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Transient Free Laminar Convection in the Vicinity of a Thermal Conductive Vertical Plate

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Abstract**—**In this paper the influence of a vertical plate's thermal capacity is numerically investigated in order to evaluate the evolution of the thermal boundary layer structure, as well as the convective heat transfer coefficient and the velocity and temperature profiles. Whereas the heat flux of the heated vertical plate is evaluated under time depending boundary conditions. The main important feature of this problem is the unsteadiness of the physical phenomena. A 2D CFD model is developed with the Ansys Fluent 14.0 environment and is validated using unsteady data obtained for plasterboard studied under a dynamic temperature evolution. All the phenomena produced in the vicinity of the thermal conductive vertical plate (plasterboard) are analyzed and discussed. This work is the first stage of a holistic research on transient free convection that aims, in the future, to study the natural convection in the vicinity of a vertical plate containing Phase Change Materials (PCM).

Keywords**—**CFD modeling**,** natural convection, thermal conductive plate, time-depending boundary conditions.

I. Introduction

THE phenomenon of natural convection with coupled heat transfer has received considerable attention due to its many applications in diverse research fields such as architectural design, chemical engineering and environmental dynamics. The heat transfer process is encountered in many engineering applications: aeronautics, fluid fuel nuclear reactors, chemical process industries and many other applications where the fluid is considered as the working medium. Nevertheless, in industrial processes, the phenomena of natural convection are presented in extremely varied forms. For example, the discharge of heat that is generated by electronic components in equipment is made due to natural convection. In a more concrete way, we can observe natural convection in everyday life, even in a room where the air circulation is generated, by upward movement along a radiator and downward movement along a closed window when outside air is colder than indoor air.

Hence, in the last 50 years, the coupled heat transfer natural convection has received considerable attention on building applications. A variety of theoretical and experimental studies focusing on this subject exists while these works are focused on the study of vertical surfaces where heat flux or temperature distributions are uniformly imposed.

More precisely, because of the fact that many transport processes are occurring in nature due to temperature Qureshi and Gebhart [1], Vliet and Liu [2] and Goldstein and Eckert [3] worked basically in the experimental investigation on laminar, transient and turbulent natural convection on a uniformly heated vertical plate. On the other hand, the transient coupled heat transfer free convection along a semi-infinite vertical isothermal plate has been studied by Gallahan and Marner [4]. In addition, Soundalgekar and Warve [5] proposed an analytical study on the unsteady free convection flow that passed through an infinite porous plate. The free convective heat transfer on a vertical semi-infinite plate has been also investigated by Berezovsky et al. [6]. Furthermore, Martynenko et al. [7] investigated the laminar free convection that occurs due to a vertical heated plate. Pohlhausen [8] developed an analytical solution and Ostrach [9]-[11] studied a numerical solution for the isothermal, vertical plate at steady state conditions. Their results are in accordance with the experimental data that Schmidth and Beckmann [12] had obtained for a uniform heat flux density at the same kind of vertical plate. Siegel [13] tried to extend this research by studying the transient case employing an integral method and he finally obtained an estimate of the time required to attain steady state. Towards this direction, Gebhart [14] developed an approximate solution for the transient behavior with a constant heat flux density at the plate. In addition, Sparrow [15] studied a laminar free convection on a vertical plate with prescribed non-uniform wall heat flux/temperature. Simultaneously, Hellum and Churchill [16] studied a complete transient and steady state natural convection problem in an unconfined fluid -initially at rest and at uniform temperature- adjacent to a semi-infinite vertical plate at a different uniform temperature. Finally, Bejan [17], Kays and Crawford [18] and Burmeister [19] based their work in theoretical, numerical and scaling analysis of laminar, transition and turbulent natural convection.

However, even if there is a huge literature regarding studies that focus on natural convection developed near vertical plates under different initial and boundary conditions, only few works treat in a holistic way the natural convection in the vicinity of a thermal conductive plate. In this paper we present the development and validation of a 2D CFD model that aims to estimate the influence of a vertical plate's thermal capacity in the thermal and kinetic boundary layer structure, the

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convective heat transfer coefficient and the phenomena that are produced in the vicinity of the thermal conductive vertical plate.

II. Numerical Analysis

A. Governing Equations

The governing equations for natural convection flow are presented in the form of coupled elliptic partial differential equations. The major problems in obtaining a solution to these equations lie in the inevitable variation of the density with temperature as well as in their partial elliptic nature. Several approximations are generally made to considerably simplify these equations. Two of the most important among these are the Boussinesq and the boundary-layer approximations [7].

The initial system of equations to describe free convection that occurs from a vertical plate in a given initial temperature is considered here to be a system of both Navier-Stokes and energy equations. For a two–dimensional (2-D) developed flow this system is written down as below ((1)-(5)). The y-axis is directed along the plate from the leading edge, the x-axis is normal to it.

The physical properties of the medium, except the density, are assumed to be constant. The natural convection flow (air) that is of interest in our study can be assumed to be nearly incompressible. Regarding the air, we chose to describe the temperature dependence of density employing the Boussinesq approximation. This choice has been made because the Boussinesq approximation correctly reflects the main specific features of coupled free-convective heat transfer for small temperature differences *($\beta\Delta T<1$)*, like in our case. The usual form of the Boussinesq approximation uses only the first-order term in the series. In this case the work of compression and viscous dissipation of energy are assumed to be negligibly small. Hence, the initial system of equations is the following [11]-[18]:

$$\frac{\partial(u)}{\partial x}+\frac{\partial(v)}{\partial y}=0 \tag{1}$$

$$\rho_0\frac{\partial u}{\partial t}+\rho_0 u\frac{\partial u}{\partial x}+\rho_0 v\frac{\partial u}{\partial y}=-\frac{\partial P_s^{'}}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\rho_0\frac{\partial v}{\partial t}+\rho_0 u\frac{\partial v}{\partial x}+\rho_0 v\frac{\partial v}{\partial y}=-\frac{\partial P_s^{'}}{\partial y}+\mu\frac{\partial^2 v}{\partial x^2}+\mu\frac{\partial^2 v}{\partial y^2}+\rho_0\beta\left(T-T_0\right)g \tag{3}$$

$$\rho_0 C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$\rho_{plate}C_p\left(\frac{\partial T}{\partial t}\right)=k_{plate}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

where ρ_{plate} is the plate's density in (kg/m^2), and ρ_0 is the air's density in (kg/m^2), C_p is the plate's thermal capacity in (J/kgK), T is the temperature in K, k is the air's thermal conductivity (W/m^{2}K), k_{plate} is the plate's thermal conductivity (W/m^{2}K) [a detailed explanation of the symbols is provided in the Appendix A section inside the nomenclature table (Table II)]. Equation (1) is the continuity equation, (2) is the x-momentum equation, (3) is the y-momentum equation, (4) is the air-energy equation and finally (5) is the plate-energy equation.

The basic parameters that characterize the process of free-convective heat transfer are the Grashof or the Rayleigh number and the Prandtl number. These numbers are calculated as follows [15]:

$$Gr_{y^*}=\frac{g\beta\phi_w y^4}{v^2 k} \tag{6}$$

$$Ra_{y^*}=\frac{g\beta\phi_w y^4}{\alpha v k}=Gr_{y^*}\Pr \tag{7}$$

$$\Pr=\frac{C_P\mu}{k} \tag{8}$$

The Prandtl number is determined by the physical parameters of the air and characterizes the similarity between the vorticity distribution and heat diffusion [6]. The Prandtl number depends on the viscosity and thermal conductivity, and for this reason it is a material property. Thus it varies from fluid to fluid. In our case the flow is the air so the Prandtl number is equal to 0.71. The Grashof number occurs in free convection and gives the relative importance of buoyancy force to the viscous force. For the laminar regime the Rayleigh number has to have the values between $10^5 \prec Ra_{y^*} \prec 10^{13}$ [17]. In our case, since we focus on laminar convection, the mean Rayleigh number of the plate is considered Ra_{y^*} ~$1.14*10^{11}$.

B. Initial and Boundary Conditions

We consider an open cavity with a thermal conductive vertical plate, which dimensions are 0.012m thickness and 1.6m height (Fig. 1). The plate is immersed in a static fluid, because of buoyancy, the plate sucks the fluid into the domain from the bottom boundary, and discharges it through the top boundary. The top boundary conditions have been placed at a distance from the plate. The reason is that the constant total pressure boundary condition requires the edge boundary to be far enough away from the heated plate in order to avoid perturbations due to the limited computational domain [20], [21].

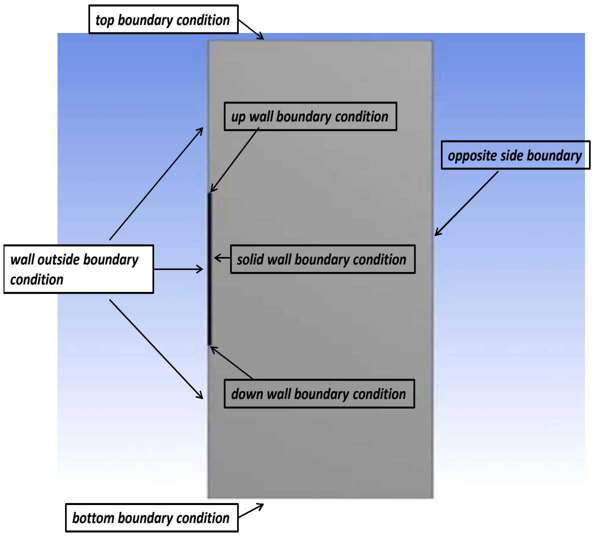

Fig. 1 The open cavity with the thermal conductive plate on the left

Otherwise, the up boundary condition influences significantly the boundary layer development. The bottom boundary condition of the cavity was also placed at a distance to prevent the disturbance of the small velocity (*v=0,02m/s*) that was imposed for the inlet of the air (see also Appendix B).

The governing equations require initial and boundary conditions. Initial conditions specify the initial values of all variables at $t=0$. In our case, as we are interested in studying the development and the evolution of natural convection in the vicinity of a thermal conductive plate with the passing of time we started from the same initial condition for both the air and the vertical plate. Hence, it is considered as *initial condition* that the plate and the flow (air) are in the same temperature:

$$t = 0,\ T_{plate} = 295{,}16K,\ T_{air} = 295{,}16K, v = u = 0 \tag{9}$$

Regarding the *top boundary condition* (see Fig. 1) we can specify the pressure taken at a point A positioned away from the plate. This pseudo-point has a modified pressure as follows:

$$P' = P_{totA} - \rho \frac{(u_A^{\,2} + v_A^{\,2})}{2} \tag{10}$$

Since the absolute value of reference pressure may be chosen arbitrary in numerical simulation, we set $P_{tot}=P_{atm}$. So, $P_{totA}=P_{atm}$ and $\rho(u_A^{\,2}+v_A^{\,2})/2$ is negligible when the point A is sufficiently far away from the plate. This is why this boundary condition requires the edge boundary to be sufficiently far away from the plate. We neglect the second term because the velocity is very small and does not influence on the results. The gradient normal to the boundary surface of the variables v,u,T are equal to 0. A detailed study on this kind of boundary conditions is provided by Xiaxiong Yuang [20], Georgantopoulou and Tsangaris [21] and Cebeci and Bradshaw [22]. Hence the *top boundary condition* is resumed as follows:

$$\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = 0, \frac{\partial T}{\partial y} = 0, P' = P_{atm}, T = T_{air} \tag{11}$$

Regarding the *bottom boundary condition* (see Fig. 1) the fluid has a very small velocity in the inlet while unsteady data are provided for the temperature which decreases by 2°C per 1h. Hence the *bottom boundary condition* is resumed as follows:

$$v = 0{,}02\,\mathrm{m/s}, u = 0m/s, T = 295.16K - 0.0005 * t \tag{12}$$

The *wall outside boundary condition* (see Fig. 1) is considered adiabatic so the boundary condition is resumed as follows:

$$\frac{\partial T}{\partial x} = 0 \tag{13}$$

Regarding the *opposite wall boundary condition* (see Fig. 1) we considered that the component of the velocity normal to the surface is set to zero, and the gradients normal to the boundary surface of all other variables v, T, P are specified as zero. Hence, we obtain:

$$u = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial T}{\partial x} = 0, \frac{\partial P}{\partial x} = 0 \tag{14}$$

Regarding the wall a *solid wall boundary condition* (see Fig. 1) is specified in order to assure that the fluid cannot flow the boundary surface. The fluid must adhere to the plate, with the no-slip condition of viscous flows:

$u = 0, v = 0$ and heat flux continuity:

$$k_{plate} \frac{\partial T}{\partial x} = k_{air} \frac{\partial T}{\partial x} \tag{15}$$

Finally, the *up and down wall boundaries* are also considered adiabatic, so:

$$\frac{\partial q}{\partial x} = 0 \tag{16}$$

C. Solution

As our geometry is simple enough we chose a structure grid: hence, our mesh follows a structured i,j quadrilateral convention in 2D, in order to assure the same connectivity between neighboring vertices (Fig. 2). The discretization of the equations was done using the finite volume method. The Finite Volume Method divides the domain into a number of finite size sub-domains (control volumes). The governing differential equations are integrated over each control volume.

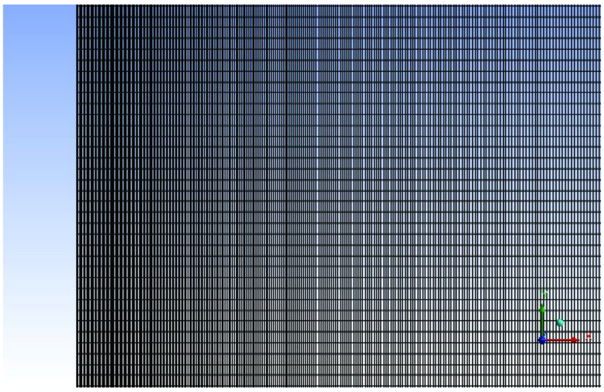

Fig. 2 Example of the problem's meshing

To estimate the evaluating of the dependent variables above integrals we considered a profile assumption. In our case we chose a Power Law scheme as the profile assumption for the momentum and energy equations. The reason is that it is the closest scheme to the exponential solution (exact solution) and it is less expensive than exponential scheme in computational time.

Furthermore, in our case the Peclet number in the surrounding cells of the plate is bigger than 0 and less than 10. So, we chose the Power Law scheme because in this range it is the closest scheme to the exact solution according to Patankar [23]. The Peclet number is equal to $P = \rho u / \left(k / \delta x \right)$, where δx is the characteristic length (the cell width). Table I shows the Peclet number values for different sizes of cells length. Table I summarizes the Peclet number values for different sizes of cells length *(δx)* when $0 \prec P \prec 10$:

TABLE I
THE PECLET NUMBER FOR DIFFERENT CELL'S SIZES

δx	Peclet number
0,00090604	6,62440937
0,00045225	3,30659687
0,00036168	2,64438146
0,00030133	2,20315361
0,00022593	1,65189886

For constructing values of a scalar at the cell faces and for computing secondary diffusion terms and velocity derivatives we chose the Green-Gauss Node-Based Gradient method. The node-based averaging scheme is known to be more accurate than the cell-based scheme for unstructured meshes, most notably for triangular and tetrahedral meshes. Hence, according to this method we obtain:

$$\bar{\phi}_f = \frac{1}{N_f} \sum_{n}^{N_f} \bar{\phi}_n \qquad (17)$$

where N_f is the number of nodes on the face and $\bar{\phi}_n$ illustrate the nodal physical quantity under investigation.

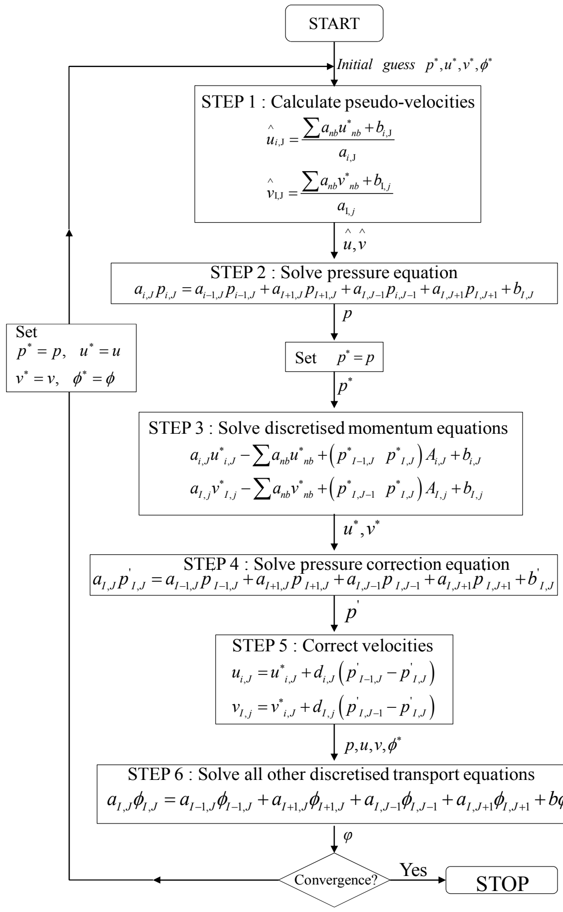

Fig. 3 The flow chart of the SIMPLEC algorithm

The nodal values $\bar{\phi}_n$ in (17) are constructed from the weighted average of the cell values surrounding the nodes, following the originally proposed approach by Holmes and Connel [24] and Rauch et al. [25]. This scheme reconstructs exact values of a linear function at a node from surrounding cell-centered values, preserving a second-order spatial accuracy.

For the solution of the system of equations the SIMPLEC (Simple-Consistent) algorithm was chosen. The SIMPLEC numerical algorithm uses a combination of continuity and momentum equations to derive an equation for pressure (or for pressure correction): it is the well known pressure-velocity coupling. SIMPLEC is preferred because it allows a faster convergence than the SIMPLE algorithm. The flow chart of the SIMPLEC algorithm is illustrated in Fig. 3.

III. VALIDATION, RESULTS AND DISCUSSION

A. Validation

The detailed numerical investigation of the evolution of the natural convection phenomena that occur in the vicinity of the vertical thermal conductive plate gives us the evolution of the plate's heat flux as a function of time. We observe that the heat flux curve has the tendency to become vertical as time passes (Fig. 4). So after simulating 4 hours (real time), the curve is almost vertical and the fluctuation is small (Fig. 4). We can assume that the system reached the steady state regime, as the air temperature and the plate temperature decrease. So, laminar free convection phenomena occur along a vertical plate with uniform surface heat flux. Employing the energy equation we can deduce the value of the heat flux when the system reaches the steady state regime. Fig. 5 shows the evolution of the heat flux curves whereas a fictive constant heat flux curve for steady state regime is fixed equal to 4,88W/m^2.

Let's explain how this value has been calculated. If we change the variable from T to ΔT we obtain the following system of 3 equations:

$$\begin{aligned} T_\infty &= T_{\infty,0} + b_\infty t \\ \Delta T &= T - T_\infty \\ T &= \Delta T + T_\infty = \Delta T + T_{\infty,0} + b_\infty t \end{aligned} \qquad (18)$$

Hence, employing the energy equation we calculate as follows the φ_{ref} that represents the value of the heat flux when the system reaches the steady state regime:

$$\begin{aligned} &\frac{\partial \Delta T}{\partial t} + u\frac{\partial \Delta T}{\partial x} + v\frac{\partial \Delta T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 \Delta T}{\partial x^2} - b_\infty \overset{x=0}{\Rightarrow} \\ &\Rightarrow \rho C_P \int_0^{e_w} \frac{\partial \Delta T}{\partial t} dt = k\int_0^{e_w} \frac{\partial^2 \Delta T}{\partial x^2} dx - \int_0^{e_w} b_\infty \rho C_P dx \Rightarrow \\ &\Rightarrow \rho C_P \frac{\partial \Delta T}{\partial t} e_w = k\left(\frac{\partial \Delta T}{\partial x}\bigg|_{e_w} - \frac{\partial \Delta T}{\partial x}\bigg|_0\right) - b_\infty \rho C_P e_w \overset{\frac{\partial \Delta T}{\partial t}=0}{\Rightarrow} \\ &\Rightarrow k\frac{\partial \Delta T}{\partial x}\bigg|_{e_w} = b_\infty \rho C_P e_w \Rightarrow \\ &\Rightarrow \phi_{ref} = b_\infty \rho C_P e_w = 4,88 W/m^2 \end{aligned} \qquad (19)$$

where x=0 is considered to be at the solid wall boundary condition.

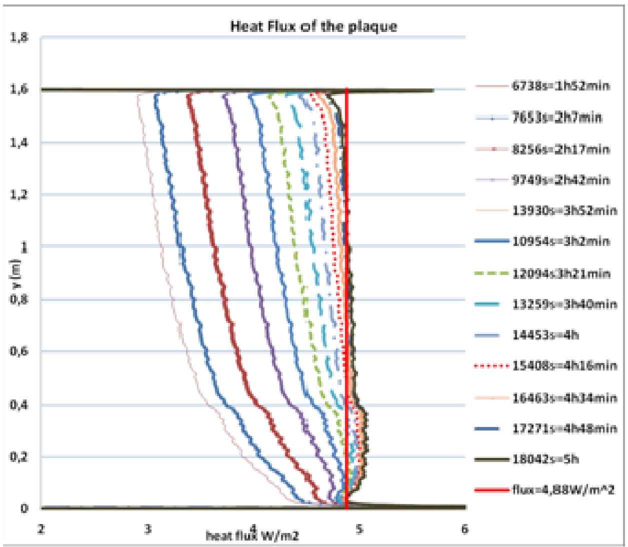

Fig. 4 The heat flux evolution with the passing of time in the vicinity of the thermal conductive plate and the constant heat flux 4.88 W/m^2 when the system reaches the steady state regime

Furthermore, to validate our results with the already existed studies we compared them to the data of Sparrow and Gregg [26] regarding laminar free convection for a vertical plate with uniform surface heat flux of the thermal boundary layer. Fig. 5 shows two identical curves with a difference of ~0.1K. This gap is due to the fact that in our case the heat flux in the level of the thermal conductive plate is not uniform along the plate on steady state conditions.

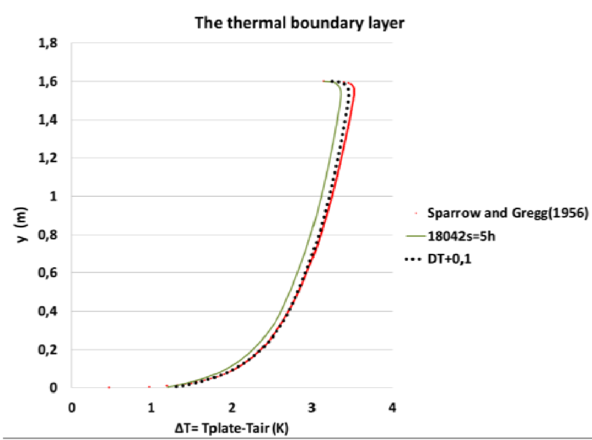

Fig. 5 Confrontation of our simulated data to the thermal boundary layers from Sparrow and Gregg [26] for heat uniform flux

B. Results

We notice that the results are in agreement with the steady state regime thus we can emphasize on the plate's behavior. In Fig. 6 we observe the development of the thermal boundary layer with the passing of time. We can see that the thermal boundary layer becomes wider when time passes. This phenomenon occurs because the air temperature in the open cavity decreases as a function of time according to the equation $T_{air} = 293.15\text{-}.0005\ t$. On the other hand the response of the plate because of its thermal inertia is late and less important. So, the difference $T_{plate} - T_{air}$ increases. We can also notice that after 4 hours (real time) the thermal boundary layer almost does not change, so it is obvious that the system reached the steady state regime.

In addition the temperature and velocity profiles are summarized on Figs. 7 and 8. The major difference that we observe in this study is the fact that the air temperature decreases with the passing of time. This is obvious especially regarding the temperature profile that is presented on Fig. 7.

On the other hand, on Fig. 8 we notice that the velocity profile does not present any important evolution after 3 hours (real time). That means that the air maximum velocity in the boundary layer increases locally however the kinetic boundary layer remains constant even if changes occur on the temperature profile and as a consequence the thermal boundary layer grows. This fact illustrates that the temperature changes are not influential enough to modify the width of the velocity's profile. Thus, the kinetic boundary layer does not present an important evolution with the passing of time.

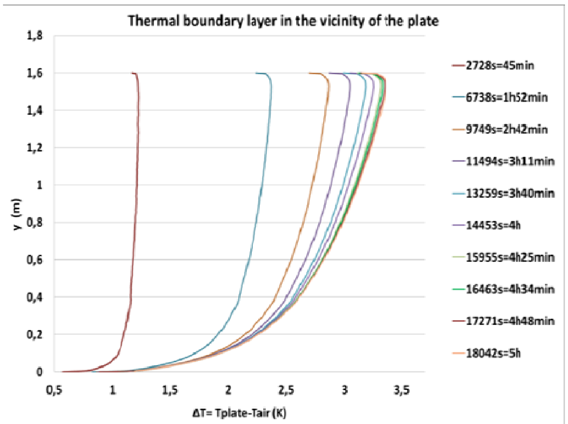

Fig. 6 Temperature profile evolution with the passing of time in the vicinity of the thermal conductive plate

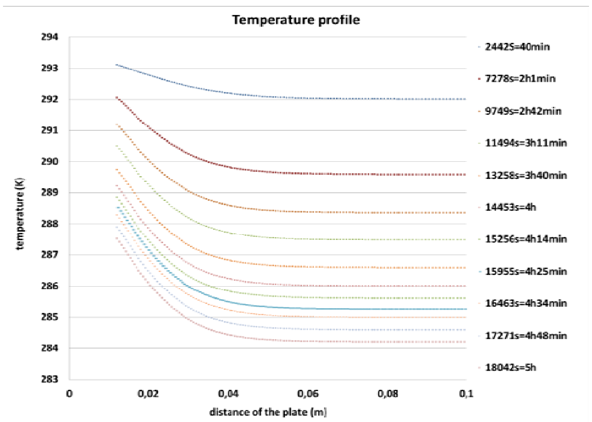

Fig. 7 Temperature profile evolution with the passing of time

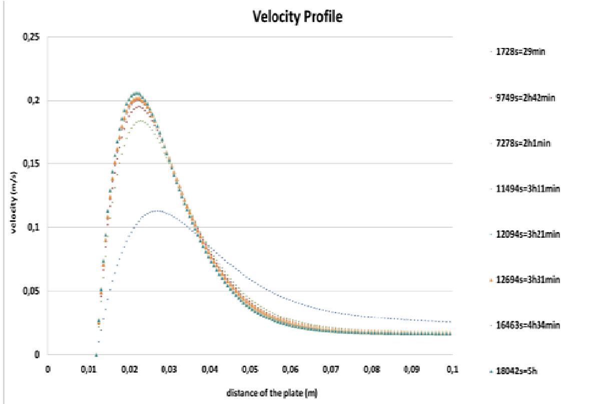

Fig. 8 Velocity profile evolution with the passing of time

Regarding the local convection coefficient we observed that it decreases in the vicinity of the higher layers of the plate (Fig. 9). This observation is in accordance with theory. Since the heat flux is almost constant (presenting very small fluctuations) all along the vertical plate and the ΔT increases from the plate's bottom to the top then the convective heat transfer coefficient decreases. This fact is also well known from theory whereas ΔT and h are inversely proportional physical quantities:

$$h = \frac{\phi_w}{\Delta T} \tag{20}$$

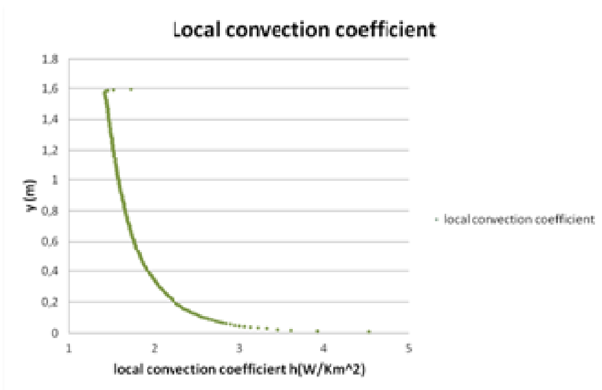

Fig. 9 The evolution of the local convective heat transfer coefficient after simulating 5h (real time) We observe that it decreases as we climb the plate

Finally concerning the temperature gradient inside the plate we noticed that temperature decreases uniformly with the passing of time (Fig. 10). We observe that the curves are identical; however they are transposed in the vertical dimension with the passing of time. This behavior is logical because the air becomes colder as time passes; hence, the plate becomes colder with the passing of time and especially it is colder in the plate-air interface than on the adiabatic side. More precisely, this happens since the $\Delta T = T_{plate} - T_{air}$ increases due to the fact that the air temperature in the open cavity varies according to the equation T_{air} = *293.15-.0005 t* as we mentioned above.

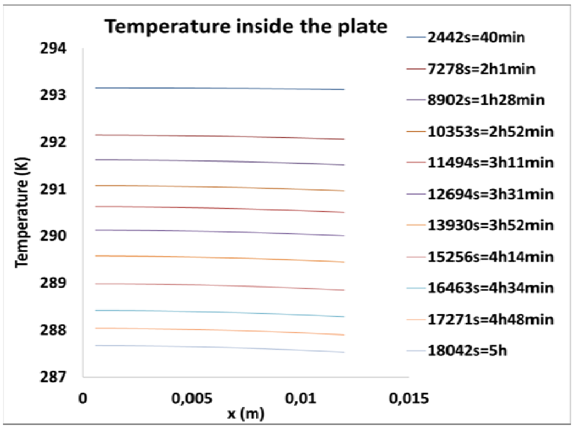

Fig. 10 Temperature evolution inside the plate with the passing of time We can see that the temperature inside the plate decreases

Furthermore, Fig. 11 illustrates more closely the temperature distribution inside the plate after 5 hours, when we consider that our system is reaching the steady state regime. Since the curves are identical and the only difference is their transposition in the *y* dimension as time passes, we chose to focus and analyze the behavior of only 1 curve in order to be able to illustrate in the best scale the simulation plots.

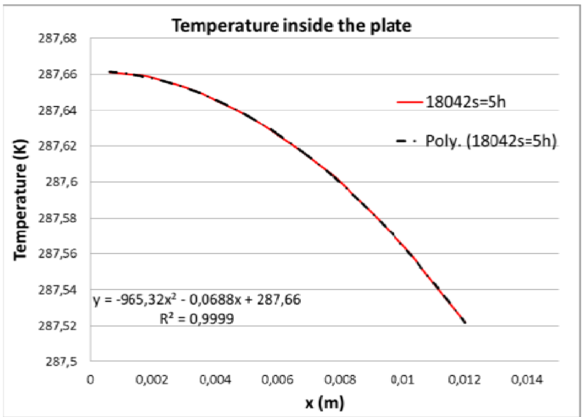

Fig. 11 Temperature evolution inside the plate after 5 hours The second order polynomial equation and the square of the linear correlation coefficient R^2 are also shown

Fig. 11 is a detail of Fig. 10 where only the temperature evolution profile inside the plate after 5 hours of simulated data is plotted. We observe an important gradient of temperature in the plate. Furthermore, the temperature is reaching its maximum values in the vicinity of the adiabatic boundary condition. Also, the minimum temperature value is observed in the plate-air interface. The difference between minimum and maximum values is about 0.14 K. Thus, we observed that the temperature evolution inside the plate follows a second order polynomial equation. We found empirically that the relation $y=-965.32x^2-0.688x+287.66$ fits perfectly to the temperature evolution curve inside the plate after 5 hours of simulated data presenting a $R^2=0.999$.

IV. Conclusions and Perspectives

In spite of the simple geometry, numerical simulation of transient laminar free convection in the vicinity of a thermal conductive vertical plate still remains a challenging problem for computational fluid dynamics. This happens because of the unsteadiness of the physical phenomena. Furthermore, in our case the problem was much more complicated since the entire numerical modeling is developed and validated using unsteady data obtained for plasterboard under a dynamical temperature evolution. This investigation is very important especially for building applications since weather data are unpredictable and so thermal engineers need to know in detail the behavior of a thermal conductive wall under transient laminar free convection to predict its insulating behavior.

Hence, in this paper a 2-D computational fluid dynamics investigation of airflow, to estimate the influence of a vertical plate's thermal capacity in the thermal and kinetic boundary layer structure, the convective heat transfer coefficient and describe the phenomena that are produced in the vicinity of thermal conductive vertical plate is conducted in the Ansys Fluent 14.0 environment. A low thermal conductive vertical plate has been placed in an open cavity, while a CFD investigation was conducted for a wide range of different sizes of cells length *(δx)* and for the related range of Peclet number values that varied from very low (1,65189886) to very high (6,62440937).

The governing equations for natural convection flow are presented in the form of coupled elliptic partial differential equations, whereas the initial system of equations to describe free convection is considered to be a system of both Navier-Stokes and energy equations. The air's density temperature dependence is described via the Boussinesq approximation since it is considered to be an approximation that correctly reflects the main specific features of coupled free-convective heat transfer. Therefore, compression and viscous dissipation of energy are assumed to be negligibly small in the present study. Appropriate initial and boundary conditions has been used. In order to anticipate perturbations due to the limited computational domain we locally applied the pseudo-cells method since previous research proved that the constant total pressure boundary condition requires the edge boundary to be far enough away from the heated plate otherwise local noise appear as a constant noise influencing the results (see figures in Appendix C).

The equations were discretized using the finite volume method. Additionally, we chose the Power Law scheme as the profile assumption for the momentum and energy equations to estimate the evaluating of the dependent variables. The Green-Gauss Node-Based Gradient method has been employed to construct values of a scalar at the cell faces and to compute secondary diffusion terms and velocity derivatives, since this nodal-based averaging scheme is considered to be more accurate than the cell-based scheme for unstructured meshes.

Finally, the SIMPLEC numerical algorithm that uses a combination of continuity and momentum equations to derive an equation for pressure is employed because it allows a faster convergence than the SIMPLE algorithm. For us quick convergence was very important since we simulated 5 hour of real time with adaptive time steps that often reach 10^{-5} sec values.

Once the numerical model was validated and evaluated according to past literature results (after the confrontation of our simulated data to past researches on the same subject), we managed to obtained some very important quantitative and qualitative conclusions regarding the behavior of a vertical thermal conductive plate placed in an open cavity:

- The velocity profile is not influenced from the decrease of the air temperature after approximately 3h (real time). Hence, the kinetic boundary layer remains constant even if changes occur on the temperature profile and as a consequence the thermal boundary layer grows.
- On the other hand the temperature profile is significantly influenced by the air temperature decrease. The thermal boundary layer grows as a function to the air temperature decrease.
- Heat flux curve has the tendency to become vertical as time passes until the system reaches steady state regime. The heat flux in the capacitive plate presents some differences of a uniform heated plate.
- The kinetic boundary layer does not present important evolution as time passes.
- The plate temperature decreases uniformly.
- The local convective heat transfer coefficient decreases as we climb the vertical plate.

Furthermore, an empirical second order polynomial equation that forecasts in an accurate way the temperature evolution curve inside the plate after 5 hours of simulated data is provided.

Since we managed to validate and sufficiently evaluate our 2-D CFD model with accuracy, our next step consists to extend the present research including Phase Change Materials in the plasterboard, which means adding the enthalpy vs. temperature curve for the material. Then we will be able to study in detail the thermal and kinetic boundary layer structure, the evolution of convective heat transfer coefficients and the phenomena that are produced in the vicinity of a building wall containing Phase Change Materials (PCM). This is an extremely important issue on building physics since the low inertia buildings generally suffer of summer discomfort however a potential solution to this problem is the use of walls containing Phase Change Materials (PCM). Finally, in the future we have the intention to work on 3-D CFD modeling in order to enhance on the understanding of the thermal and kinetic boundary layer structure and formulation in the vicinity of a vertical plates in scale 1:1, as well as studying the evolution of turbulent flow in the open cavity.

Appendix A

A nomenclature table is provided here. All the symbols employed in the paper, as well as their units in the SI system are tabulated below.

TABLE II
NOMENCLATURE

Symbol	Quantity	SI Units
u	x-velocity	*m/s*
v	y-velocity	*m/s*
K	thermal conductivity	W/Km^2
h	convective heat transfer coefficient	W/Km^2
Cp	Heat capacity	*J/kgK*
ρ	density	Kg/m^3
T	temperature	*K*
P	pressure	*atm*
Pr	Prundtl number	*1*
Gr_{y*}	Grashof number	*1*
Ra_{y*}	Rayleigh number	*1*
ν	kinetic viscosity	m^2/s
μ	dynamic viscosity	*kg/ms*
g	gravity	m/s^2
β	extension coefficient	*1/T*
φ_w	wall's heat flux	W/m^2
t	time	*s*
β_∞	temperature rate	*K/s*

Appendix B

The following graphical images show the thermal and kinetic distribution inside the plate and the open cavity after having simulated 2h 7min (real time). We observe that the low plate part has already lost some of its thermal energy in contrast to the up plate part. The heat discharge advances progressively inside the plate from the bottom to the top.

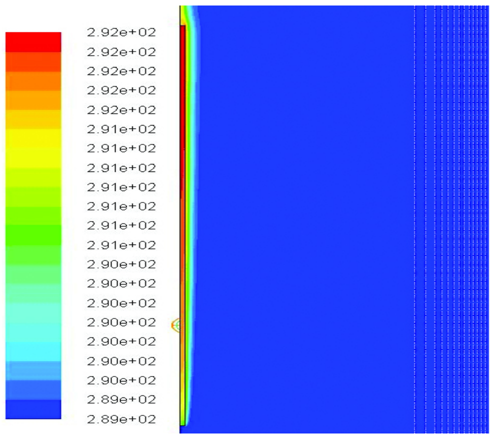

Fig. 12 The plate thermal discharge and the thermal boundary layer in the vicinity of the plate after 2h 7min

As about the velocity inside the open cavity we focus on the kinetic boundary layer in the vicinity of the plate. The kinetic boundary layer presents a significant growth on the top plate part and very small on the down plate part. The fact that the plate temperature is lower on the down part leads to the small buoyancy phenomena which mean very small y-velocity. The influence from the plate to the boundary layer and vice-versa is mutual.

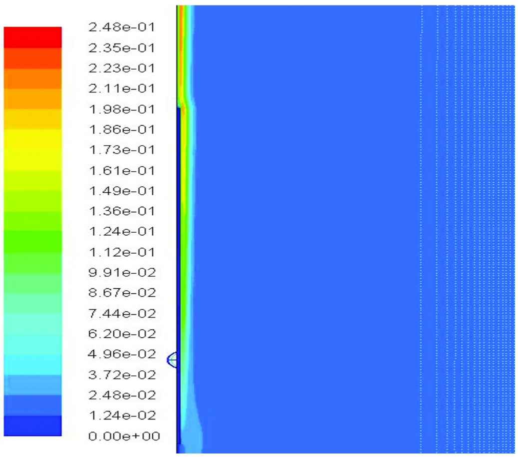

Fig. 13 The Kinetic boundary layer after 2h7min

APPENDIX C

The following graphical images show that the top boundary condition's placement can generate a turbulent flow inside the open cavity.

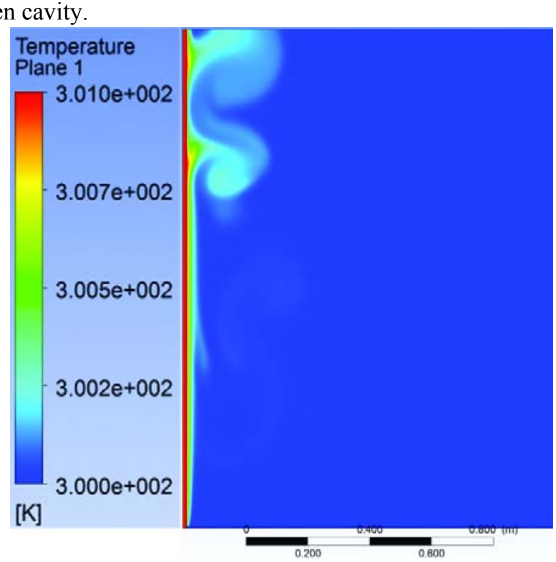

Fig. 14 Temperature distribution in the simulation domain

This is why we chose to work employing the pseudo-cells method [20], [21] putting the edge boundary far enough from the heated plate to avoid the appearance of local turbulences. The following figures present the temperature and velocity distribution when no pseudo-cells are added. The top boundary position is imposed directly at the end of the vertical plate. As we can see local turbulences are present on the top and influence the accuracy of the simulated data.

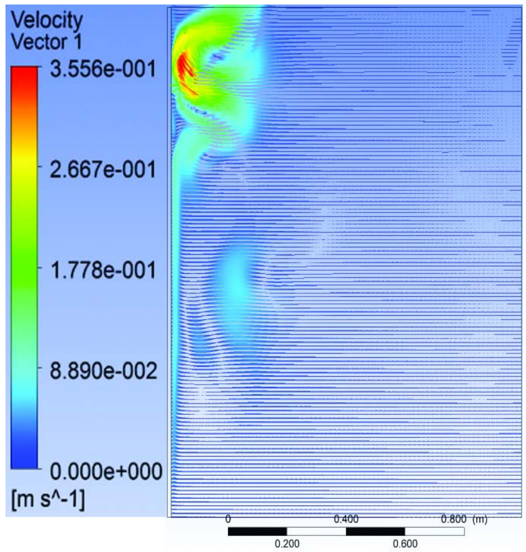

Fig. 15 Velocity distribution in the simulation domain

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