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The Influence of Internal and External Damping on Turbocharger Stability

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Abstract—This paper presents the mathematical description of the high-speed rotating system taking into account the influence of internal and external damping. The mathematical model is obtained by using the finite element method. The analyzed system is an automotive turbocharger understood as a rotor-bearing system. The circular cross-section shaft is equipped with one compressor wheel, one turbine wheel and is supported by two floating ring bearings. Based on the model, the dynamical analysis of a turbocharger is performed and stability conditions are evaluated.

Keywords—External damping, internal damping, journal bearing, stability, turbocharger.

I. Introduction

Rotating machines are essential parts of equipments used in many industrial areas. During the operation it is necessary to monitor their properties and behavior due to the need of detection of unbalance, failure and some damage. Each detected adverse effect is reflected by changes in dynamic characteristics of a rotating system [3]. In order to predict their behavior in case of failure, the sufficiently accurate mathematical model of the rotating system must be derived. The finite element method (FEM) is frequently used tool for the description of the rotating machines behavior. Therefore, the principle of this approach is clarified in many publications, for example in [1].

The turbochargers are regarded as rotating machines. As indicated in [2], turbochargers have earned a lot of attention due to wide use of diesel engines in automotive, locomotive, marine and aerospace applications. Although their function is primarily to increase the engine performance, the turbochargers differ in size. While the turbochargers used in automotive and aerospace applications are light and operate at the speed around 100 000 rpm, the turbochargers used in marine and locomotive applications are heavy and operate at the speed around 30 000 rpm.

In this paper the automotive turbocharger undestood as rotor-bearing system is analyzed. The structure of a general automotive turbocharger rotor is shown in Fig. 1.

The automotive turbocharger rotors are distinguished by a short and relatively thin shaft whose ends are equipped by massive large wheels (C, T). The characteristic design allow using the floating-ring bearing system that is of a simple structure and relatively cheap.

According to [2], a typical automotive turbocharger rotor weights about 1 kg and is of 200 mm length and of 5-15 mm journal diameter. The bearing subsytem of an automotive turbocharger is often represented by floating ring bearings as the least expensive solution. The floating ring is put in the bearing clearance between the rotating journal and the stationary bearing housing. The floating ring bearings are of two oil-films, the inner film between the journal and the floating ring and the outer film between the floating ring and the stationary housing. The stiffness and damping characteristics of a bearing is affected not only by the oil films characteristics but also by the rotational speed of the floating ring. Some assumptions and methods used for modeling of floating ring bearings are discussed in [4]. In systems that include turbochargers the engine foundation may occur and subsequenty the non-linear forces may be induced [5]. This fact results in the more complicated behavior of a turbocharger and may even cause unpredictable states.

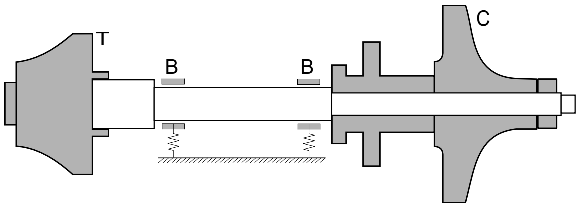

Fig. 1. Turbocharger configuration scheme.

As in the case of large rotating machines used for example in power plants the dynamic behavior of turbocharger is affected by some damping effect, too. Hence, in this paper the methodology used for building of a turbocharger mathematical model is explained, then the stability conditions are defined and the influence of internal and external damping is evaluated in case of an automotive turbocharger. The methodology for external and internal damping modeling is described in detail in [1] .

II. Equations of motion for turbocharger rotor-bearing system

The rotor-bearing system of turbocharger can be modeled using the finite element (FE) approach. Therefore, the rotor bearing system can be divided into subsystems - the shaft, the added masses and the bearings. In this paper, each subsystem is modeled independently and the derived single models are connected, as mentioned in [1].

A. Flexible shaft model

The flexible rotor shaft of a continuously distributed mass and of a circular cross-section area $A(x)$, that rotates by

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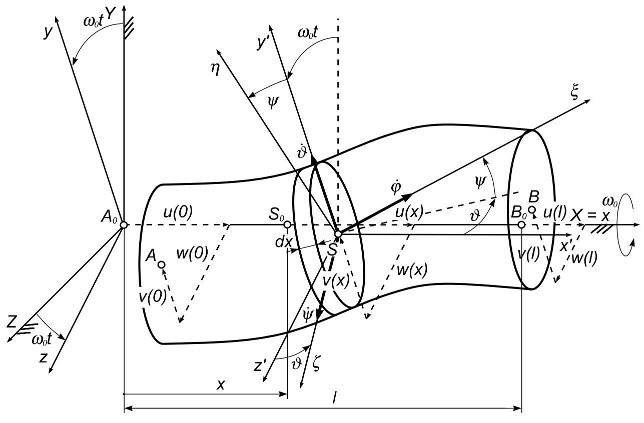

Fig. 2. Shaft element scheme.

constant angulat velocity ω, is discretized by nodes into finite elements, see Fig. 2.

Considering bending vibrations of the system, each node location is described by two lateral displacements $v_i(x)$, $w_i(x)$ and by two angular displacements $\vartheta_i(x)$, $\psi_i(x)$. Then the position of the shaft element e is given by displacements of end nodes i and $i+1$ and the generalized coordinates vector in fixed coordinates can be written as

$$\mathbf{q}_{XYZ}^{(e)} = [v_i, \psi_i, v_{i+1}, \psi_{i+1}, w_i, \vartheta_i, w_{i+1}, \vartheta_{i+1}]^T \ . \quad (1)$$

Based on kinetic energy (2) and potential energy (3) of the shaft finite element

$$E_k^{(e)} = \frac{1}{2}\int_0^l \left[A\mathbf{v}^T\mathbf{v} + \boldsymbol{\omega}^T\mathbf{J}\boldsymbol{\omega}\right] dx \ , \quad (2)$$

$$E_p^{(e)} = \frac{1}{2}\int_0^l \int_{(a)} E\varepsilon_x^2 dA dx \ , \quad (3)$$

where $\mathbf{v}$ is the velocity vector of a FE lateral displacement, $\boldsymbol{\omega}$ is the angular velocity vector of a spherical movement, $\mathbf{J}$ is the inertia matrix, E is Young modulus and ε_x is a proportional elongation of the FE. Considering the approximation along the shaft element e by cubic polynomials, then the mass matrix $\mathbf{M}^{(e)}$, the stiffness matrix $\mathbf{K}^{(e)}$ and the gyroscopic matrix $\mathbf{G}^{(e)}$ describing the FE can be obtained from Lagrange's equations in form

$$\mathbf{M}^{(e)} = \begin{bmatrix} \mathbf{S}_1^{-T}(\mathbf{I}_1 + \mathbf{I}_2)\mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_2^{-T}(\mathbf{I}_1 + \mathbf{I}_2)\mathbf{S}_2^{-1} \end{bmatrix} , \quad (4)$$

$$\mathbf{G}^{(e)} = \begin{bmatrix} \mathbf{0} & -2\mathbf{S}_1^{-T}\mathbf{I}_2\mathbf{S}_2^{-1} \\ -2\mathbf{S}_2^{-T}\mathbf{I}_2\mathbf{S}_1^{-1} & \mathbf{0} \end{bmatrix} , \quad (5)$$

$$\mathbf{K}^{(e)} = \begin{bmatrix} \mathbf{S}_1^{-T}\mathbf{I}_3\mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_2^{-T}\mathbf{I}_3\mathbf{S}_2^{-1} \end{bmatrix} , \quad (6)$$

where $\mathbf{I}_i$ (i=1,2,3) are auxiliary integral matrices of constant value and $\mathbf{S}_j$ (j=1,2) are matrices resulting from the polynomial approximation of displacements. The specific form of matrices $\mathbf{I}_i$, $\mathbf{S}_j$ and the detailed derivation are presented in [1].

Furthermore, the effect of damping forces acting on the FE is suitable for inclusion in the mathematical model. Damping forces are induced by rotor working conditions (so-called external damping) and by material deformations (so-called internal damping). Assuming the external damping forces depending on lateral absolute velocity of FE the Rayleigh dissipation function of external damping can be expressed as

$$R_E^{(e)} = \frac{1}{2}\int_0^l \left(b_{EY}\dot{v}^2(x,t) + b_{EZ}\dot{w}^2(x,t)\right) dx \ . \quad (7)$$

The b_{EY} , b_{EZ} $\left[kgm^{-1}s^{-1}\right]$ are coefficients of viscous damping per unit lenght. Using the simplifying assumption of isotropic external damping the external damping matrix corresponding with bending oscillations can be derived in fixed coordinate XYZ frame as

$$\mathbf{B}_E^{(e)} = b_E \begin{bmatrix} \mathbf{S}_1^{-T}\mathbf{I}_\Phi\mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_2^{-T}\mathbf{I}_\Phi\mathbf{S}_2^{-1} \end{bmatrix} . \quad (8)$$

On the other hand internal damping forces are induced by deformations of the shaft element, therefore it is needed to be described in the rotational coordinate system xyz connected with the shaft element. Then the normal stress is proportional to the longitudinal strain velocity and the Railegh dissipation function of isotropic internal damping can be expresed as

$$R_I^{(e)} = \frac{1}{2}\int_0^l \int_{(A)} b_I E \varepsilon_x^2 dA dx \quad (9)$$

where the b_I $[s]$ is the viscous proportional internal damping coefficient. Then the internal damping matrix of shaft element can be written as

$$\bar{\mathbf{B}}_I^{(e)} = b_I E J \begin{bmatrix} \mathbf{S}_1^{-T}\mathbf{I}_{\Phi''}\mathbf{S}_1^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_2^{-T}\mathbf{I}_{\Phi''}\mathbf{S}_2^{-1} \end{bmatrix} . \quad (10)$$

in the rotating coordinate frame xyz.

Because the other matrices describing the shaft element are derived in the fixed coordinate frame XYZ the internal damping matrix $\bar{\mathbf{B}}_I^{(e)}$ must be transformed from the rotating coordinate frame xyz into fixed coordinate frame XYZ. Using the relation between vectors of generalized coordinates in fixed frame $\mathbf{q}_{XYZ}^{(e)}$ and the vector of generalized coordinates in rotating frame $\mathbf{q}_{xyz}^{(e)}$

$$\mathbf{q}_{XYZ}^{(e)} = \mathbf{T}(t)\,\mathbf{q}_{xyz}^{(e)} \quad (11)$$

where $\mathbf{T}(t)$ is a time-dependent transformation matrix. Then the internal damping effect is described by two time-dependent matrices - the internal damping matrix $\mathbf{B}_I^{(e)}(t)$ and circulation matrix $\mathbf{C}_I^{(e)}(t)$ in the fixed coordinate frame XYZ.

All matrices describing the shaft element are derived in the fixed coordinate frame XYZ that is characterized by generalized coordinates vector $\mathbf{q}_{xyz}^{(e)}$ in form (1). Therefore the motion of each shaft element can be described by matrix equation of motion

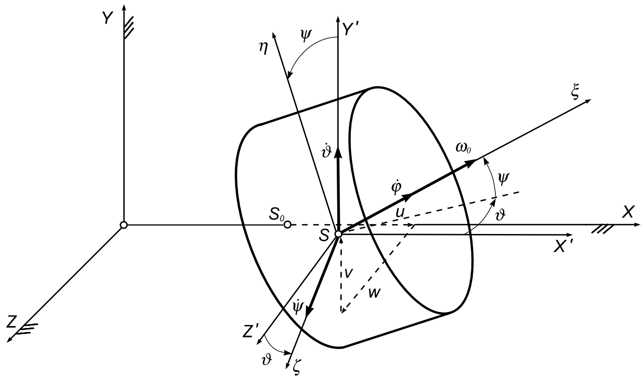

Fig. 3. Rigid disc sheme.

$$\mathbf{M}^{(e)}\ddot{\mathbf{q}}^{(e)}_{XYZ}+\left(\mathbf{B}^{(e)}_{E}+\mathbf{B}^{(e)}_{I}+\omega_0\mathbf{G}^{(e)}\right)\dot{\mathbf{q}}^{(e)}_{XYZ}+\left(\omega\mathbf{C}^{(e)}_{I}+\mathbf{K}^{(e)}\right)\mathbf{q}^{(e)}_{XYZ}=\mathbf{0}\ . \tag{12}$$

B. Rigid discs

In this paper each added mass attached to the shaft is regarded as an axisymmetric rigid disc that is fixed to the shaft in the chosen node i, see Fig. 3.

Using Lagrange's approach the mathematical description is based on the rigid disc kinetic energy E^d_k

$$E^{(d)}_k=\frac{1}{2}m\mathbf{v}^T_S\mathbf{v}_S+\frac{1}{2}\boldsymbol{\omega}^T_i\mathbf{I}\boldsymbol{\omega_i}\ . \tag{13}$$

Using the adjustments specified in [1] and Lagrange's equations the mass matrix $\mathbf{M}^{(d)}$ and the gyroscopic matrix $\mathbf{G}^{(d)}$ are derived in the fixed coordinate frame XYZ defined by the vector of generalized coordinates

$$\mathbf{q}_i=[v_i\quad w_i\quad \vartheta_i\quad \psi]^T\ . \tag{14}$$

Then the matrices can be written in the following form

$$\mathbf{M}^{(d)}=\begin{bmatrix} m & 0 & 0 & ma \\ 0 & m & -ma & 0 \\ 0 & -ma & I+ma^2 & 0 \\ ma & 0 & 0 & I+ma^2 \end{bmatrix}, \tag{15}$$

$$\mathbf{G}^{(d)}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ & 0 & 0 & I_0 \\ 0 & 0 & -I_0 & 0 \end{bmatrix}. \tag{16}$$

In contrast to the shaft element modeling, the rigid disc is affected only by isotropic external damping forces acting on the surface of the body. In case of bending vibrations the external damping matrix can be expressed as

$$\mathbf{B}^{(d)}_E=\begin{bmatrix} b^{(d)}_{EY} & 0 & 0 & 0 \\ 0 & b^{(d)}_{EZ} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{17}$$

where $b^{(d)}_{EY}$, $b^{(d)}_{EZ}$ are the external damping coefficients. The external damping matrix (17) is derived in fixed coordinate frame described by vector of generalized coordinates (14) too.

C. Bearings

As already mentioned, in practice turbocharger rotors are supported using floating ring bearings that can be understood as bearing two oil film in the series [4]. To determine their stiffness and damping characteristics at a given speed, it is necessary to take into account not only the stiffness and damping parameters of both oil films but also to determine the effect of the floating ring speed [4]. If the bearing is simplified as a short journal bearing and the linear dependency between induced forces, deformations and damping is assumed, then the forces induced in the bearing b can be expressed as

$$\begin{bmatrix} P_v \\ P_w \\ P_\vartheta \\ P_\psi \end{bmatrix}=\underbrace{\begin{bmatrix} b_{yy} & b_{yz} & 0 & 0 \\ b_{zy} & b_{zz} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{B}_i}\begin{bmatrix} \dot{v}_i \\ \dot{w}_i \\ \dot{\vartheta}_i \\ \dot{\psi}_i \end{bmatrix}+ \underbrace{\begin{bmatrix} k_{yy} & k_{yz} & 0 & 0 \\ k_{zy} & k_{zz} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{K}_i}\begin{bmatrix} v_i \\ w_i \\ \vartheta_i \\ \psi_i \end{bmatrix}, \tag{18}$$

where $\mathbf{B}_i$ is the damping matrix and $\mathbf{K}_i$ is the stiffness matrix of the bearing placed in i^{th} node. The parameters b_{ij}, k_{ij} are dependent onthe shaft rotational speed ω.

D. Whole rotor-bearing system model

As mentioned above, the rotating system is composed of three types of elements - the shaft, the rigid discs and the bearing supports. These components must be joined, therefore the matrices describing the shaft element must be transformed from fixed coordinate system $X\tilde{Y}Z$ characterized by (1) into the system characterized by (14). This operation can be performed using permutation matrix that ensures the required rearrangement of elements of vectors and of matrices. Then the mathematical model of whole rotation system can be expressed in form

$$\mathbf{M}_\Sigma\ddot{\mathbf{q}}\,(t)+\underbrace{(\mathbf{B}_E+\mathbf{B}_I+\mathbf{B}_B+\omega\mathbf{G})}_{\mathbf{B}_\Sigma}\dot{\mathbf{q}}\,(t)+\underbrace{(\mathbf{K}+\mathbf{C}_I+\mathbf{K}_B)}_{\mathbf{K}_\Sigma}\mathbf{q}\,(t)=\mathbf{0}\ . \tag{19}$$

The mass matrix $\mathbf{M}_\Sigma$, the damping matrix $\mathbf{B}_\Sigma$ and the stiffness matrix $\mathbf{K}_\Sigma$ are composed from matrices describing single elements according to Fig. 4.

III. Modal properties specifying and Stability assessment

According to [1] the first step in determining the dynamic properties is the identification of modal properties. It involves

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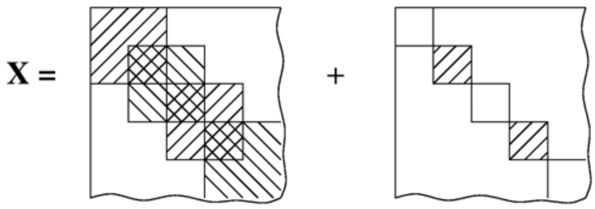

Fig. 4. Block matrix structure.

in calculation of eigenvalues and of eigenvectors. The next step is stability assessment on the basis of computed eigenvalues. The eigenvalue problem solution is based on the equation (19) describing the analyzed system. If at least one of the matrices is asymmetrical then the model composed of transposed matrices must be set up, too. Considering solutions of equations describing the system motion in form

$$\mathbf{q}\left(t\right) = \mathbf{q} \cdot e^{\lambda t} \text{ , resp. } \mathbf{r}\left(t\right) = \mathbf{r} \cdot e^{\lambda t} \text{ ,} \tag{20}$$

the equations of motion can be modified as

$$\left(\lambda^2 \mathbf{M}_\Sigma + \lambda \mathbf{B}_\Sigma + \mathbf{K}_\Sigma\right) \mathbf{q} = \mathbf{0} \text{ resp. } \left(\lambda^2 \mathbf{M}_\Sigma^T + \lambda \mathbf{B}_\Sigma^T + \mathbf{K}_\Sigma^T\right) \mathbf{r} = \mathbf{0} \text{ .} \tag{21}$$

The nontrivial solution of problems (21) exists for

$$\det\left(\lambda^2 \mathbf{M}_\Sigma + \lambda \mathbf{B}_\Sigma + \mathbf{K}_\Sigma\right) = 0 \text{ resp. } \det\left(\lambda^2 \mathbf{M}_\Sigma^T + \lambda \mathbf{B}_\Sigma^T + \mathbf{K}_\Sigma^T\right) = 0 \text{ .} \tag{22}$$

It is known that the roots of equations (22) are identical and are called eigenvalues λ_ν. They may be complex conjugate in pairs ($\lambda_\nu = -\alpha_\nu \pm \beta_\nu$) or real ($\lambda_\nu = -\alpha_\nu$). The unique eigenvector can clearly be assigned to each eigenvalue, in detail in [1].

Many criterions of stability of dynamic systems are defined. Therefore, if the stability of a rotating system is evaluated the criterion should be mentioned for clarification. In this case the stability of a rotating system is evaluated based on computed eigenvalues.

Generally, the eigenvalue λ_ν is a complex number. Whereas the imaginary part provides some information on the natural frequency of vibrations, the stability of a rotating system is affected by the real part of the eigenvalue. Three different situations may occur:

1) all eigenvalues are of negative real parts ($\alpha_\nu > 0$), then the analyzed system is considered asymptotically stable,

2) at least one eigenvalue or a couple of complex conjugate eigenvalues is of zero real part ($\alpha_\nu = 0$), then the analyzed system is on the stability threshold,

3) at least one eigenvalue or a couple of complex conjugate eigenvalues is of positive real part ($\alpha_\nu < 0$), then the analyzed system is considered unstable.

IV. Numerical simulations and results analysis

The objective of this paper is to analyze the influence of internal and external damping on the dynamic properties of the high-speed rotating turbocharger and assess the impact on the stability. The turbocharger rotor-bearing system shown in Fig. 5 is composed of the flexible steel shaft fitted with the added masses understood as rigid discs and of two non-identical hydrodynamic bearings.

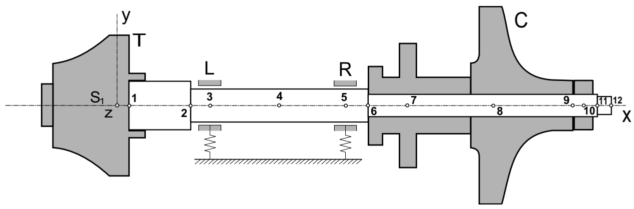

Fig. 5. Turbocharger sheme after discretization.

A. Turbocharger parameters

With regard to the position of connected rigid discs the rotor shaft was discretized using 12 nodes. Thus, 11 finite elements of parameters specified in the table Tab. I were defined by its end nodes. The parameters of 4 added masses are mentioned in Tab. II. It is necessary to note that the geometric parameters of the shaft finite elements listed in Tab. I and added masses in Tab. II are normalized.

TABLE I
Shaft elements parameters.

e	$\tilde{D}^{(e)}$	$\tilde{l}^{(e)}$
1	1	0.66
2	0.69	0.18
3	0.69	0.68
4	0.69	0.68
5	0.69	0.18
6	0.48	0.28
7	0.48	0.81
8	0.48	1
9	0.48	0.03
10	0.48	0.27
11	0.37	0.15

TABLE II
Added masses parameters.

i	$\tilde{m}^{(i)}$	$\tilde{I}_0^{(i)}$	$\tilde{I}^{(i)}$	$\tilde{a}^{(i)}$	$\tilde{D}_{xy}^{(i)} = \tilde{D}_{xz}^{(i)}$	$\tilde{D}_{yz}^{(i)}$
1	1	0.98	1	0.91	0	$9.1 \cdot 10^{-12}$
7	0.10	0.16	0.01	0	0	0
8	0.27	0.32	0.23	0	0	0
9	0.02	0.002	0.001	0	0	0

The modeled turbocharger rotor is supported by two bearings of different stiffness and damping characteristics depending on rotational velocity ω. For the left bearing (L) the dependencies of parameters are shown in Fig. 6, 7, for the right bearing (R) in Fig. 8, 9. The values of bearing coefficients were calculated from measured parameters of real bearings.

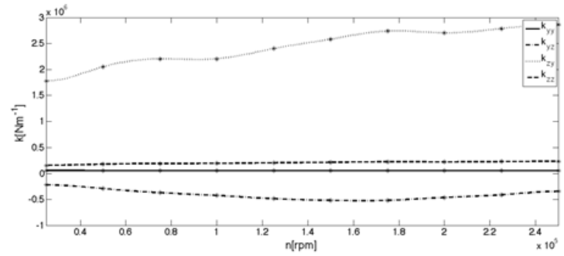

Fig. 6. Stiffness parameters dependency- bearing L.

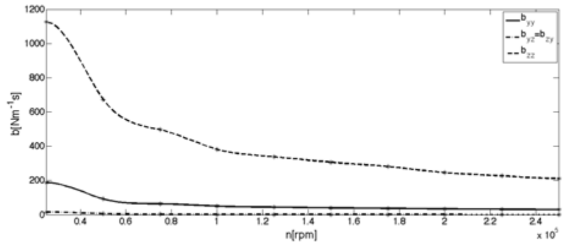

Fig. 7. Damping parameters dependency- bearing L.

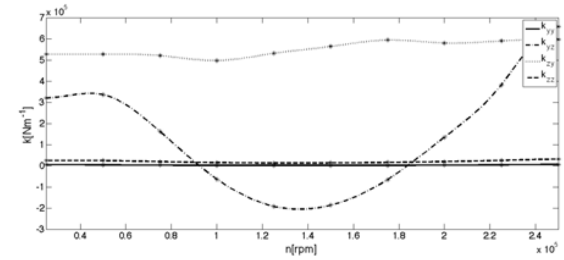

Fig. 8. Stiffness parameters dependency- bearing P.

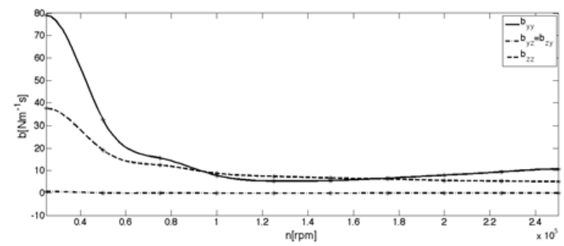

Fig. 9. Damping parameters dependency- bearing R.

B. Computed results

The turbocharger rotor-bearing system stability is evaluated for the speed range $n \in< 25000, 250000 >$. The internal and external damping acting on the steel shaft is characterized by coefficients $b_I = 4 \cdot 10^{-3} s$ and $b_E = 6 \cdot 10^{-3} kgm^{-1}s^{-1}$. Values of damping coefficients are determined on the basis of known damping ratio value for rotors at $n = 0\, rpm$. Using the same procedure the external damping coefficient $b_E^{(d)} = 5 \cdot 10^{-2} kgm^{-1}s^{-1}$expressing the influence on rigid discs is computed for $n = 0\, rpm$.

For the turbocharger decsribed above the dependencies between real parts of eigenvalues $Re\{\lambda_\nu\}$ and rotating speed n were obtained. Firstly, the system without internal and external damping effect was analyzed, see Fig. 10. In this case only the damping effect of bearing supports represented by matrix $\mathbf{B}_B$ is included. As shown inn Fig. 10, the threshold of stability is located nearby 160000 rpm.

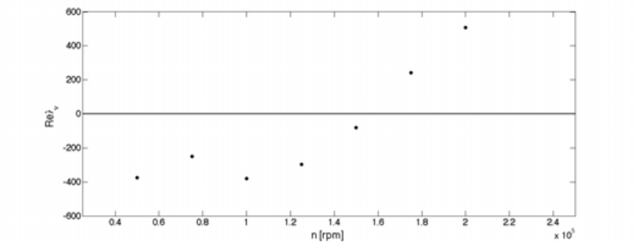

Fig. 10. Dependency of real parts of eigenvalues on rotating speed for undamped system.

Then the system including internal and external damping effect was analyzed, see Fig. 11. In this case all the damping effect mentioned in finite element model (19) is included. As shown in Fig. 11, the threshold of stability is located nearby 40000 rpm. It is mu lower rotating speed n than in the first case.

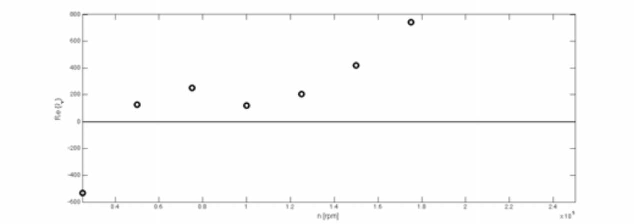

Fig. 11. Dependency of real parts of eigenvalues on rotating speed for damped system.

V. Conclusion

The aim of this paper was to analyze the internal and external damping effects on dynamical characteristics of the high-speed rotating machine called the turbocharger. This type of devices is widely used in automotive industry. Although the detailed analysis is performed before the start of production, the undesirable behavior can cause during its operation. Therefore the appropriate model of analysed system must be set up and all impacts affect the system must be evalued. Therefore the finite element model was updated by internal and external damping effects. The computed results show that damping effects play an important role in the field of the high-speed working system stability. Based on computed results, in this case the damping effect consideration results in decrease of threshold stability value.

It is very important to note that the presented finite element model is only the basic model. In the case of turbocharger, the bearing supports are of more fragmented construction including so-called a floating ring. Therefore this approach is designed only for zoom in the dynamical characteristics of turbocharger and it will be refined.

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