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A New Developed Formula to Determine the Shear Buckling Stress in Welded Aluminum Plate Girders

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***Abstract*—**This paper summarizes and presents main results of an in-depth numerical analysis dealing with the shear buckling resistance of aluminum plate girders. The studies conducted have permitted the development of a simple design expression to determine the critical shear buckling stress in aluminum web panels. This expression takes into account the effects of reduction of strength in aluminum alloys due to welding process. Ultimate shear resistance (USR) of plate girders can be obtained theoretically using Cardiff theory or Höglund's theory. USR of aluminum alloy plate girders predicted theoretically using BS8118 appear inconsistent when compared with test data. Theoretical predictions based on Höglund's theory, are more realistic. Cardiff theory proposed to predict the USR of steel plate girders only. Welded aluminum alloy plate girders studied experimentally by others; the USR resulted from tests are reviewed. Comparison between the test results with the values obtained from Höglund's theory, BS8118 design method and Cardiff theory performed theoretically. Finally, a new equation based on Cardiff tension-field theory, proposed to predict theoretically the USR of aluminum plate girders.

***Keywords*—**Shear resistance, Aluminum, Cardiff theory, Höglund's theory, Plate girder.

I. INTRODUCTION

THE usual slender design of the web panels in plate structures makes these structural elements, most times, susceptible to instability phenomena: web buckling. This situation makes it imperative to evaluate accurately the shear buckling strength in order to optimize their design. Historically, elastic shear buckling in steel plates has been determined assuming that web panels are simply supported at the juncture between flanges and web. This assumption has turned out to be conservative since the geometrical properties of the plate girder modify the boundary conditions and influence the web behavior in shear.

Moreover, the analysis of web buckling on steel plates implies also the study of the material nonlinearity effects. In most current standards the effect of the material nonlinearity is introduced in the formulation of the critical shear buckling stress by including a plasticity reduction factor. Ultimate-limit-state design methods and codes of practice [1], [2] indicated that web plates may exhibit a significant post-buckling reserve of strength and stiffness under adequate web boundary constraint. Aluminum alloys are used in variety of structural engineering applications due to their high strength/weight ratio and durability. These characteristics are of particular significance in the design of lightweight and transportable bridges, for which easy and speed of construction, low maintenance and long service life are important considerations. However it is well known that the heat of welding reduces the strength of aluminum alloys in the vicinity of the weld. The heat-affected zone (HAZ) extends illustrated in Fig. 1, within the parent metal, approximately 25mm in all directions from the weld [3].

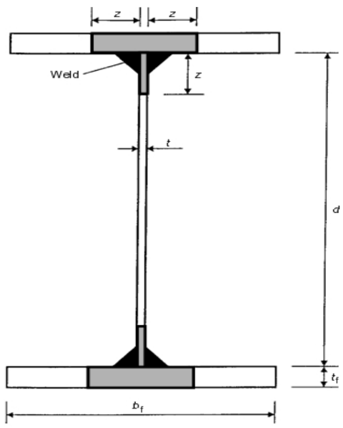

Fig. 1 Extent of heat-affected zone

Experimental studies of the ultimate shear resistance of welded aluminum alloy plate girders have indicated that failure generally occurs due to rupture of the welded web boundary during the development of a typical shear failure mechanism [2]. The ultimate shear resistance of steel plate girders has been studied extensively, both experimentally and theoretically, resulting in the development of the now well-established Cardiff tension-field theory, or Höglund's theory [4]-[7]. An idea about each theory will be pointed out in this paper.

II. SHEAR STRENGTH OF PLATE GIRDER USING CARDIFF TENSION FIELD THEORY

Cardiff theory is based on equilibrium stress field (tension field) in the girder, which satisfies the theoretical conditions for lower bound strength prediction, provided the material possesses sufficient ductility for the stress filed to develop. For the assumed failure mechanism shown in Fig. 2, the ultimate shear resistance V_s of a transversely stiffened girder

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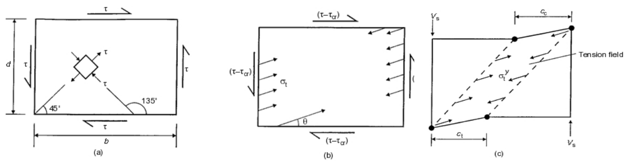
can be expressed in three different stages; pre-buckling stage, post-buckling stage and collapse stage.

For a plate girder subjected to a small shear load, bending theory can be used to determine how the internal forces are carried by the web and the flanges. When the applied load is increased, the failure mode of a plate girder will depend largely on the panel aspect ratio (*b/d*) and the web slenderness ratio (*d/t)*, where *b* is the clear distance between vertical stiffeners, and *d* and *t* are the clear depth and the thickness of the web panel, respectively.



(a) Pre-buckling (b) Post-buckling (c) Collapse

Fig. 2 Shear Failure mechanism in Cardiff tension Field theory

When the panel is stocky, the web will fail by yielding in shear, which is governed by the theoretical shear yield strength $\tau_{yw} = \sigma_{yw}/\sqrt{3}$, where σ_{yw} is the uniaxial tensile yield strength of the web. For most practical plate girders, however, web panels are generally thin and tend to buckle first before yielding. The overall behavior of a web panel is thus divided into three stages, (1) pre-buckling, (2) post-buckled, and (3) collapsed stage.

A. Pre-Buckling Stage

If a uniform shear stress is applied to the web, there will be a principal tensile stress of magnitude τ acting throughout the whole web. This stress state will continue until the applied shear stress reaches the critical shear strength τ_{cr} which can be determined from classical stability theory for plates by Timoshenko and Gere [8]:

$$\tau_{cr} = K\left[\frac{\pi^2 E}{12(1-\mu^2)}\right]*\left(\frac{t}{d}\right)^2 \le \tau_{yw} \tag{1}$$

The buckling coefficient *K* is obtained from the following equation

$$K = 5.35 + 4*\left(\frac{d}{b}\right)^2, \text{ for } \frac{b}{d} > 1 \tag{2}$$

$$K = 5.35*\left(\frac{d}{b}\right)^2 + 4, \text{ for } \frac{b}{d} < 1$$

where: *E* the modulus of elasticity, and μ is the Poisson's ratio.

Therefore, the shear load that causes the web plate to buckle is given by:

$$V_{cr} = \tau_{cr} dt. \tag{3}$$

Recent studies show that the restraint provided by the flanges could enhance the buckling coefficient *K*, which would lead to an enhancement in shear strength [1], [4], [8]. In this paper the boundary condition has been arbitrarily and conservatively assumed to be fixed.

B. Post-Buckling Stage

Once the critical shear strength is reached, the web cannot carry any increase in shear load. Additional shear force will be supported by the mobilization of tensile membrane stress σ_t in the diagonal band of the web. For a web panel subjected to pure shear, the value of σ_t that causes the web to yield can be written as

$$\sigma_t^y = -\frac{3}{2}\tau_{cr}\sin 2\theta + \sqrt{\sigma_{yw}^2 + \tau_{cr}^2\left[\left(\frac{3}{2}\sin 2\theta\right)^2 - 3\right]} \tag{4}$$

where the angle θ is the inclination of the membrane tensile yielding strength σ_t^y. It should be mentioned that, the above equation is applied only if τ_{cr} less than or equal τ_{yw}.

C. Collapsed Stage

Failure of the plate girder occurs when sufficient numbers of hinges have formed in the top and bottom flanges; together with the diagonal yield zone, the web panel forms a plastic sway mechanism. The additional shear load V_f sustained by the web panel until it collapses is determined from a consideration of virtual work applied to the sway mechanism [5], [6]:

For the assumed failure mechanism, the ultimate shear resistance V_f. of transversely stiffened girders can be expressed as:

$$V_f = \sigma_t^y * t * \sin^2\theta * \left(d * \cot\theta - b + \frac{c_c}{2} + \frac{c_t}{2}\right) + \frac{2 * M_{pfc}}{c_c} + \frac{2 * M_{pft}}{c_t} \tag{5}$$

$$c = \frac{2}{\sin\theta}\sqrt{\frac{M_{Pf}}{\sigma_t^y t}} \quad \text{where} \quad M_{pf} = \frac{c^2 * \sin^2\theta * \sigma_t^y * t}{4} \tag{6}$$

$$V_f = 2 * c * t * \sigma_t^y * \sin^2\theta + \sigma_t^y * d * t * \sin^2\theta * (\cot\theta - \cot\theta_d) \tag{7}$$

The first term on the right hand side of (7) represents the contribution of flanges to panel shear strength. The value of *c* is obtained by considering the equilibrium of the panel between the two plastic hinges in the flange, and θ_d is governed by the panel aspect ratio ($Cot\ \theta_d = b/d$).

$$V_{ult} = V_{cr} + V_f \tag{8}$$

$$V_{ult} = \tau_{cr} * d * t + 2 * c * t * \sigma_t^y * \sin^2\theta + \sigma_t^y * d * t * \sin^2\theta * (\cot\theta - \cot\theta_d)$$

In the above equations the ultimate shear load V_{ult} and the inclination of principal tensile stress θ are unknown. A parametric study shows that the variation of V_{ult} with θ is not abrupt [8], [9]. It was suggested that the assumption of $\theta = 2\theta_d/3$, in order to maximize V_{ult} [4].

Equation (8) can be expressed in non-dimensional from by dividing by the shear yield resistance of the web:

$$V_{yw} = \tau_{yw} * d * t = \left(\sigma_{yw} \big/ \sqrt{3} \right) * d * t \cdot$$

Hence

$$\frac{V_s}{V_{yw}} = v_1 + v_2 + m * v_3 \le 1.0 \qquad (9)$$

In which v_1 represent the shear buckling resistance of the web, v_2 are represents the post-buckling resistance derived from that part of the tension field supported by the transverse web stiffeners, and mv_3 represents the flange contribution to the shear resistance these terms may be expressed as

$$v_1 = \frac{\tau_{cr}}{\tau_{yw}} \qquad (10)$$

$$v_2 = \sqrt{3} * \sin^2\theta * \left(\cot\theta - \frac{b}{d} \right) * \left(\frac{\sigma_t^y}{\sigma_{yw}} \right) \qquad (11)$$

$$v_3 = 2\sqrt{3} * \sin\theta * \sqrt{\frac{\sigma_t^y}{\sigma_{yw}}} \qquad (12)$$

Since m is a function of c, it is taken as the lesser of m_1 and m_2 given by:

$$m_1 = \sqrt{\frac{4 * M_{pf}}{\sigma_{yw} * t * d^2}} \quad \text{and} \quad m_2 = \frac{b}{d} * \sin\theta * \sqrt{\frac{\sigma_t^y}{\sigma_{yw}}} \qquad (13)$$

To allow for the reduction in material strength due to welding, Evans et al. [3] incorporated a material strength reduction or softening factor k_z in the Cardiff tension field theory to predict the ultimate shear resistance of aluminum alloy plate girders. It was argued that, as the tension field must pass through the welded web boundary to anchor on to the boundary members, the material softening factor should be applied to all the components of shear resistance except the buckling resistance. This recommendation was subsequently incorporated in BS8118: 1991: part 1, [9]. BS8118 also recommends that the allowable yield stress in the HAZ be taken as $k_z\ \sigma_y$; where σ_y is the yield stress of the parent material, and k_z is equal to 0.5 and 0.6 for 6000 and 7000 series aluminum alloys, respectively [9].

III. BS 8118 Design Procedure

The design procedure incorporated in BS8118 is based on the modification of Cardiff tension-field theory proposed by Evans et al. [3]. The code also introduces a partial material safety factor $\gamma m = 1.2$ in the calculation of the ultimate shear resistance Vrs equation (9) is rewritten as

$$\frac{V_{rs}}{V_{yw}} = \frac{\left[v_1 + k_z * \left(v_2 + m * v_3 \right) \right]}{\gamma_m} \le 1 \qquad (14)$$

V_{rs} is based on nominal materials properties (E= 70 kN/mm^2 and μ = 0.34) and design curves are provided for the evaluation of v1, v2 and v3.

IV. Shear Strength of Plate Girder Web Using Höglund Theory

Höglund rotating- stress-field theory is based on a system of perpendicular bars in compression and tension, which are assumed to represent the web panel. In pure shear the absolute value of the principal membrane stresses σ1 and σ2 are the same as long as no buckling occurred (τ < τcr). After reaching buckling load (Vcr= dw * tw *τcr) the web plate will buckle and redistribution of stresses starts. Increase in tensile stress σ1 due to increasing in applied load where slightly increase in compressive stress σ2 may be occurred as shown in Fig. 3.

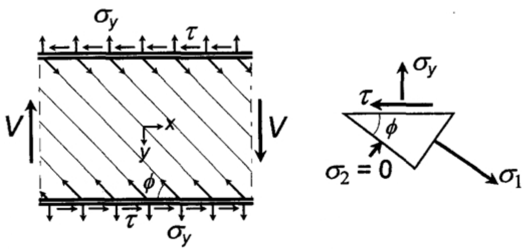


Fig. 3 State of stress in web

From Fig. 3;

$$\tau = \sigma_1 * \sin\phi * \cos\phi = 0.50 * \sigma_1 * \sin 2\phi \qquad (15)$$

where the direction of the tensile stresses is chosen to give maximum value of τ. When σ1 equal to the yield strength of the web, f_{yw}, then

$$\frac{\tau_u}{f_v} = \frac{0.5 * f_{yw}}{f_v} = \frac{\sqrt{3}}{2} \quad \text{for} \quad \phi = 45^\circ \qquad (16)$$

where

$$f_v = \frac{f_{yw}}{\sqrt{3}} \qquad (17)$$

This theory is called ideal tension field theory, and is valid only if the flanges are prevented from moving towards each other by an external structure such as an inner panel in a plate with rigid cross beams and stringers [10].

Furthermore, the slenderness parameter λ_w is introduced

$$\lambda_w = \sqrt{\frac{f_v}{\tau_{cr}}} \quad (18)$$

From above equations the ultimate strength τu = τ can be derived as a function of λw as follow:

$$\frac{\tau_u}{f_v} = \frac{\sqrt[4]{3}}{\lambda_w} * \sqrt{\sqrt{1 - \frac{1}{4\lambda_w^4}} - \frac{1}{2\sqrt{3}\lambda_w^2}} \quad \text{for } \lambda w \geq 1.00 \quad (19)$$

if λw ≥ 2.5 leads to

$$\frac{\tau_u}{f_v} \cong \frac{1.32}{\lambda_w} \quad (20)$$

The shear buckling capacity can be obtained from

$$V_w = \rho_v * f_{yw} * d * t_w \quad (21)$$

where ρ_v can be defined as shear buckling reduction factor which is given in Table I:

TABLE I
REDUCTION FACTOR ρ_v FOR SHEAR BUCKLING

λ_w	Rigid end Post (Aluminum)	Non-Rigid end Post (Aluminum)
$\lambda_w < 0.48/\eta$	η	η
$0.48/\eta \leq \lambda_w < 0.95$	$\frac{0.48}{\lambda_w}$	$\frac{0.48}{\lambda_w}$
$\lambda_w \geq 0.95$	$\frac{1.32}{(1.66 + \lambda_w)}$	$\frac{0.48}{\lambda_w}$

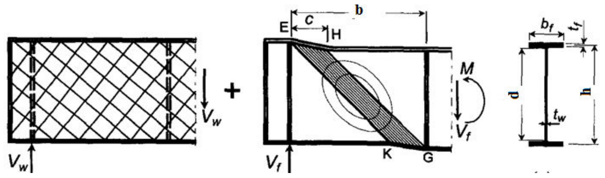
The value of ρ_v is a reduced value related to scatter in test results such as initial imperfections [10]. For small slenderness ratios, $\lambda_w < 0.48/\eta$, Strain hardening in shear can take place, giving larger strength than corresponding to initial yielding

$$\eta = 0.40 + 0.20 * \frac{f_{uw}}{f_{yw}} \quad (22)$$

A. Transversely Stiffened Web

Transverse stiffeners welded to the web have two main effects on the behavior and strength of a girder in shear : first, they prevent the web from out-of-plane deflections, thus increasing the elastic buckling strength, and second, they prevent the flanges from coming closer to each other. As mentioned, transverse stiffeners prevent the web from deflecting and prevent the flanges from coming nearer to each other at the stiffeners in the stage of failure. Four hinges denoted E, H, G, and K, formed in the top and bottom flanges respectively as shown in Fig. 4 (b) perform the tension stress field, EHGK. The contribution of the flanges in shear force, Vf, which is transmitted by the tension stress field is obtained from the equation of equilibrium of the flanges portion c as follow:

$$V_f = \frac{4 * Z * f_{yf}}{c} \quad (23)$$



(a) Shear force carried by web (b) Shear force carried by truss action (c) Cross Section

Fig. 4 Model of web in post buckling range

The tensile stresses in the tension stress field produce a stiffening effect on the web [10]. This effect is favorable to the load-carrying capacity. However, it is assumed that the shear resistance of the web, V_w is not changed by the formation of the tension field between flanges. Then the shear resistance of the girder, Vu, is the sum of the shear resistance of the web, Vw, and the shear resistance contributed by the flanges Vf.

$$V_u = V_w + V_f. \quad (24)$$

The distance c is estimated for aluminum plate girders as follow:

$$c = \left(0.08 + \frac{4.4 * b_f * t_f^2 * f_{yf}}{t_w * d_w^2 * f_{yw}}\right) * b \quad (25)$$

V. PROPOSED DESIGN METHOD

The proposed design method differs from the modified Cardiff tension-field theory proposal by considering that the influence of the reduction in material strength due to welding is taken as having a uniform effect on the overall shear resistance provided by the web and flanges. In-spite of the HAZ is limited to a small region in the vicinity of welds, the extent of which can be considered to affect all web panel and flanges to take into account the initial imperfection and any other disfiguration may be performed during fabrication process. The proposed equation may be rewritten as:

$$\frac{V_{rs}}{V_{yw}} = \frac{[v_1 + (v_2 + m * v_3)]}{\gamma_m} \leq 1$$

where

$$V_{yw} = \tau_{yw} * d * t$$

and γm is partial material safety factor equal 1.2 for aluminum Alloy 6000 and 1.1 for aluminum Alloy 7000.

VI. Experimental Results and Comparison with Theoretical Predictions

Extensive experimental studies have been conducted in Cardiff on the ultimate shear resistance of aluminum alloy plate girders, a summary of which have been presented by Evans and Lee [3] listed in Table II. A summary of these tests conducted on transversely stiffened, 6000 and 7000 series aluminum alloy plate girders, subjected to predominantly shear loading.

VII. Comparison Between Design Methods

The results of experimental study conducted by Evans and Lee [3] to obtain the ultimate shear resistance of aluminum alloy plate girders are summarized in Table I. The table also described a summary of material properties, samples dimensions and test results conducted on transversely stiffened, 6000 and 7000 series aluminum alloy plate girders and subjected to predominantly shear loading.

For aluminum alloys 6000 and 7000, Figs. 5 and 6 show the comparison between the ultimate shear resistance which can be obtained by applying the different theories, code and the proposed equation (Cardiff theory, Höglund theory, BS8118, Proposed equation). Tables III and IV also summarized the mean, standard deviation and coefficient of variation for the results of the ratio between ultimate shear resistance obtained experimentally compared with the value of ultimate shear resistance obtained from the each of the four methods described above. From Fig. 1 and Table II, it can be summarized that, BS8118, using nominal values of material properties and including an additional partial material safety factor 1.2, appear inconsistent and unduly conservative when compared with the available test data. Theoretical predictions based on the procedure proposed by Höglund's theory and proposed equations are in good agreement with test results.

TABLE II
Details of Test Girders and Test Results

Girder reference	b	d	tw	bf	tf	E	fyw	fyf	Vu kN	Vexp./VS Cardiff	Vexp./VS BS8118	Vexp./VS Hoglund	Vexp./VS Proposed
6000 series aluminum alloy plate girders													
AG1	669	455	1.6	101	9.6	69500	283	300	56.3	0.96	2.14	1.09	1.15
AG2	671	455	1.21	101	9.6	69300	260	301	41.4	0.94	2.18	1.27	1.13
AG3	672	456	1.21	102	6.5	68100	259	285	33.9	0.94	2.15	1.15	1.13
AG4	669	455	1.64	101	6.5	67800	285	286	52.3	1.02	2.26	1.07	1.23
AG5	667	457	1.61	101	12.6	67500	287	295	69.3	1.01	2.29	1.26	1.21
AGS1-T1	442	454	1.2	102	6.3	71500	279	239	47	0.97	2.22	1.30	1.17
AGS2-T1	218	456	1.2	104	6.3	71500	252	239	62	0.96	2.08	1.11	1.15
AGS2-T2	218	456	1.2	104	6.3	71500	252	239	63.5	0.98	2.13	1.14	1.18
AGS3-T2	218	456	1.2	103	6.3	71500	252	239	59	0.91	1.98	1.06	1.09
AGS4-T1	218	456	1.2	104	6.3	71500	252	239	60.5	0.93	2.02	1.08	1.12
AGS5-T1	144	455	1.2	102	6.3	71500	245	239	62	0.85	1.70	0.84	1.02
AGS5-T2	143	455	1.2	102	6.3	71500	245	239	68.5	0.93	1.87	0.92	1.12
AGCS1-T1	219	455	1.2	102	6.4	67500	268	277	57.9	0.83	1.83	0.99	1.00
AGCS1-T2	215	455	1.2	102	6.4	67500	268	277	59.7	0.85	1.87	1.00	1.02
AGCS2-T1	218	454	1.2	102	6.4	67500	268	277	61.7	0.89	1.95	1.05	1.07
AGCS2-T2	216	454	1.2	102	6.4	67500	268	277	61.8	0.89	1.94	1.04	1.06
AGCS3-T1	219	455	1.2	103	6.4	67500	268	277	56.7	0.81	1.79	0.96	0.98
AGCS3-T2	216	455	1.2	103	6.4	67500	268	277	60.6	0.87	1.90	1.02	1.04
A6G72s-T2	444	898	3.2	174	12.7	68400	287	311	330	0.88	1.82	0.97	1.06
TAG1	900	900	3.02	175	12.8	70000	303	308	277.5	1.09	2.45	1.32	1.31
7000 series aluminum alloy plate girders													
A7G1	943	602	3.2	115	12.9	75200	344	307	185.3	1.11	2.01	0.99	1.22
A7G2	624	602	3.2	105	10.1	75200	344	307	234.2	1.16	2.09	1.15	1.28
A7G3	478	601	3.2	209	9.5	75200	344	307	285.9	1.10	1.97	1.15	1.21
A7G4	304	601	3.2	125	12.9	75200	344	307	454.9	1.37	2.29	1.38	1.50
A7G5	201	602	3.2	90	15.9	75200	344	307	447.9	1.12	1.68	1.04	1.23
A7G6	302	317	3.2	138	6.1	75200	344	307	192.3	1.25	1.91	1.19	1.38
A7G7	885	894	3.2	175	12.9	75200	344	307	298.9	1.00	1.90	1.19	1.10
A7G8	1039	1052	3.2	222	16	75200	344	307	360.7	0.98	1.89	1.27	1.08
A7G9	624	603	3.2	130	9.6	75200	344	307	236.2	1.15	2.07	1.14	1.27
A7G10	626	602	3.2	116	15.9	75200	344	307	236.2	0.99	1.81	1.04	1.09
A7G11	624	602	3.2	165	18.7	75200	344	307	354.2	1.28	2.36	1.46	1.41

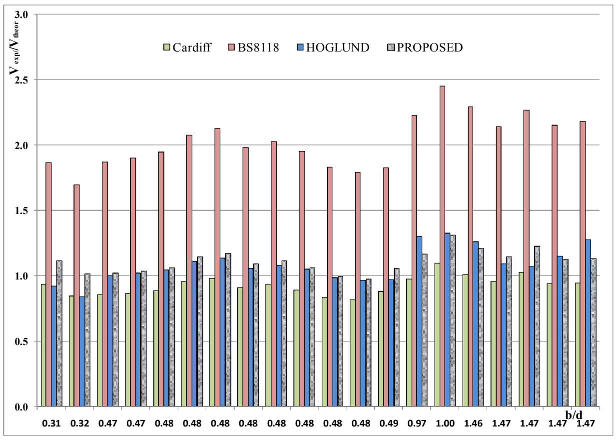


Fig. 5 Comparison between V_{exp}/V_S for Alloy 6000

TABLE III
MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION FOR 20 SAMPLES ALLOY 6000

Method of analysis	ALLOY 6000 - Sample Size (S.Size) = 20		
	Mean	Standard deviation	Coefficient of variation
Cardiff	**0.93**	**0.07**	**0.07**
BS8118	**2.03**	**0.19**	**0.09**
HOGLUND	**1.08**	**0.13**	**0.12**
Proposed equation	**1.11**	**0.08**	**0.07**

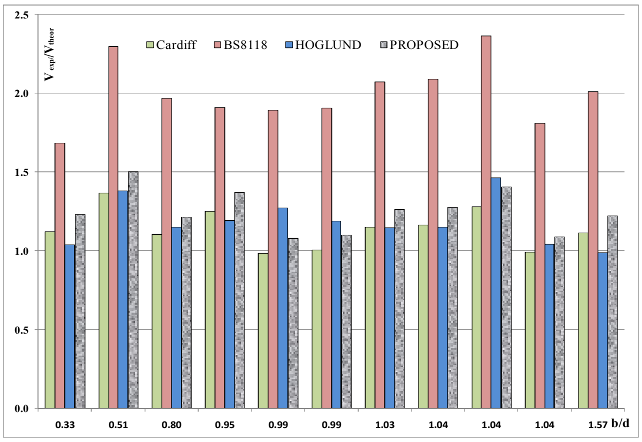


Fig. 6 Comparison between V_{exp}/V_S for Alloy 7000

TABLE IV
MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION FOR 11 SAMPLES ALLOY 7000

Method of analysis	ALLOY 7000 - Sample Size (S.Size) = 17		
	Mean	Standard deviation	Coefficient of variation
Cardiff	**1.14**	**0.12**	**0.10**
BS8118	**2**	**0.19**	**0.10**
HOGLUND	**1.18**	**0.14**	**0.12**
Proposed equation	**1.25**	**0.13**	**0.10**

VIII. CONCLUSION

- Cardiff theory can be used to predict the ultimate shear resistance of steel plate girders having intermediate transverse stiffeners only and not applied for aluminum plate girders.
- BS8118, using nominal values of material properties and including an additional partial material safety factor 1.2, appears inconsistent and unduly conservative.
- Theoretical predictions based on the procedure proposed by Höglund's theory, for aluminum plate girders are more realistic than BS8118.
- The tension field must pass through the welded web boundary to anchor on to the boundary members, the material softening factor should be applied to all the components of shear resistance (i.e. Web and flange components).
- To allow for the reduction in material strength due to welding in the Cardiff tension field theory, the material safety factor is equal to 1.2 and 1.1 for 6000 and 7000 series aluminum alloys, respectively.

The formula proposed to predict the values of ultimate shear resistance using Cardiff theory are in good agreement with test results.

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