]. Thus accurate prediction of the material behavior is significant. Several methods have been used to determine the stresses in cylinders. Zhang and Hoa presented the stress analysis of composite cylinders by using Taylor series expansion to solve the equations [3]. Fourier integral transformation is used to solve the stress problem for a long hollow cylinder under the residual strains [4]. Zimmerman and Lutz derived the solutions of thermal stresses in circular cylinders by using Frobenius series method [5]. Stress analysis is one of the major problems in engineering field and studied by many researchers [6]-[10]. Afzall et al. analyzed the temperature distribution of the cylinders based on finite element analysis [11]. Su and Bhuyan studied the elastic stress analysis and deformation analysis of steel cylinder using three dimensional finite element model. The thermal stress analysis of the cylinders based on ANSYS is performed in this study [12]. Laplace transform technique is employed to solve the ordinary differential equations. Elastic-plastic stress analysis of a cylinder with fix ends is examined in [13]. Their analyses were based on Tresca's yield criterion. Sayman studied the stress analysis of composite cylinders subjected to the hygrothermal loadings [14]. The results are carried out for plane strain assumption and checked with the finite element method. Jabbari et al. investigated the analytical solution to calculate the thermal and mechanical stresses in a hollow cylinder [15]. Direct method is used to obtain the solutions.

The aim of this study is to perform elastic stress analysis of laminated cylinders subjected to the mechanical, thermal and thermomechanical loads. Material properties are assumed to vary along to the radial direction. Based on the analytical results, stresses and displacement are affected significantly by the material combination.

II. Analysis

Governing differential equations of equilibrium for cylinders in 2D are [16];

$$\begin{aligned} &\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + R = 0 \\ &\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{2\tau_{r\theta}}{r} + R = 0 \end{aligned} \qquad (1)$$

Stress components are independent of θ because of the symmetry and can be only functions of r. Also, the shear stress, $\tau_{r\theta}$, equals to zero. Hence, (1) with mass force becomes to

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} + \rho(r)\omega^2 r = 0 \qquad (2)$$

where σ_r and σ_θ are radial and circumferential stresses, respectively. $\rho(r)$ is the density of the material in the radial direction and ω is the angular velocity.

For plain strain case, the relation between strain and stress under the temperature can be described by Hooke's Law

$$\begin{aligned} \varepsilon_r &= \frac{1+\nu}{E(r)}\left[(1-\nu)\sigma_r - \nu\sigma_\theta + E(r)\alpha(r)T(r)\right] \\ \varepsilon_\theta &= \frac{1+\nu}{E(r)}\left[(1-\nu)\sigma_\theta - \nu\sigma_r + E(r)\alpha(r)T(r)\right] \end{aligned} \qquad (3)$$

where ν, E, α, T are Poisson's ratio, elasticity modulus, thermal expansion coefficient and temperature, respectively.

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Strain-displacement relations are the following;

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r} \tag{4}$$

where u is the displacement component in the radial direction. The compatibility equation can be expressed as,

$$\varepsilon_r = \frac{d}{dr}\left(r\,\varepsilon_\theta\right) \tag{5}$$

stress components for rotating cylinder in terms of the stress function can be described as

$$\sigma_r = \frac{F}{r}, \quad \sigma_\theta = \frac{dF}{dr} + \rho\omega^2 r^2, \sigma_r = \frac{F}{r} = C_1 + \frac{C_2}{r^2} + Ar^2 \tag{6}$$

applying (6) into the (3) and then substituted into the (5) gives

$$r^2\frac{d^2F}{dr^2} + r\frac{dF}{dr} - F = -\rho(r)\omega^2 r^3\left(\frac{\nu}{1-\nu}+3\right) - \frac{E(r)\alpha(r)}{1-\nu}r^2T'(r) \tag{7}$$

Equation (7) is the second order differential equation and applying r = e^t transformation yields

$$\frac{d^2F}{dt^2} - F = -\rho(r)\omega^2 e^{3t}\left(\frac{\nu}{1-\nu}+3\right) - \frac{E(r)\alpha(r)}{1-\nu}e^{2t}T'(r) \tag{8}$$

A. Analysis for Mechanical Loading

For rotating cylinder which has an angular velocity, (8) reduces to

$$\frac{d^2F}{dt^2} - F = -\rho(r)\omega^2 e^{3t}\left(\frac{\nu}{1-\nu}+3\right) \tag{9}$$

Equation (9) is solved for F and the stress function, F, can be written as

$$F = C_1 r + \frac{C_2}{r} + Ar^3 \tag{10}$$

where A is

$$A = -\frac{\rho\omega^2(\frac{\nu}{1-\nu}+3)}{6} \tag{11}$$

and stress components are

$$\sigma_r = \frac{F}{r} = C_1 + \frac{C_2}{r^2} + Ar^2 \tag{12}$$

$$\sigma_\theta = \frac{dF}{dr} + \rho\omega^2 r^2 = -\frac{C_2 - r^2C_1 - 3r^4A}{r^2} + \rho\omega^2 r^2 \tag{13}$$

to determine C_1 and C_2, the boundary conditions are

$$\sigma_r = 0, \ r = r_i \text{ and } \sigma_r = 0, \ r = r_o \tag{14}$$

where r_i and r_o are inner and outer radius of laminated cylinders, respectively. By using these conditions

$$C_1 = \frac{\rho\omega^2(r_o^{\,2} + r_i^2)(2\nu-3)}{6(\nu-1)} \tag{15}$$

$$C_2 = -\frac{\rho\omega^2 r_o^{\,2} r_i^2)(2\nu-3)}{6(\nu-1)} \tag{16}$$

B. Analysis for Thermal Loading

Under the linearly-decreasing temperature which is

$$T = T_0(\frac{r_o - r}{r_o - r_i}) \tag{17}$$

for thermal loading (8) becomes

$$\frac{d^2F}{dt^2} - F = -\frac{E(r)\alpha(r)}{1-\nu}e^{2t}T'(r) \tag{18}$$

Equation (18) is solved for F and the stress function, F, can be written as

$$F = C_1 r + \frac{C_2}{r} + Sr^2 \tag{19}$$

where S is

$$S = -\frac{E\alpha T_0}{3(1-\nu)(r_o - r_i)} \tag{20}$$

and the stress components are

$$\sigma_r = \frac{F}{r} = C_1 + \frac{C_2}{r^2} + Sr \tag{21}$$

$$\sigma_\theta = \frac{dF}{dr} = C_1 - \frac{C_2}{r^2} + 2Sr \tag{22}$$

using the boundary conditions (14), C_1 and C_2 are determined as

$$C_1 = \frac{ET_0\alpha(r_o^2 + r_o r_i + r_i^2)}{3(r_o^2 - r_i^2)(\nu-1)} \tag{23}$$

$$C_2 = -\frac{ET_0\alpha(r_o^2 r_i^2)}{3(r_o^2 - r_i^2)(\nu-1)} \tag{24}$$

C. Analysis for Thermomechanical Loading

For rotating cylinder subject to the linearly-decreasing temperature (8) is solved for F and the stress function, F, can be written as

$$F = C_1 r + \frac{C_2}{r} + Kr^3 + Lr^2 \tag{25}$$

where K and L are

$$K = -\frac{\rho\omega^2\left(\frac{\nu}{1-\nu}+3\right)}{8}, \quad L = \frac{E\alpha T_0}{3(1-\nu)(r_o - r_i)} \tag{26}$$

and stress components are

$$\sigma_r = \frac{F}{r} = C_1 + \frac{C_2}{r^2} + Kr^2 + Lr \tag{27}$$

$$\sigma_\theta = \frac{dF}{dr} + \rho\omega^2 r^2 = C_1 - \frac{C_2}{r^2} + 3Kr^2 + 2Lr + \rho\omega^2 r^2 \tag{28}$$

using the boundary conditions C_1 and C_2 are determined as

$$C_1 = \frac{9\rho r_i^4\omega^2 - 9\rho r_o^4\omega^2 + 8ET_0\alpha r_o^2 + 8ET_0\alpha r_i^2 - 6r_o^4\rho\nu\omega^2 - 6r_i^4\rho\nu\omega^2 + 8ET_0\alpha r_o r_i}{24(r_o^2 - r_i^2)(\nu-1)} \tag{29}$$

$$C_2 = \frac{r_o^2 r_i^2\left(8ET_0\alpha - 9\rho r_o^2\omega^2 + 9\rho r_i^2\omega^2 + 6r_o^2\rho\nu\omega^2 - 6r_i^2\rho\nu\omega^2\right)}{24(r_o^2 - r_i^2)(\nu-1)} \tag{30}$$

radial displacement can be written as

$$\varepsilon_\theta = r\left[\frac{1+\nu}{E(r)}\left((1-\nu)\sigma_\theta - \nu\sigma_r + E(r)\alpha(r)T(r)\right)\right] \tag{31}$$

III. Results and Discussions

In this paper, stress analyses of laminated cylinders are investigated analytically subject to the mechanical, thermal and thermomechanical loads, respectively. Mechanical properties of the cylinder such as Poisson's ratio, Young's modulus, density and thermal expansion coefficient are given in Table I. The inner radius of the cylinder (r_i) is 20 mm and the outer radius of the cylinder (r_o) is 100 mm.

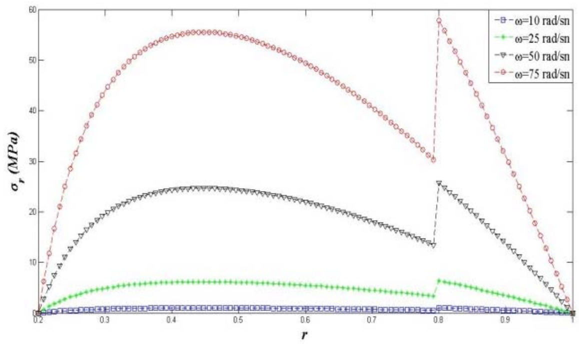
TABLE I
Material Properties of the Cylinder

Material Number	E (MPa)	α (1/°C)	ν	ρ (kg/m^3)
I (20-80 mm)	70000	23×10^{-6}	0.3	2700
II (80-100 mm)	151000	10×10^{-6}	0.17	5700

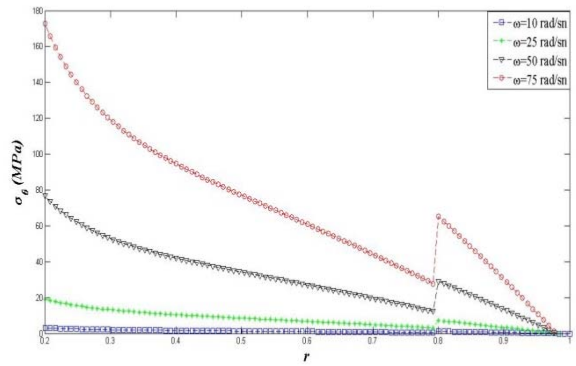
A. Results for Mechanical Loading

In Fig. 1, variation of the radial stress, circumferential stress, longitudinal stress and displacement along the cylinder subjected to the inertia force are presented (ω=10,25,50,75 rad/sn).

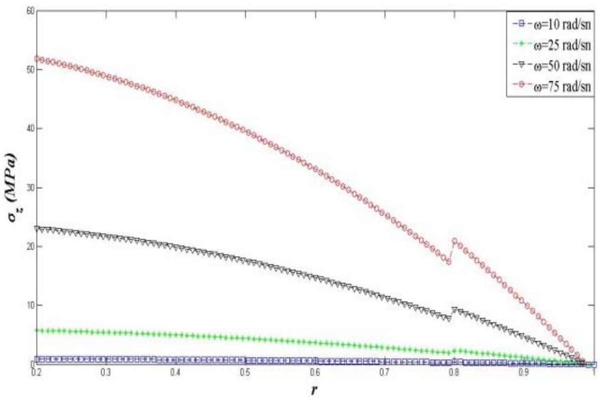
As seen from Fig. 1 (a), radial stress value is proportional to the angular velocity. The radial stress is zero inside and outside of the cylinder for different angular velocity values. Maximum stress occurs at the beginning of the second material. The stresses are tensile stresses. Stress values jump at the transition from first material to the second material. Fig. 1 (b) shows that circumferential stresses are the maximum at the beginning of the cylinder and decrease gradually up to the second material. Stresses increase sharply at the transition to the second material. Fig. 1 (c) demonstrates that longitudinal stresses decrease gradually from inside of the cylinder to the outside of the cylinder. It's clear from Fig. 1 (d) that, the displacement decreases gradually from first material to the second material. The radial displacement reaches the minimum value at the outside of the cylinder.



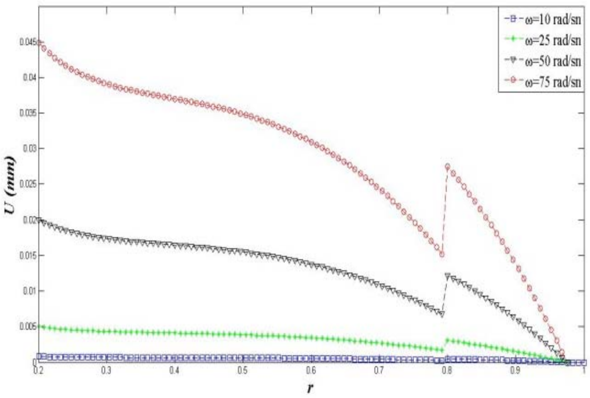
(a)



(b)



(c)



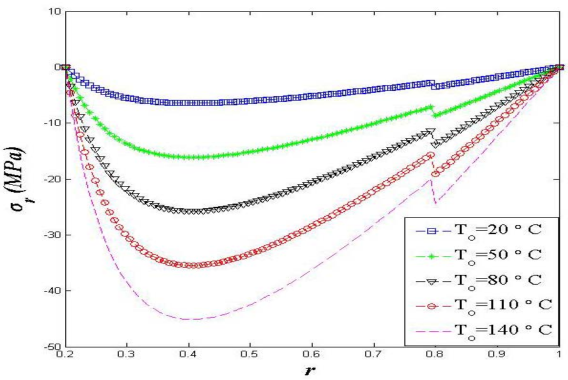
(d)

Fig. 1 Variations of the radial stresses (a), circumferential stresses (b), longitudinal stresses(c), displacement (d) subjected to inertia force due to rotating, ω=10,25,50,75 rad/sn

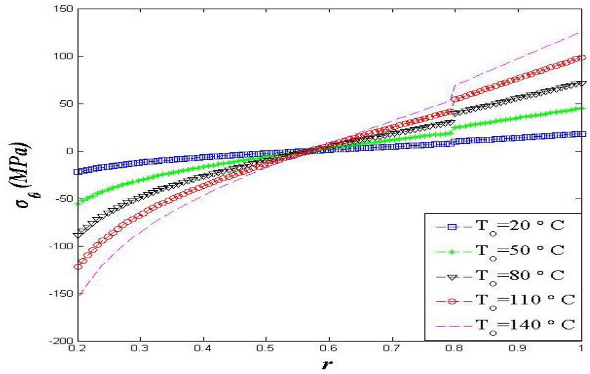
B. Results for Thermal Loading

In Fig. 2, variations of the radial stresses, circumferential stresses, longitudinal stresses and displacements subjected to the variable temperatures, T_0=20, 50, 80, 110, 140ºC, are presented.

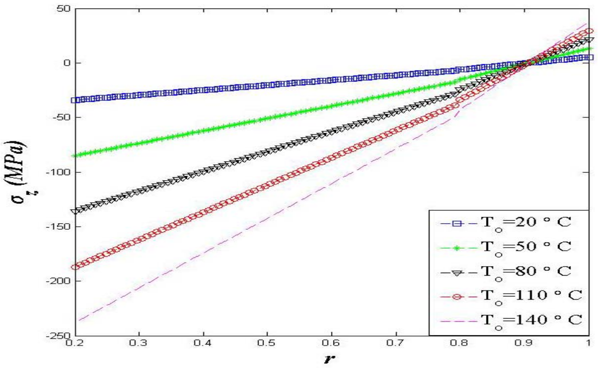
Fig. 2 (a) shows that the radial stresses are compressive along to the cylinder. Radial stress value starts from zero and ends at zero for each temperature values. It is clear from Fig. 2 (b) that, the circumferential stresses are compressive at the inner surface and becomes tensile at the outer surface of the cylinder. Fig. 2 (d) shows that, the radial displacement increases gradually and reaches the maximum value at the outer surface of the cylinder. There is a harp increase at the beginning of the second material.



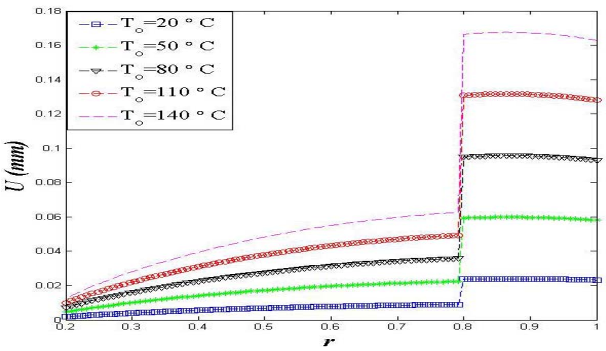
(a)



(b)



(c)



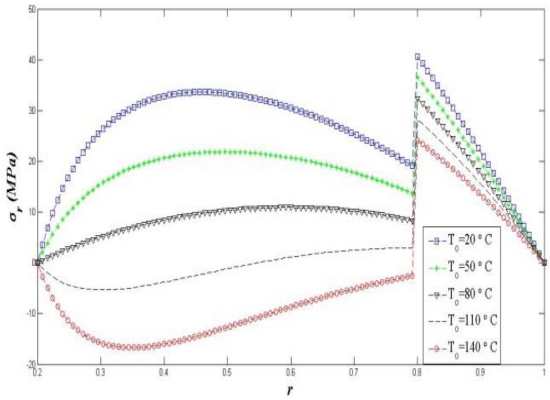
(d)

Fig. 2 Variations of the radial stresses (a), circumferential stresses (b), longitudinal stresses (c), displacement (d) subjected to variable temperatures, T_0=20,50,80,110,140 ºC

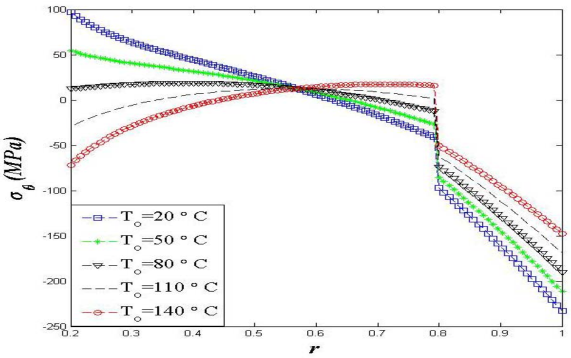
C. Results for Thermomechanical Loading

In Fig. 3, variations of the radial stresses, circumferential stresses, longitudinal stresses and displacement subjected to inertia force due to rotating, ω=75 rad/sn and subjected to variable temperatures, T_0=20, 50, 80, 110, 140ºC, are presented.

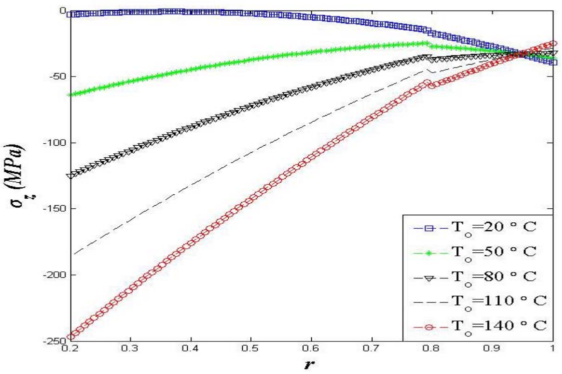
As seen in Fig. 3 (a), the radial stresses are tensile for T_0=20, 50 and 80ºC. On the contrary, the radial stresses are compressive for T_0=110, 140ºC up to the second material. There is a sharp increase at the beginning of the second material and stresses become tensile. In addition, radial stresses are zero at the inner and outer surfaces of the cylinder. Fig. 3 (c) shows that longitudinal stresses are compressive along the cylinder for each temperature value. Fig. 3 (d) shows the distribution of the radial displacement. Radial displacement increases gradually and starts to decrease at some point up to the second material for each temperature value. There is a sharp fall at the beginning of the second material and then radial displacement continues to decline.



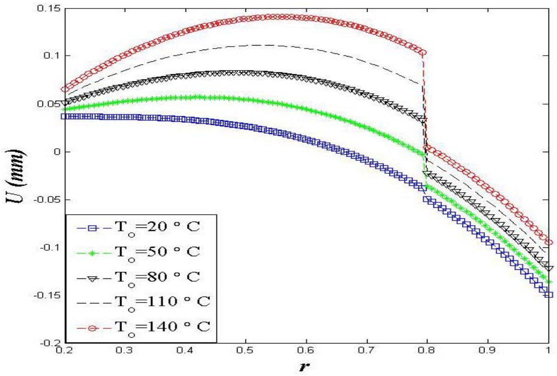
(a)



(b)



(c)



(d)

Fig. 3 Variations of the radial stresses (a), circumferential stresses (b), longitudinal stresses (c), displacement (d) subjected to inertia force due to rotating, ω=75 rad/sn and subjected to variable temperatures, T_0=20, 50, 80, 110, 140 ºC

IV. Conclusions

This paper has presented the elastic stress analysis of laminated cylinders subject to the mechanical, thermal and thermomechanical loads. Numerical analyses of the cylinders lead to the following observations:

- Material properties, inertia force and temperature have significant effects on the stress distribution.
- Radial stresses are tensile when subjected to the mechanical loading. However, radial stresses are compressive under the thermal loading. Furthermore, radial stresses start from zero value and ends at the zero value under mechanical, thermal and thermomechanical loading.
- The radial displacements reach the minimum values at the inner surface and reach the maximum values at the outer surface of the cylinder under thermal and thermomechanical loads.

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