

Mathematical Model for Progressive Phase Distribution of Ku-band Reflectarray Antennas

M. Y. Ismail, M. Inam, A. F. M. Zain, N. Misran

Abstract—Progressive phase distribution is an important consideration in reflectarray antenna design which is required to form a planar wave in front of the reflectarray aperture. This paper presents a detailed mathematical model in order to determine the required reflection phase values from individual element of a reflectarray designed in Ku-band frequency range. The proposed technique of obtaining reflection phase can be applied for any geometrical design of elements and is independent of number of array elements. Moreover the model also deals with the solution of reflectarray antenna design with both centre and off-set feed configurations. The theoretical modeling has also been implemented for reflectarrays constructed on 0.508mm thickness of different dielectric substrates. The results show an increase in the slope of the phase curve from 4.61°/mm to 22.35°/mm by varying the material properties.

Keywords—Mathematical modeling, Progressive phase distribution, Reflectarray antenna, Reflection phase.

I. INTRODUCTION

THE two main components of a reflectarray antenna are the feed antenna and the planar reflector. Reflectarray feed can be located in the centre or at an off-set position. The feed antenna used is based on the same technology as the feed horn used in parabolic reflectors. However the planar reflector concept is relatively new which started with the work of D.G. Berry, R.G. Malech and W.A. Kennedy in 1963 [1]. The individual elements of the periodic array have to be designed in such a way that they can convert a spherical beam into a planar wave front. The required reflection phase from an individual element of an array also depends on the location of the feed horn. For proper phase requirements, different techniques such as, identical patches of variable-length stubs [2], square patches of variable sizes [3], identical planar elements of variable rotation [4] and identical rectangular patches with different types of slot configurations have been used [5], [6]. All these phasing techniques increase the possibility of reflectarrays to become an alternative option to the parabolic reflectors. However one of the main concerns of a reflectarray antenna is its limited bandwidth performance as compared to the parabolic reflector antennas [7]-[9].

Different configurations have been proposed by researchers in the past few years for the bandwidth improvement of reflectarray antennas [10]-[12] but considerable efforts are still

required for reflectarrays to reach the parabolic reflector bandwidth.

This work provides a simple and detailed technique for the design of a reflectarray with progressive phase distribution. The algorithm provides an opportunity to design reflectarrays with any configuration and hence offers an opening to improve the reflectarray bandwidth. For the comparison, the reflectarray bandwidth is characterized in this work using FoM which is slope of the reflection phase curve.

II. THEORETICAL MODEL

The basic design principle of reflectarray requires the phase Ψ_i of the field reflected from the element to be chosen in such a way that the total phase delay from the feed to a fixed aperture plane in front of reflectarray is constant for all elements [13]. This constant total phase delay can be obtained by the progressive phase distribution of the reflectarray elements and can be given by:

$$k_0(R_i - \bar{r}_i \cdot \hat{r}_0) - \Psi_i = 2\pi N \quad (1)$$

where; k_0 is free space wave number at design frequency f_0 , R_i is the distance from the phase centre of feed to the centre of the i^{th} element, \bar{r}_i is the position vector of the centre of i^{th} element from array centre and N is an integer.

For the design and analysis of a reflectarray, a plane incident wave can be used which is given by;

$$\bar{E}_i = \bar{E}_0 e^{jk_0(xu_i + yv_i - z\cos\theta_i)} \quad (2)$$

where; \bar{E}_0 defines amplitude and polarization of the incident field and u_i and v_i are direction cosines of the wave which are given by:

$$u_i = \sin\theta_i \cos\varphi_i \quad (3)$$

$$v_i = \sin\theta_i \sin\varphi_i \quad (4)$$

When an incident electric field of the form given in (2) is impinged on a unit cell without patch element, the specular reflection from the ground plane occurs with reflection coefficient ($\bar{\Gamma}$) as:

$$\bar{\Gamma}(\theta_i, \varphi_i) = \bar{R}(\theta_i, \varphi_i) \quad (5)$$

While in the presence of a patch element with dimension L and W (length and width respectively), the reflection coefficient has an additional scattered component and can be

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written as:

$$\bar{\Gamma}(\theta_i, \varphi_i) = \bar{R}(\theta_i, \varphi_i) + \bar{S}(\theta_i, \varphi_i, L_i, W_i) \quad (6)$$

where; θ_i and φ_i are the two components of reflection phase while L_i and W_i are length and width of the i^{th} (rectangular or square) patch element respectively. It can be observed from (6) that the dyad \bar{R} does not need the factors L_i and W_i which are needed to be included in dyad \bar{S} . This is because reflection from ground plane depends only on the properties of dielectric substrate and ground plane while the scattered field depends on the patch element dimensions. Therefore the total electric field becomes:

$$\bar{E}_t = \bar{E}_i + \bar{E}_r + \bar{E}_s \quad (7)$$

where; \bar{E}_i is the incident field given by (2) while \bar{E}_r and \bar{E}_s are the reflected and scattered electric fields respectively which can be obtained by using (8) and (10).

$$\bar{E}_r = \bar{R}(\theta_i, \varphi_i) \cdot \bar{E}_i \quad (8)$$

$$\bar{E}_r = \bar{R}(\theta_i, \varphi_i) \cdot \bar{E}_0 e^{jk_0(xu_i + yv_i - z\cos\theta_i)} \quad (9)$$

$$\bar{E}_s = \bar{S}(\theta_i, \varphi_i, L_i, W_i) \cdot \bar{E}_i \quad (10)$$

$$\bar{E}_s = \bar{S}(\theta_i, \varphi_i, L_i, W_i) \cdot \bar{E}_0 e^{jk_0(xu_i + yv_i - z\cos\theta_i)} \quad (11)$$

Equations (9) and (11) can further be expanded into θ and φ components as:

$$\begin{bmatrix} E_{r\theta} \\ E_{r\varphi} \end{bmatrix} = \begin{bmatrix} R_{\theta\theta} & 0 \\ 0 & R_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} E_{0\theta} \\ E_{0\varphi} \end{bmatrix} e^{jk_0(xu_i + yv_i - z\cos\theta_i)} \quad (12)$$

$$\begin{bmatrix} E_{s\theta} \\ E_{s\varphi} \end{bmatrix} = \begin{bmatrix} S_{\theta\theta} & S_{\theta\varphi} \\ S_{\varphi\theta} & S_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} E_{0\theta} \\ E_{0\varphi} \end{bmatrix} e^{jk_0(xu_i + yv_i - z\cos\theta_i)} \quad (13)$$

where; $R_{\theta\theta}, R_{\varphi\varphi}, S_{\theta\theta}, S_{\theta\varphi}, S_{\varphi\theta}$ and $S_{\varphi\varphi}$ are the plane wave coefficients of reflected and scattered electric field components. Using (8) to (11) with (7), total electric field can now be written as:

$$\bar{E}_t = [1 + \bar{R}(\theta_i, \varphi_i) + \bar{S}(\theta_i, \varphi_i, L_i, W_i)] \cdot \bar{E}_0 e^{jk_0(xu_i + yv_i - z\cos\theta_i)} \quad (14)$$

In the case of a waveguide simulator technique, general relation for the total electric field can be given as:

$$\bar{E}_{tw} = \bar{G} \cdot \bar{J} + \bar{E}_t \quad (15)$$

where; \bar{G} is the Green's function and \bar{J} is the current density. If the electric fields are excited in the Y-direction then the electric field for the wave guide simulator can be given by:

$$\bar{E}_{tw} = \bar{G}_{YY} \cdot \bar{J}_Y + \bar{E}_{Yinc}(l + \bar{G}_{YY}) \quad (16)$$

where; l is the length of the unit cell patch element and \bar{J}_Y can be given by:

$$\bar{J}_Y = \sum_n A_n \varphi_n(x, y) \quad (17)$$

where; A_n is the unknown vector coefficient and φ_n is the required phase from an individual patch element of a reflectarray in order to form a progressive phase distribution.

III. PROGRESSIVE PHASE DISTRIBUTION

In (17), φ_n has both x and y components. In this work, φ_n will be calculated for the elements placed on the $Y=0$ line. Based on the required phase shift of reflectarray elements shown in Fig. 1 and provided in [14]-[16], it has been observed that the required phase shift remains constant for elements lying on circles of radius (r) in a periodic array. Hence if φ_n is calculated for the elements lying on the X -axis with radius $r = x_i$, the required phase shift of all the elements of the periodic array can be effectively approximated. In order to calculate the phase shift for the elements on X -axis trigonometric identities can be used as:

$$\varphi = -\frac{2\pi}{3} \cot^{-1} \frac{f}{x_i} \quad (18)$$

where; f is the vertical distance of feed from surface of the array and x_i is the distance between the center of i^{th} element and the point perpendicular to the feed and φ is in degrees. Once φ is calculated for different values of $x=x_i, y=0$, the phase shift for all the array elements can be obtained. This method simplifies the calculation of the required phase shift from each of the array elements and reduces the complexity and time required for the periodic reflectarray design. This technique can also be used for the progressive phase distribution of off-set feed reflectarrays.

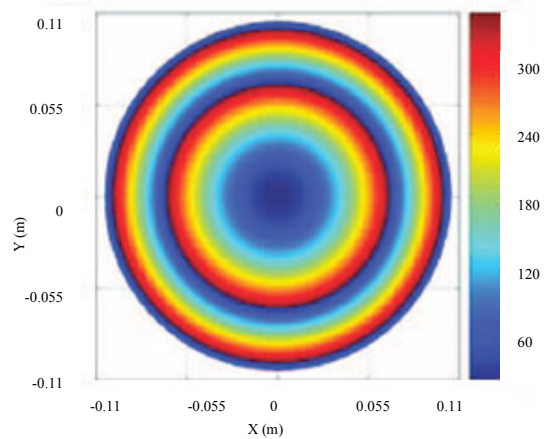


Fig. 1 Required phase shift at reflectarray surface using ray tracing [14]

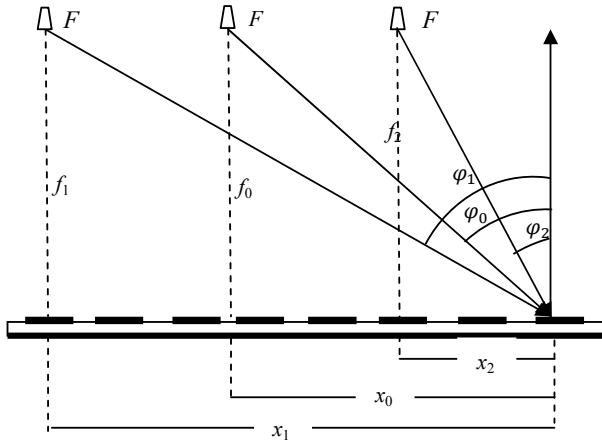


Fig. 2 Reflection phase from different feed points in a reflectarray

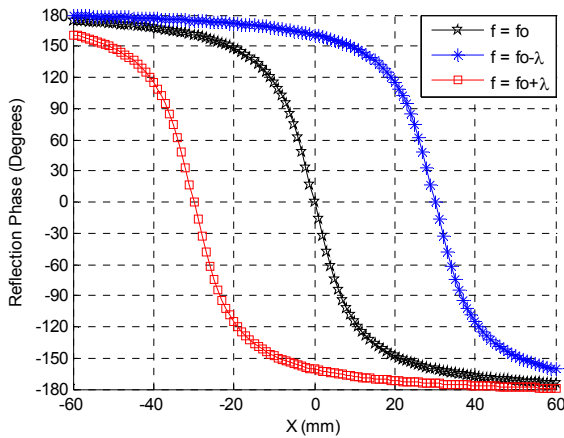


Fig. 3 Reflection phase curves for progressive phase distribution of reflectarrays with different feed positions

In the case of off-set feed reflectarrays ΔX_f has to be introduced as the distance between the feed and the line perpendicular to the array centre. Then the required reflection phase can be calculated by:

$$\varphi = -\frac{2\pi}{3} \cot^{-1} \frac{f}{x_i \pm \Delta X_f} \quad (19)$$

Fig. 2 shows the geometry of the centre feed and off-set feed reflectarrays for different reflectarray designs. The feeds F1 and F2 are placed at the offset distance of $\Delta X_f = X_0 - X_1$ and $\Delta X_f = X_0 - X_2$ respectively. Fig. 3 shows the progressive phase distribution obtained for the design of reflectarrays with center and off-set feed positions. In the case of off-set feed, the feeds were placed at a distance of one wavelength (λ) away from the centre of the array on both sides.

IV. EFFECTS OF MATERIAL PROPERTIES

Equation (19) provides a general formula for the design of reflectarray with progressive phase distribution for any dielectric material and either centre or off-set feed

configuration. In order to obtain the progressive phase distribution of reflectarray designed with different dielectric substrates, the material properties should be incorporated in (19). The material properties effect on the reflection coefficient (Γ) which effects the reflection phase of the reflectarray. In the case of reflectarrays, Γ depends on the attenuation due to dielectric and conductor loss which are given by:

$$\alpha_d = \frac{\omega}{2} \sqrt{(\mu_0 \epsilon_0 \epsilon_r)} \tan \delta \quad (20)$$

$$\alpha_c = \frac{8.68}{W Z_m} \sqrt{\left(\frac{\omega \mu_0}{2 \sigma_c}\right)} \quad (21)$$

where; α_d and α_c are attenuation due to dielectric and copper loss respectively, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$), ϵ_0 is the permittivity of free space (8.854×10^{-12}), σ_c is the conductivity of copper (5.96×10^7 S/m) and Z_m is the impedance of free space (376.73Ω). After incorporation of effects of dielectric and copper attenuation on Γ and reflection phase of reflectarray, (19) can be written as:

$$\varphi = -\frac{2\pi}{3} \cot^{-1} \frac{f}{K(x_i \pm \Delta X_f)} \quad (22)$$

In (22), K is a variable which relates φ with Γ and depends on resonant frequency and material properties which affects the radiated and scattered fields given by (9) and (10). The value of K will be higher for the materials with higher values of dielectric permittivity and loss tangent. Therefore K is directly proportional to attenuation due to dielectric and conductor given by (20) and (21) respectively or K can be given by:

$$K \propto \alpha_d + \alpha_c \quad (23)$$

$$K = C.(\alpha_d + \alpha_c) \quad (24)$$

where; C is a compensation variable and varies with different design requirements and materials used. Finally (24) can be written as:

$$\varphi = -\frac{2\pi}{3} \cot^{-1} \frac{f}{C.(\alpha_d + \alpha_c)(x_i \pm \Delta X_f)} \quad (25)$$

Fig. 4 shows the effects of different values of K on the reflection phase of reflectarrays in the case of centre feed ($\Delta X_f = 0$) while Fig. 5 shows the reflection phase curves for different materials and different feed positions. It can be observed from Figs. 4 and 5 that as the value of K is increased from 0.3 to 1.9, the slope of the reflection loss curve gets steeper which shows a lower bandwidth value in reflectarray design.

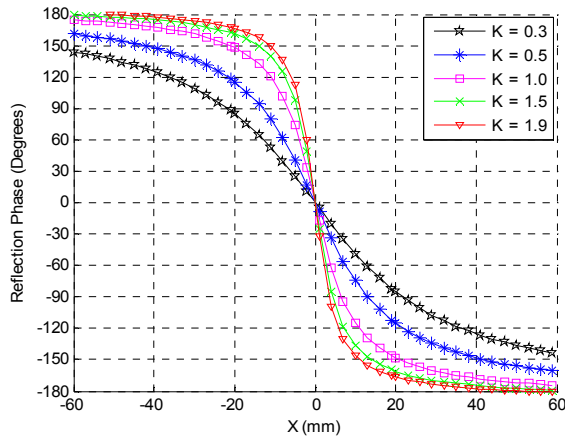


Fig. 4 Effects of different material properties on the reflection phase of reflectarray design

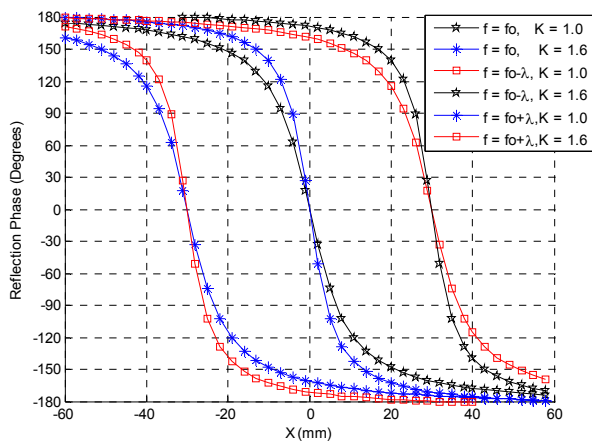


Fig. 5 Effect of different material properties on the reflection phase obtained for different feed positions

In order to characterize the bandwidth performance using different material properties, a Figure of Merit (FoM) is defined as the slope of reflection phase curve with respect to the dimensions of the reflectarray and is given by:

$$\text{FoM} = \frac{\Delta\theta}{\Delta X} \text{ } ^\circ/\text{mm} \quad (26)$$

From the investigations of FoM, it has been demonstrated that FoM increases from $4.61^\circ/\text{mm}$ to $22.35^\circ/\text{mm}$ as the value of K is increased from 0.3 to 1.9 which shows a degradation in the reflector antenna bandwidth in the case when materials with higher dielectric permittivity and loss tangent values are selected. Moreover for lower values of K, the reflection phase range is shown to decrease much lower than 360° which indicates the rise of phase errors. This trade-off between bandwidth and phase errors for the periodic array follows the same trend as given in [17] and [18] for the unit cell reflectarray elements designed in an infinite array environment.

V. CONCLUSION

A technique based on the calculation of required reflection phase of reflectarray elements lying on the X-axis is presented for the progressive phase distribution of reflectarray antenna design. It has been shown that the reflection phase is dependent on the material properties and configuration of elements of a reflectarray. Moreover the selection of dielectric materials is critically important to be considered in order to achieve enhanced bandwidth and phase range performance.

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