

# Load Transfer Mechanism Based Unified Strut-and-Tie Modeling for Design of Concrete Beams

Ahmed, M., Yasser A., Mahmoud H., Ahmed, A., Abdulla M. S., Nazar, S.

**Abstract**—Strut-and-Tie Models (STM) for the design of concrete beams, comprising of struts, ties, nodes as the basic tools, is conceptually simple, but its realization for complex concrete structure is not straightforward and depends on flow of internal forces in the structure. STM technique has won wide acceptance for deep member and shear design. STM technique is a unified approach that considers all load effects (bending, axial, shear, and torsion) simultaneously, not just applicable to shear loading only. The present study is to portray Strut-and-Tie Modeling based on Load-Transfer-Mechanisms as a unified method to analyze, design and detailing for deep and slender concrete beams. Three shear span- effective depth ratio ( $a/d$ ) are recommended for the modeling of STM elements corresponding to dominant load paths. The study also discusses the research work conduct on effective stress of concrete, tie end anchorage, and transverse reinforcement demand under different load transfer mechanism. It is also highlighted that to make the STM versatile tool for design of beams applicable to all shear spans, the effective stress of concrete and, transverse reinforcement demand, inclined angle of strut, and anchorage requirements of tie bars is required to be correlated with respect to load transfer mechanism. The country code provisions are to be modified and updated to apply for generalized design of concrete deep and slender member using load transfer mechanism based STM technique. Examples available in literature are reanalyzed with refined STM based on load transfer mechanisms and results are compared. It is concluded from the results that proposed approach will require true reinforcement demand depending on dominant force transfer action in concrete beam.

**Keywords**—Deep member, Load transfer mechanism, Strut-and-Tie Model, Strut, Truss.

## I. INTRODUCTION

STRUT-and-Tie models (STM) are the distinct depiction of statically equivalent distributed stress field, developed in a continuum structural domain due to combined action of load of bending, axial, shear, torsion, by resultant straight lines and concentrate the curvature of the stress field in nodes. STM is essentially a truss analogy. Truss members that are in compression, termed as struts, are made up of concrete, while truss members that are in tension consist of steel reinforcement. Transverse reinforcement may be necessary to prevent splitting caused by transverse tension due to high compressive stress in the strut. The complexity of strut-and-tie

method involve in transforming a continuous description of a structural domain to a discrete strut-and-tie model and in specification of concrete effective strength under different force flow conditions. The several empirical approaches are proposed to aid in generating STM. Schlaich et al. [1] suggested that one possible approach for the development of suitable strut and tie models is to orient the struts and the ties members within 15 degrees from the principal stress path obtained from elastic un-cracked stress field and the most valid model tends to be the one that minimize the amount of reinforcement. The crack patterns, if available, are helpful in laying out best strut-and-tie models. It is suggested [2] that a STM developed with struts parallel to the orientation of initial cracking will behave very well. The computer-aided topology optimization for continuum structures [3] is also utilized for the development of the strut-and-tie models where the lowliest stressed portions from the structural concrete member are gradually removed.

The concept of strut-and-tie modeling was first introduced by Ritter [4] and Morsch [5]. Considerable contributions are under way for further development of strut-and-tie modeling and for application of strut-and-tie technique to shear critical regions and deep structures. Zhu et al. [6] conducted experiments for crack width prediction in concrete beams using a compatibility-aided strut-and-tie model. The strut-and-tie model technique is applied by Ahmed et al. [7] to design the T-cantilever deep beam. To validate the design of concrete structures using STM, an automated finite-element-based validation method is presented by Park et al. [8]. Macro-mechanical strut and tie model has been presented by Khalifa [9] for analysis of fibrous high-strength concrete corbels. Over the past decade, the concept of strut-and-tie approach has been incorporated into several codes of practice and design guidelines. An experimental investigation, aimed at evaluating the adequacy of the strength factors for concrete struts in ACI building code, is carried out by Febres et al. [10]. Rathie [11] investigates the proposed strut-and-tie section of draft code DR05252, revision to Australian code - AS 3600 [12]. The ACI318-05 and AASHTO-LRFD code provisions related to STM are evaluated and new recommendations are suggested, using published database of large number of tests, by Brown and Bayrak [13]. Park and Kuchma [14] have predicted strength of deep beam using STM. They consider cracked concrete constitutive law for wide range of provided horizontal and vertical web reinforcement ratios, concrete compressive strengths, and shear span-to-depth ratios ( $a/d$ ). On the basis of transfer mechanism of shear loads, an analytical method has been proposed by Kuo et al. [15] to

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define the boundaries of Bernoulli (B) regions and disturbed (D) regions. They emphasized to include dimension of member, strength of material and reinforcement details to define the D-regions, and classified the beams as deep beams, slender (DBD) beams and short (DD) beams based on the length of D-regions. Kim and Jeong [16] have developed a model to decouple the shear contribution by the tied arch action in concrete beams subjected to combined shear and bending.

Various researchers have contributed to study the effect of the Shear span - Effective depth ( $a/d$ ) ratio on the shear strength and overall behavior of reinforced concrete beams. Foster and Gilbert [17] suggested to select different STM for design based on the shear span to flexure lever arm ratio ( $a/z$ ) i.e. direct strut-and-tie model for  $a/z < 1$ , combined strut-and-tie model for  $1 < a/z < 1.73$  and two panel truss model for  $a/z > 1.73$ . Russo et al. [18] have proposed for high strength concrete deep beam, a critical ( $a/d$ )<sub>c</sub> ratio as  $\frac{0.57 \rho^{0.19} f_y^{0.41}}{f_c^{0.05}}$  such that if ( $a/d$ ) < ( $a/d$ )<sub>c</sub>, arch action will dominant in the beam while if ( $a/d$ ) > ( $a/d$ )<sub>c</sub>, truss action will dominant in the beam. Concrete beam specimens with  $a/d$  ratio ranged from approximately 1 to 8 were tested by Ahmad and Lue [19]. It was observed that the capacity and the failure mode of the test specimens were largely a function of the  $a/d$  ratio. The failure was by crushing of the arch at  $a/d$  ratio less than 1.5, whereas at an  $a/d$  ratio between 1.5 and 2.5, the beams failed by shear compression of the web of the beam. The shear behavior of the beam transitioned from a shear-compression type failure to a diagonal-tension type failure at an  $a/d$  ratio of approximately 2.5. At an  $a/d$  ratio between 2.5 and 6, the failure of the beams was due to a diagonal tension crack that originally propagated from a flexural crack (flexure-shear crack) whereas the beams generally failed in flexure at an  $a/d$  ratio more than 6. Experimental database are compiled by Brown et al. [28] to examine the effects of position of the load on the load transfer mechanism of reinforced concrete beams. The experiments were conducted by Bircher et al. [20] to study the effect of minimum reinforcement and  $a/d$  ratio on strength and serviceability of deep beam. Test specimens were of  $a/d$  ratios as 1.2, 1.85, and 2.5 with 0.2% web reinforcement in each direction. With increasing  $a/d$  ratio, a change in failure mode was observed. At  $a/d$  ratio of 1.2, the failure was the result of crushing along the diagonal strut but at an  $a/d$  ratio of 2.5, it was completely due to diagonal tension failure. At  $a/d$  ratio of 1.85, the load is transferred partly by direct-strut mechanism and partly by sectional shear or diagonal tension mechanism. They also suggested that a multiple-panel STM should be used when the  $a/d$  ratio exceeds 2.

The development of suitable layout of strut and tie for concrete structures is the major task in the Strut-and-Tie technique. An ( $a/d$ ) ratio, a factor for the development of load transfer action, is proposed in the present study for selection of a unified Strut-and-Tie Models for concrete beams. Based on cited reference and literature survey (Fig. 1, [2]), three shear span- effective depth ratio ( $a/d$ ) are recommended for the modeling of STM elements corresponding to dominant load

paths, namely,  $a/d \leq 1$  (Arch action),  $1 < a/d < 2.5$  (Arch - Truss transition or combined action) or  $a/d \geq 2.5$  (Truss action). It should be noted that the strut-and-tie models provides only the locations of its components. The exclusive recommendation based on the strength performance criteria should be framed for the concrete effective stress, strut angle, minimum reinforcement requirements and anchorage requirements of tie correlating with different ( $a/d$ ) ratios to evolve a unified STM technique of concrete beams.

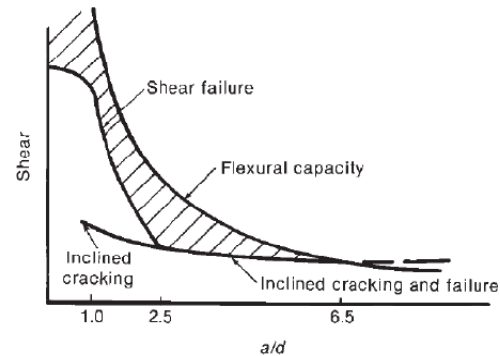
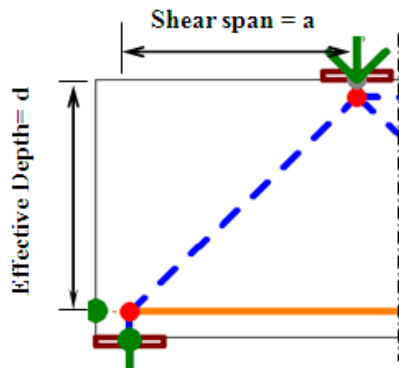
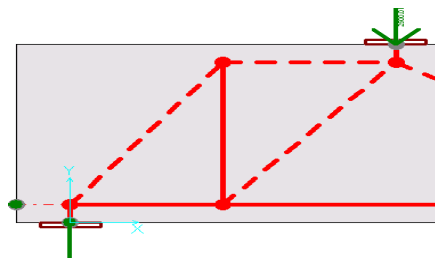
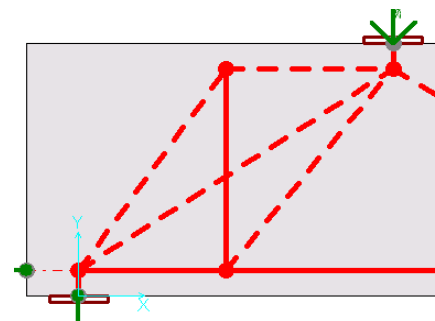


Fig. 1 Shear at failure with  $a/d$  ratio

## II. STM AND LOAD TRANSFER MECHANISM

Most of studies using STM for concrete members consider two basic types of load transfer actions only, i.e. direct strut or truss action, and transition force transfer or combined action is generally ignored. There are also disagreements on distribution of load transferred in combined force transfer mechanism. Separate configurations of STM are necessitated for individual force transfer actions and combination of force actions. For the beams with ( $a/d$ )  $\leq 1$ , direct strut or arch transfer action will govern i.e. major portion of the load is directly transferred to support through single inclined strut. For this mechanism, one panel STM will be required for load transmission. For the beams with ( $a/d$ )  $\geq 2.5$ , flexural action will govern i.e. a two or multiple panel STM will be required for load transmission. In the truss model, an additional tie is required to balance the transverse component of the force in the flatter inclined strut. For the beams with  $1 < (a/d) < 2.5$ , transition force transfer actions is existed. For this mechanism, combined STM, i.e. two-panel STM with one direct strut from load to support is needed. The possible configurations of Strut-Tie Model based on force transfer action are shown in Figs. 2-4. Fig. 2 shows Strut-Tie Model for ( $a/d$ )  $\leq 1$  considering tied-arch action. Fig. 3 depicts Strut-Tie Model for ( $a/d$ )  $\geq 2.5$ , considering truss action. Strut-and-Tie Model for  $1 < (a/d) < 2.5$ , representing combined arch-truss action is shown in Fig. 4.

Fig. 2 Strut-and-Tie Model for  $a/d \leq 1$  (Arch action)Fig. 3 Strut-and-Tie Model for  $a/d \geq 2.5$  (Truss action)Fig. 4 Strut-and-Tie Model for  $1 < a/d < 2.5$  (Arch-Truss transition action)

The national standards guidance except FIP recommendations [21] permits the layouts representing either arch or truss action in Strut-Tie Model with a restriction on the inclined angle of the strut. FIP recommendations, which are adopted from the Schafer [22] proposals, consider load transferred to support by combination of arch and truss action in all shear span- depth ratios condition and discard the direct arch action or pure truss action. According to FIP recommendations, the fraction of load transferred to support by the transverse ties truss depends on the shear span- internal lever arm ratio ( $a/z$ ) and is represented as  $(1/3) [(a/z) - 1]$ . The remainder of the load is transferred directly to the support by the direct strut action. Foster and Gilbert [23] have also proposed similar expression. According to them, the load contribution to support in concrete member by the transverse ties truss is given by  $(1/2) [\sqrt{3} \cdot (a/z) - 1]$ . More recently, Zhi et al. [24] derive the relations for fraction of load transfer between direct strut and truss mechanism in combined force

transfer action, separately for corbel and deep beams. They suggested that combined mechanism will exist for  $a/d$  ratio between 1 and 2 and combined mechanism will consist of vertical tie truss mechanism, horizontal tie truss mechanism and direct compression strut mechanism. The portion of load transferred by the vertical tie truss mechanism and by the horizontal tie truss mechanism in deep beam shall be  $(4/3) [1 - 3/2(a/z)^2]$  and  $(4/3) [1 - 3(a/z)^2 / 2]$  respectively. The remainder of the load is transmitted directly to the support by direct strut mechanism. It is interesting to note that upper limit of  $a/z$  ratio given by them for absolute dominance of vertical tie truss mechanism is when  $a/z \geq 2.45$ . It is found that that the contribution of the truss action by such recommendations is overestimated. The effect of transverse reinforcement has not been duly accounted in calculating the load transfer fraction between the direct strut and truss mechanism.

### III. CONCRETE EFFECTIVE STRESS OF STRUT

Cracked concrete subjected to high tensile strain in the direction normal to the compression is softer than concrete in a standard cylinder test, in other words concrete element in STM under loading is stressed to an effective concrete strength [25]. The major factors affecting the compressive strength are width and orientation of crack, longitudinal reinforcement and lateral confinement. The compressive strength of concrete also affects the load transfer characteristics in beams. The STM element subjected to different load action will have different characteristics and behavior such as cracking and degree of softening of concrete strength, and mode of failures. The severity of strength softening is highest in the truss action region and least in direct arch action. Standards and researchers recommend two approaches to introduce effective concrete strength in STM, one is single capacity reduction factor and other is material capacity reduction factors based on actual tensile and compressive strain in concrete and reinforcement. Standards, such as ACI318, recommend a single capacity reduction factor for concrete strength while other standards, namely AASHTO, Australian code etc., recommend using a material capacity reduction factors. It will be helpful in generalizing the STM if strength reduction factors are integrated with load transfer mechanism i.e relationship in terms of  $a/d$  ratios. Wang et al [26] has recommended to include  $a/d$  ratio for determining the effective strength of concrete deep beams. The following are the capacity reduction or so-called efficiency factor, to account for concrete softening in terms of  $a/d$  ratio, (applicable to  $a/d$  ratio  $< 2$  and irrespective of dominant force transfer action) proposed by various authors.

Foster and Gilbert [23]:

$$\frac{1}{1.14 + (0.64 + f'_c/470) (a/d)^2} \quad (1)$$

Wang et al. [26]:

$$(0.8 - f'_c/200)[1.25 - 0.25(a/h)] \quad (2)$$

Collins et al. [27]:

$$\frac{1}{0.8+170\epsilon_1} \quad (3)$$

Brown et al. [28]:

$$36.67(a/d \sqrt{f'_c})^{-0.97} \quad (4)$$

Arabzadeh et al. [29]:

$$\frac{f'_c{}^{-0.3}}{0.5+0.1(a/d)^2} + 0.09 \rho_p^{0.65} \frac{f_y}{f'_c} \frac{a}{a_s} \frac{1}{\sin \theta} \quad (5)$$

Sahoo et al. [30]:

$$\left(0.6 + \frac{0.05}{r_c} + 55\rho_T\right) \frac{\alpha_s}{90} \quad (6)$$

Rao and Sundareson [31]:

$$\frac{A_1 \rho^{a_1} j_o \sqrt{f'_c}}{1+B_1(a/d)} \quad (7)$$

where:  $\alpha_s$  or  $\theta$  = Smallest angle between the compressive strut and adjoining tie;  $\epsilon_1$  = Tensile strain that can be expressed as

$$\epsilon_1 = \epsilon_s + (\epsilon_s + 0.002) \cot^2 \alpha_s$$

$\epsilon_s$  = Tensile strain in the concrete in the direction of the tension tie usually taken as 0.002;  $f'_c$  = Specified concrete compressive strength;  $\rho_T$  = effective transverse ratio;  $r_c$  = Concentration ratio of the load resisted by the diagonal;  $\rho$  = longitudinal reinforcement ratio;  $j_o$  = the neutral axis depth

#### IV. INCLINED ANGLE OF STRUTS

The degree of strength softening of concrete will dictate the minimum angle of strut permitted for STM analysis. Potential compatibility problems in STM are also avoided with limits on angles between struts and ties and minimum reinforcement requirements. Bentz et al. [32] have given the formula to determine the angle ( $\theta$ ) of strut as

$$\theta = (29 \text{ deg.} + 7000 \epsilon_x)(0.88 + \frac{S_{xe}}{2500}) \quad (8)$$

Kim and Mander [33] have calculated the cracked angle of concrete for shear and flexure region and have developed the following relation to calculate the cracked angle using energy minimization method

$$\theta = \tan^{-1} \left[ \frac{\rho_v n + \zeta \frac{\rho_v A_v}{\rho_v A_g}}{1 + \rho_v n} \right]^{1/4} \quad (9)$$

where  $A_g$  = gross section area of concrete element;  $A_v$  = shear area of concrete section;  $n = E_s/E_c$  = Modular ratio;  $E_c$  = Elasticity modulus of concrete;  $E_s$  = Elasticity modulus of

steel;  $\rho_v$  = Volumetric ratio of shear steel;  $\rho_l$  = Volumetric ratio of longitudinal steel;  $\zeta$  = End Fixity Factor

The country standards have recommended a varying minimum angle of concrete strut. The minimum angle recommended for strut in Euro code, ACI318, Swiss code and AS3600 are  $22^\circ$ ,  $25^\circ$ ,  $26^\circ$ ,  $30^\circ$  respectively. There is no limitation placed on angle of strut in AASHTO code. For generalization of Strut-tie-model technique, the specific recommendation for angle should be proposed based on the dominant load transfer mechanism in the concrete beams.

#### V. TRANSVERSE REINFORCEMENT

The load transfer mechanism and redistribution of internal forces is depends on the amount of transverse reinforcement in addition to shear span-depth ratio ( $a/d$ ) and compressive strength of concrete. The purpose of web reinforcement, in direct arch mechanism is to resist the transverse tensile forces developed in a concrete strut. As the  $a/d$  ratio increases and the behavior of the load transfer in beams transitions from an arch action to a truss action, the effectiveness of vertical reinforcement increases. Many studies have been devoted to web reinforcement requirements in STM techniques. Ghoneim [34] based on experimental studies has concluded that the beneficial effect of the vertical web reinforcement is more significant at  $a/d$  ratio of 0.95 and confirmed the established knowledge that the effect of horizontal web reinforcement diminishes with the increase of  $a/d$  ratio. Based on the Comparisons of results obtained from the experimental program, STM, Conventional Sectional Method and Non-Linear Finite Element analysis, Shuraim [35] concluded that 0.003% web reinforcement recommended as minimum web reinforcement by codes will not ensure ductile behavior in the reinforced concrete deep beams. Sahoo et al. [36] has experimentally investigated that relative effectiveness of vertical and horizontal reinforcement in beams depends on strut inclination ( $a/d$  ratio). Horizontal reinforcement may be more effective in beams with steeply inclined diagonal strut and vice versa. They also modified the ACI318 web reinforcement transformation equation to represent a clear relationship between transverse reinforcement and strut efficiency factor. The proposed modified equation is represented as

$$\rho_{\perp} = \sum \frac{A_{si} \sin^2 \alpha_i}{b_s s_i} \quad (10)$$

where  $A_{si}$  is the cross-sectional area of each layer of web reinforcement in the  $i$ -th orientation;  $b_s$  is the strut or beam thickness (out-of-plane);  $s_i$  is the spacing of web reinforcement in the  $i$ -th orientation; and  $\alpha_i$  is the angle between the strut and the bars in the  $i$ -th orientation.

Martin and Sanders [37] has proposed crack control web reinforcement as

$$0.003[\sin(\alpha_s) + \cos(\alpha_s)] \quad (11)$$

The minimum web reinforcement equation sensitive to load transfer mechanism i.e.  $a/d$  ratio is proposed by Brown et al. [28] and is given as,

$$\rho_{\perp, \min} \geq \frac{v f_c' A_c \sin \theta}{f_y b d m} \quad (12)$$

where,  $v$  = efficiency factor for reinforced struts,  $A_c$  = minimum cross-sectional area of the strut,  $\theta$  = angle of strut with respect to the horizontal  $f_y$  = yield strength of web reinforcement,  $b$  = width of strut,  $d$  = effective depth of the strut;  $m$  = slope of the angle of dispersion,  $\rho$  = reinforcement ratio perpendicular to the splitting crack

The country codes specify varying minimum reinforcement for strut in transverse directions. The recommended minimum reinforcement across the strut in different codes varies from 0.0015 to 0.003%. There is need to unify the recommendations for transverse reinforcement considering load transfer mechanism and other related factors. The proposals for relative proportion of reinforcement demand in vertical and horizontal direction of the member should also be based on prevailing load transfer mechanism.

#### VI. TIE BARS END ANCHORAGE

The transverse confining conditions that develop from the reaction and diagonal strut at the support of concrete members will affect the load transfer mechanism in the beams. The end anchorage requirements will be different over the support to develop the probable load transfer mechanism. Rogowsky et al. [38] reported that for beams with small shear span-to-depth ratios, compressive stresses at the supports can greatly reduce the anchorage length of the bars. Bou Saleh Ahmed [39] shows that the confining pressure at the support reactions will assist in developing the required yield strength in the steel bars even though the provided development length is below code requirements. Moran et al. [40] has experimentally studied the concrete beam reinforced with high strength steel and provided with ACI318 code straight anchorage having ( $a/d$ ) ratio as 1.2, 1.8 and 2.4 and no anchorage failure was observed at beams end. They concluded that existing code provisions for development length are feasible for beam design using STM method. Roy and Brena [41] and Brena and Roy [42] has also examined and discussed end anchorage requirements of beams with different  $a/d$  ratios. They concluded that beams with straight bar anchorages of only approximately 50% of the length required by provisions of ACI 318-08 was able to develop peak loads comparable with beams where longer anchorages were provided. For arch action type force transfer action, a full anchorage of tie bars may be needed but partial anchorage could be provided for other type of load transfer action. For generalized design of beams with STM technique, clear guidance should be framed for bar anchorage requirement correlated to load transfer action.

#### VII. EXAMPLE PROBLEMS

Example problems of concrete deep beam with eccentric loading having  $a/d$  ratio as  $\approx 1$  and  $\approx 2$  available in literature [43] is considered for demonstration of the force transfer mechanism ( $a/d$  ratio) based Strut-and-Tie Models formulation. The details of the beam are shown in Fig. 5. The finite element procedure based computer program (CAST-Computer Aided Strut-and-Tie) developed by Tjhin and Kuchma [44] is used to evaluate STM element forces of the problems. The design of the beam is based on ACI code provisions. CAST is an integrated analysis and design tool for structural concrete using strut-and-tie models. CAST provides a graphical interactive environment for all aspects of the design process, including definition of the D-region, selection of the strut-and-tie model, truss analysis and member definitions. CAST designs the STM elements for multiple load cases in which equilibrium is satisfied between the applied loadings and forces in the strut-and-tie model components. The loading acting on the D-regions is limited to static monotonic. The idealized shape of struts and ties is prismatic. The nodal zones are formed by the intersection of effective widths of the framing struts and ties. The primary failure modes of the D-regions are the yielding of ties, crushing of struts or nodal zones and diagonal splitting of struts.

The concrete grade of the example beam is M 28 and steel grade is Fe 410. The conventional STM is modified to suit the load transfer action. The left span of the beam is under possible arch type force transfer action and right span is under transition arch-truss action as  $a/d$  ratios are 1 and 2 respectively. The conventional STM and proposed STM based on load transfer mechanism are shown in Figs. 6, 7.

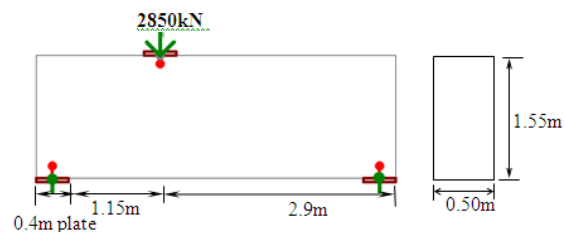


Fig. 5 Loading and Geometry of the deep beam

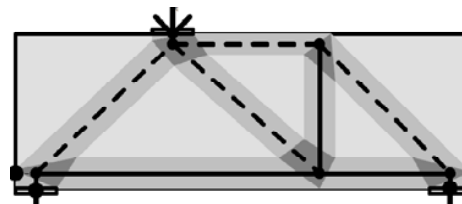


Fig. 6 Layout of Conventional Strut-and-Tie Model assumed in available literature



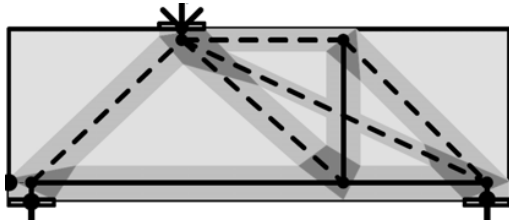


Fig. 7 Layout of Proposed Load Transfer Mechanism based Strut-and-Tie Model (Left side span: direct strut ( $a/d \approx 1$ ); Right side span: Arch-Truss transition ( $a/d \approx 2$ ))

The conventional STM and proposed STM based on load transfer mechanism for the beam along with STM elements stress demand and corresponding demand-capacity ratio (in brackets) are shown in Figs. 8-10. The comparison of reinforcement demand using conventional STM and load transfer mechanism based STM for left span of the beam are given in Table I. It is clear from the Table I that horizontal reinforcement requirement is much more than the minimum reinforcement code requirements and additional truss should be included to account for actual reinforcement demand. In direct strut dominant action, the more horizontal reinforcement and less vertical reinforcement appear to be more appropriate distribution. The comparison of reinforcement demand using conventional STM and proposed STM for right span of the beam are given in Table II. It is evident from the table that longitudinal reinforcement requirement in right span of the beam with conventional STM is about 35% less than the load transfer mechanism based STM and it is about 32% more transverse reinforcement requirement than the load transfer mechanism based or proposed STM. Table III shows the comparison of load fraction distribution between arch and truss action in combined force transfer mechanism. The load fractions transfer obtained in proposed approach are similar to the other proposed methods. It can be concluded that the load transfer action based STM is providing a realistic reinforcement demand for the concrete beam.

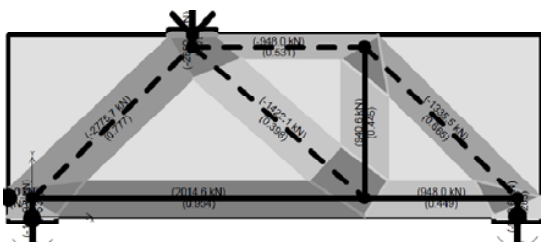


Fig. 8 Conventional STM stress demand and demand-capacity ratio

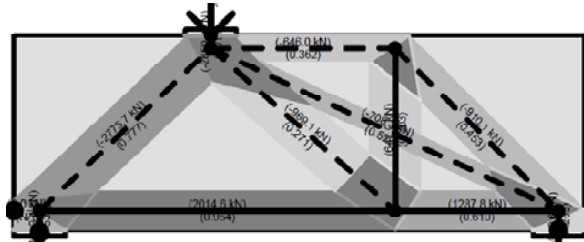


Fig. 9 Proposed or Load transfer mechanism based STM stress demand and demand-capacity ratio

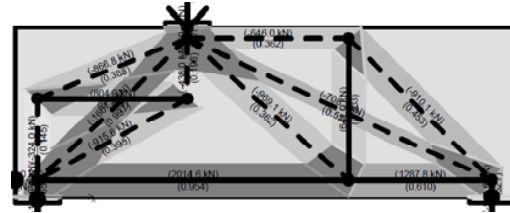


Fig. 10 Load transfer mechanism based STM stress demand and demand-capacity ratio (Left side span: direct strut with additional truss element to account for horizontal reinforcement)

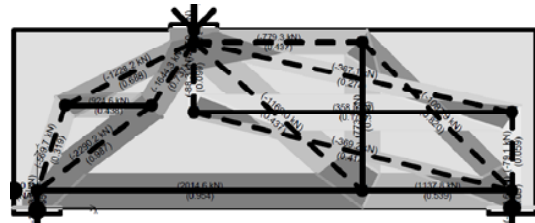


Fig. 11 Conventional STM stress demand and demand-capacity ratio (Left side span: direct strut as bottle shape with single tie to account for horizontal reinforcement; Right side span: Additional Truss Element to account for horizontal reinforcement)

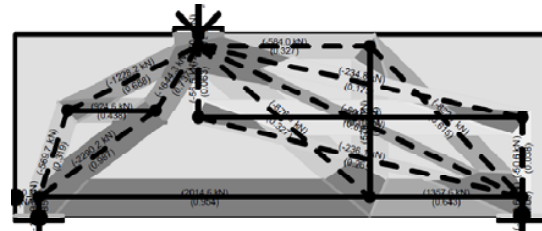


Fig. 12 Load transfer mechanism based STM stress demand and demand-capacity ratio (Right side span: Additional Truss Element to account for horizontal reinforcement)

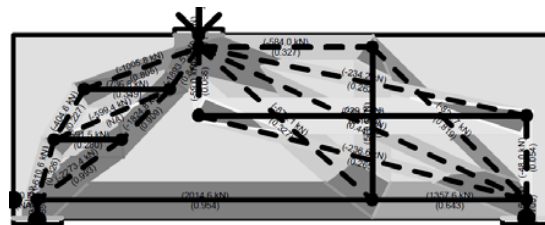


Fig. 13 Load transfer mechanism based STM stress demand and demand-capacity ratio (Left side span: bottle shaped strut with double tie to account for horizontal reinforcement)

TABLE I  
COMPARISON OF TRANSVERSE REINFORCEMENT DEMAND IN DIRECT STRUT MECHANISM (RIGHT SPAN OF THE SS DEEP BEAM)

Type of Strut	Transverse Reinforcement (mm <sup>2</sup> )		Remarks
	Vert.	Hor.	
Typical STM strut , ACI Code web reinforcement requirement§11.8.4/5	1690	1015	
Proposed STM strut with additional horizontal tie truss	1690	2615	Fig. 10
Proposed STM bottle shape strut with single tie	1690	3005	Fig. 11
Proposed STM bottle shape strut with double tie	1690	2395	Fig. 12
Reinforcement provided in literature [43]	2700	1690	

TABLE II  
COMPARISON OF REINFORCEMENT DEMAND OF PROPOSED COMBINED DIRECT STRUT AND TRUSS MECHANISM (RIGHT SPAN OF THE SS DEEP BEAM)

Type of Reinforcement	Conventional STM (Fig. 8)	Conventional STM with horizontal tie truss (Fig.11)	Proposed STM (Fig. 9)	Proposed STM with horizontal tie truss (Fig.13)
Longitudinal Reinforcement (mm <sup>2</sup> )	3085	3700	4185	4415
Vertical Reinforcement (mm <sup>2</sup> )	3060	2515	2085	1890
Horizontal Reinforcement (mm <sup>2</sup> )	1015	1165	1015	745

TABLE III  
COMPARISON OF DISTRIBUTION OF LOAD BETWEEN DIRECT STRUT AND TRUSS MECHANISM FOR COMBINED MECHANISM (RIGHT SPAN OF THE SS DEEP BEAM)

Propose method	a/d Ratio	Strut Mechanism (%)	Truss Mechanism (%)
Load transfer mechanism based STM using CAST program [44]	1	100	0
	2	25	75
FIP recommendation	1	67	33
	2	0	100
Foster & Gilbert	1	64	36
	2	0	100
Zie et al.	1	100	0
	2	17	83

## VIII. CONCLUSION

The STM follows the Lower Bound Theorem of Plasticity, in which only equilibrium and yield criteria are fulfilled and the strain compatibility is not required. As a result of this relaxation, more than one admissible Strut-and-Tie Model may be developed. The research work in cited literature indicates that shear span- effective depth ratio (a/d), factor for the development of load transfer action, can be taken as guiding factor for selection of most appropriate and for unification of Strut-and-Tie Model. The strut-and-tie model 'method (STM) is an effective tool for the design and detailing of shear critical regions as well as the Bernoulli's regions of the concrete structures as STM method considers all load effects (bending, axial, shear, and torsion) simultaneously. In the present study, the various idealization of Strut-and-Tie Model to describe the different load-transfer-mechanisms is presented. It is also concluded from the results that Strut-and-Tie Model based on load transfer action will provide realistic reinforcement demand. It is discussed that concrete member under different force action will have different characteristics and behavior, and for rationalizing the Strut-and-tie modeling, the guidelines for efficiency factor of concrete, tie bar anchorage value, transverse reinforcement and angle of strut should be correlated to (a/d) ratios according to dominant load transfer mechanisms. Dedicated analytical and experimental investigations seem required for further understanding the load transfer mechanism of the beam and to develop inter related correlations based on a/d ratio including final recommendations for various factor affecting the dominant

load transfer mechanism based strut-and-tie model method.

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