

# Thinned Elliptical Cylindrical Antenna Array Synthesis Using Particle Swarm Optimization

Rajesh Bera, Durbadal Mandal, Rajib Kar, Sakti P. Ghoshal

**Abstract**—This paper describes optimal thinning of an Elliptical Cylindrical Array (ECA) of uniformly excited isotropic antennas which can generate directive beam with minimum relative Side Lobe Level (SLL). The Particle Swarm Optimization (PSO) method, which represents a new approach for optimization problems in electromagnetic, is used in the optimization process. The PSO is used to determine the optimal set of ‘ON-OFF’ elements that provides a radiation pattern with maximum SLL reduction. Optimization is done without prefixing the value of First Null Beam Width (FNBW). The variation of SLL with element spacing of thinned array is also reported. Simulation results show that the number of array elements can be reduced by more than 50% of the total number of elements in the array with a simultaneous reduction in SLL to less than -27dB.

**Keywords**—Thinned array, Particle Swarm Optimization, Elliptical Cylindrical Array, Side Lobe Label.

## I. INTRODUCTION

USUALLY the radiation pattern of a single element is relatively wide and each element provides low values of directivity. Antenna arrays increase the directivity without enlarging the size of single element. Generally, the overall array properties such as directivity and gain, direction of maximum directivity, SLL, half power beam-width etc. can be controlled and optimized by adjusting the number of elements, the spacing between them, their excitation coefficients, their relative phases, the geometrical configuration of the overall array (linear, circular, elliptical, etc.) and the relative pattern of the individual elements [1].

Nowadays, one of the most recent innovations for wireless communications to overcome the problem of increasing demand for capacity is to deploy smart antenna [2], which can be visualized as the antenna directing a beam toward the communication partner only. Smart antennas not only increase the capacity, they have also potential to introduce new services, increased range, more security, reduced multipath propagation etc. Narrower main-beam and more nulls in the pattern can resolve the Signals-Of-Interest (SOI) more accurately and allow the smart antenna system to reject more Signals-Not-Of-Interest (SNOI).

A linear array has excellent directivity and it can form the narrowest main-lobe in a given direction, but it does not work well in all azimuth directions. Since a circular array does not have edge elements, directional patterns synthesized with a circular array can be electronically rotated in the plane of the array without a significant change of the beam shape [3]. Also

circular array pattern has no nulls in azimuth plane [1]. In smart antenna applications to reject SNOI the array pattern should have several nulls in the azimuth plane. This can be implemented by the use of elliptical arrays instead of circular arrays [4].

The circular array is of high side-lobe geometry. If the distance of array elements is decreased to reduce the side lobes, the mutual coupling influence becomes more significant. For mitigating high side-lobe levels, concentric arrays are utilized in [5]. Also concentric circular array antennas have several advantages including the flexibility in array pattern synthesis and design both in narrowband and broadband beam forming applications [6].

Thinning an array means switching off some elements in a uniformly spaced or periodic array to generate a pattern with low SLL. In the proposed method, the locations of the elements are fixed and all the elements have two states, either “on” or “off” (Similar to Logic “1” and “0” in digital domain), depending on whether the element is connected to the feed network or not. In the “off” state, either the element is passively terminated to a matched load or open circuited. If there is no matching between the elements, it is equivalent to removing “off” element from the array. There are many published articles [7]-[9] dealing with the synthesis of thinned array. Element behavior in a thinned array is described in [10].

Classical optimization methods have several disadvantages such as: i) highly sensitive to starting points when the number of solution variables and hence the size of the solution space increase, ii) frequent convergence to local optimum solution or divergence or revisiting the same suboptimal solution etc. But, there are various evolutionary optimization tools for thinning, such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO) [11]-[13] etc. which do not suffer from above disadvantages. The PSO algorithm has proven to be a better alternative to other evolutionary algorithms such as Genetic Algorithms (GA), Ant Colony Optimization (ACO) etc. for optimal design of antenna array.

## II. DESIGN EQUATION

In this section, through the analysis of Elliptical and Concentric Elliptical array factor, the expression for the array factor of ECA is derived by the combination of linear and elliptical array properties.

We know, the array factor of elliptical array [4],

$$AF(\theta, \varphi) = \sum_{n=1}^N A_n e^{jk \sin \theta (a \cos \varphi_n \cos \varphi + b \sin \varphi_n \sin \varphi)} \quad (1)$$

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In the structure of ECA, all ellipses are of equal semi-major axis and semi-minor axis  $a$ ,  $b$ , respectively, and they are placed one above the other, with an equal vertical spacing  $d$  between them (Fig. 1). Thus, the elements along a vertical line on the cylinder surface form a linear array and those in a transversal plane constitute an elliptical array. For  $M$  identical elliptical arrays of ECA, the total array factor is obtained by the summation of the  $M$  elliptical array factors given by,

$$AF(\theta, \varphi) = \sum_{m=1}^M \sum_{n=1}^N C_{nm} e^{jk \sin \theta (a \cos \varphi_n \cos \varphi + b \sin \varphi_n \sin \varphi)} \quad (2)$$

If we consider the far-field region, the array factor of each elliptical array is the same as that of the elliptical array in the  $x$ - $y$  plane. However, in the vertical direction there is a phase difference between the elements of a linear array [1], hence

$$C_{nm} = A_n e^{j\alpha_n} \cdot B_m e^{j(m-1)(kd \cos \theta + \beta)} \quad (3)$$

where  $d$  is the spacing between elements laid in vertical direction, the term  $B_m e^{j(m-1)(kd \cos \theta + \beta)}$  arises from the  $m$ -th vertical element and  $A_n e^{j\alpha_n}$  is the excitation coefficient of  $n$ th element of elliptical array.

Substituting (3) into (2) yields

$$AF(\theta, \varphi) = \sum_{m=1}^M B_m e^{j(m-1)(kd \cos \theta + \beta)} \times \sum_{n=1}^N A_n e^{j\alpha_n} e^{jk \sin \theta (a \cos \varphi_n \cos \varphi + b \sin \varphi_n \sin \varphi)} \quad (4)$$

then

$$AF(\theta, \varphi) = \sum_{m=1}^M B_m e^{j(m-1)(kd \cos \theta + \beta)} \times \sum_{n=1}^N A_n e^{j(\alpha_n + k \sin \theta (a \cos \varphi_n \cos \varphi + b \sin \varphi_n \sin \varphi))} \quad (5)$$

we know, the array factor of linear array,

$$AF_{linear} = \sum_{m=1}^M B_m e^{j(m-1)(kd \cos \theta + \beta)} \quad (6)$$

from (1), the array factor of elliptical array,

$$AF_{elliptical} = \sum_{n=1}^N A_n e^{j(\alpha_n + k \sin \theta (a \cos \varphi_n \cos \varphi + b \sin \varphi_n \sin \varphi))} \quad (7)$$

So, from (5), the array factor of ECA can be written as,

$$AF(\theta, \varphi) = AF_{linear} \times AF_{elliptical} \quad (8)$$

This shows that the total array factor of elliptical cylindrical array is the product of linear and elliptical array factors.

Normalized power pattern in dB can be expressed as follows:

$$P(\theta, \varphi) = 10 \log_{10} \left[ \frac{|AF(\theta, \varphi)|}{|AF(\theta, \varphi)|_{\max}} \right]^2 = 20 \log_{10} \left[ \frac{|AF(\theta, \varphi)|}{|AF(\theta, \varphi)|_{\max}} \right] \quad (9)$$

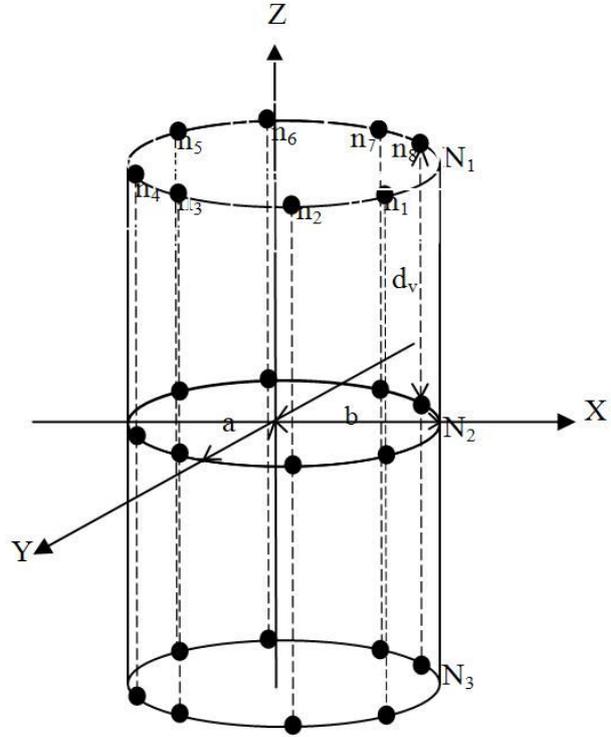


Fig. 1 Elliptical cylindrical array (ECA) structure

### III. PARTICLE SWARM OPTIMIZATION

PSO is a flexible, robust population-based stochastic search or optimization technique with implicit parallelism, which can easily handle with non-differential objective functions, unlike traditional gradient based optimization methods. PSO is less susceptible to getting trapped on local optima unlike GA, Simulated Annealing etc. Eberhart et al. developed PSO concept similar to the behaviour of a swarm of birds. PSO is developed through simulation of bird flocking and fish schooling in multi-dimensional space. Bird flocking optimizes a certain objective function. Each particle knows its best value so far (pbest). This information corresponds to personal experiences of each particle. Moreover, each particle knows the best value so far in the group (gbest) among all pbests. Namely, each particle tries to modify its position using the following information:

- The distance between the current position and the pbest.
- The distance between the current position and the gbest.

After finding the two best values, the particle updates its velocity and positions with following (10) and (11) shown below.

$$V_{j,i+1} = (w_{i+1} * V_{j,i} + C_1 * rnd_1 * (pbest_j - Position_{j,i}) + C_2 * rnd_2 * (gbest_i - Position_{j,i})) \quad (10)$$

$$Position_{j,i+1} = Position_{j,i} + V_{j,i+1} \quad (11)$$

where  $V_{j,i}$  is the velocity of vector  $j$  at iteration  $i$ ;  $w$  is the weighting function;  $C_1$  and  $C_2$  are called social and cognitive constants, respectively;  $rnd_i$  is the random number between 0 and 1;  $Position_{j,i}$  is the current position of vector  $j$  at iteration  $i$ ;  $pbest_j$  is the pbest of vector  $j$ ;  $gbest_i$  is the gbest of the group of vectors at iteration  $i$ . Normally,  $C_1=C_2=1.5-2.05$ .

Inertia weight ( $w_{i+1}$ ) at  $(i+1)$ th cycle is given by

$$w_{i+1} = w_{\max} - \frac{w_{\max} - w_{\min}}{i_{\max}}(i + 1)$$

where  $w_{\max}=1.0$ ;  $w_{\min}=0.4$ ;  $i_{\max}$  = Maximum number of iteration cycles.

The pseudo code of the procedure is as follows

For each particle

Initialize the particle

End

Do

For each particle

Calculate the fitness value

If the fitness value is better than the best fitness value (pBest) in history

Set current value as the new pBest

End

Choose the particle with the best fitness value of all the particles as the gBest

For each particle

Calculate particle velocity according to (10)

Update particle position according to (11)

End

While maximum iterations or minimum error criteria is not attained.

The population size using PSO, by which the antenna array is optimized, is 40 and maximum number of iteration cycle is 60.

Optimization task in the array designing problem is modelled as a min-max problem, i.e., minimization of maximum SLL. Cost Function (CF) is formulated to meet the corresponding designing goal as follow.

$$CF = \min(\max\{20 \log |AF(\theta)|\}) \quad (12)$$

$$\text{Subject to } \theta \in \{-90^\circ, -|FN|^\circ\} \& \{|FN|^\circ, 90^\circ\}$$

where, 'FN' is the first null in degree in a particular array pattern.

The minimum CF values against number of iteration cycles are recorded to get the convergence profile of the algorithm.

#### IV. RESULTS AND DISCUSSIONS

The paper describes the application of particle swarm optimization algorithm for reduction of the maximum Side Lobe Level (SLL) without and with prefixing the value of First Null Beam Width (FNBW) of thinned three-ring uniform Elliptical Cylindrical Arrays (UCCA) of isotropic elements.

In the numerical analysis, we consider an Elliptical Cylindrical Array (ECA) of three identical elliptical rings where each ring having 12 elements is considered in all the cases, i.e.,  $N=12$  and  $M=3$ , so the total number of isotropic elements in such an array is 36. The eccentricity of each ring of the array antenna is made equal to  $e=0.6$ . Here three cases are considered for three different values of inter-element arc spacing in the array. In the first case, spacing between elements in radial direction is  $0.5\lambda$ . Similarly, for the second and third cases, spacing is  $0.6\lambda$  and  $0.7\lambda$ , respectively.

For ECA, all ellipses are of equal semi-major axis  $a$ ,  $b$  and they are placed one above the other, with an equal vertical spacing between them. For each of three cases stated above, vertical spacing is optimized by hit and trial method by fixing other parameters of ECA. In this method, we give ranging the value of vertical spacing ( $d_v$ ) is  $[0.5 \ 1]$  by the equation given below.

$$d_v = (0.5 + \text{rand} * 0.5)\lambda \quad (13)$$

For each of the three cases, after 50 iterations, optimized value of vertical spacing is tabulated in Table I.

Sl. No.	Inter-element spacing( $\lambda$ )	Vertical Spacing( $\lambda$ )
Case I	0.5	0.9295
Case II	0.6	0.9372
Case III	0.7	0.7980

Particle swarm optimization algorithm is used for reduction of the maximum Side Lobe Level (SLL) without prefixing the value of First Null Beam Width (FNBW) of thinned array. Number of 'ON' elements, maximum SLLs and FNBWs for thinned array and fully populated array are tabulated in Table II for various element spacing.

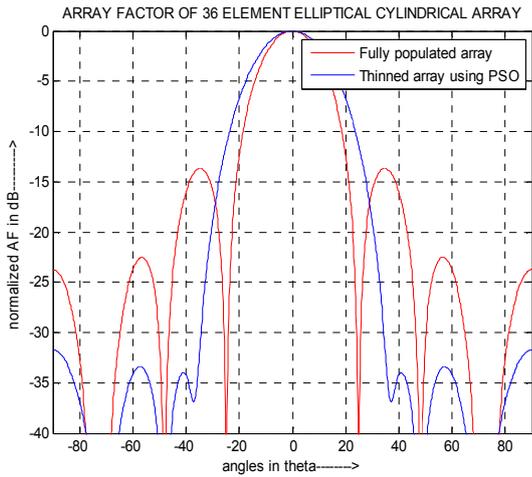


Fig. 2 Normalized absolute power patterns in dB for fully populated array and thinned array with fixed inter-element spacing  $d=0.5\lambda$

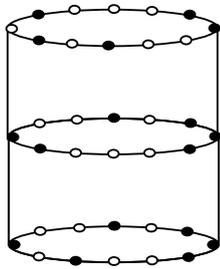


Fig. 3 PSO optimized thinned array (47.22% filling)

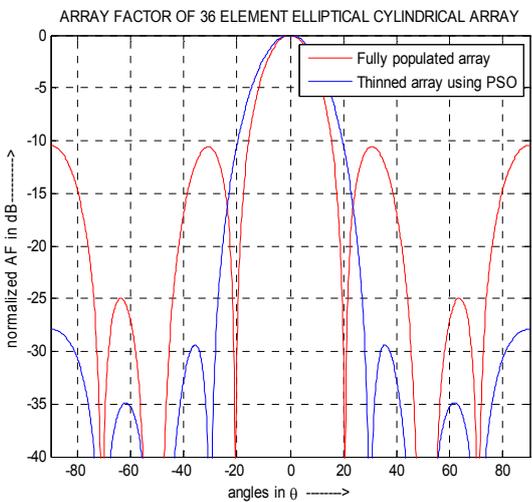


Fig. 4 Normalized absolute power patterns in dB for fully populated array and thinned array with fixed inter-element spacing  $d=0.6\lambda$

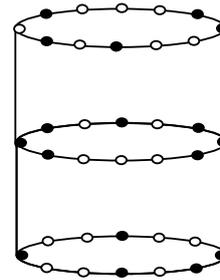


Fig. 5 PSO optimized thinned array (47.22% filling)

In the first case, i.e., element spacing  $d=0.5\lambda$ , the value of maximum SLL using PSO is -31.72 dB, shown in Fig. 2 and the corresponding array arrangement shown in Fig. 3. In the second case, i.e.  $d=0.6\lambda$ , maximum SLL is -27.89 dB which is shown in Fig. 4 and the corresponding array arrangement shown in Fig. 5. In third case, Figs. 6 and 7 show the maximum SLL which is -29.81dB and corresponding array arrangement, respectively.

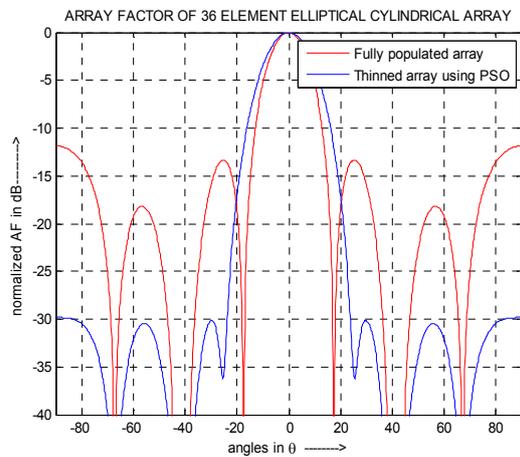


Fig. 6 Normalized absolute power patterns in dB for fully populated array and thinned array with fixed inter-element spacing  $d=0.7\lambda$

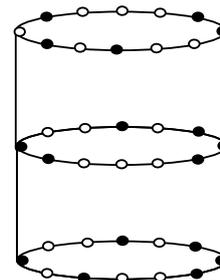


Fig. 7 PSO optimized thinned array (47.22% filling)

TABLE II  
PERFORMANCE OF THINNED ECA FOR VARIOUS ELEMENT SPACING

Sl. No.	Number of 'ON' elements		Maximum SLL(dB)		First Null Beam Width(FNBW) in degree	
	Thinned Array	Fully Populated Array	Thinned Array	Fully Populated Array	Thinned Array	Fully Populated Array
Case I	17	36	-31.72	-13.68	74.12	49.80
Case II	17	36	-27.89	-10.44	60.00	41.08
Case III	17	36	-29.81	-13.36	50.84	34.82

The filled ratio of the array is also calculated. Filled ratio is the ratio of the number of 'ON' elements to the total number of elements in the array.

The minimum CF values against number of iteration cycles are recorded to get the convergence profile of each set of element spacing. Fig. 8 shows the convergence profile of PSO. All computations were done in MATLAB 7.5 on core (TM) 2 duo processor, 3.00 GHz with 4 GB RAM.

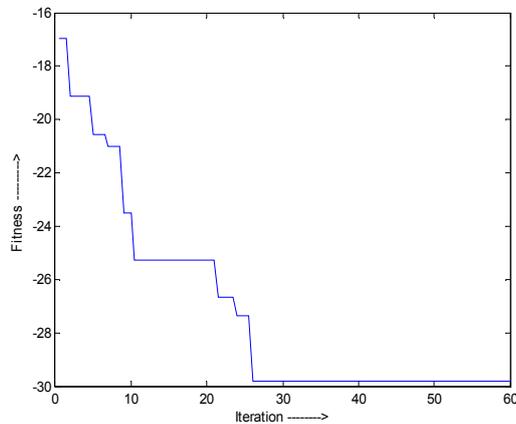


Fig. 8 Convergence curve of the thinned array shown in Fig. 7 using PSO

## V. CONCLUSIONS

In this paper, thinning of one hybrid elliptical antenna array called Elliptical Cylindrical Array (ECA) is considered for three different values of inter-element spacing ( $d=0.5\lambda$ ,  $0.6\lambda$  and  $0.7\lambda$ ) in an ellipse. We found that if the spacing between elements in an ellipse is about  $0.8\lambda$  or more, then the grating lobes appear. The PSO algorithm can efficiently handle the thinning of a ECA of 36 isotropic elements with a reduction to more than 50% of the total elements as used in case of fully populated array with a simultaneous reduction in SLL to less than  $-27\text{dB}$  with compromise of FNBW. The comparison shows a significant improvement for SLL with significant reduction in the number of elements which will reduce the cost of designing the antenna array substantially.

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