On the Parameter Optimization of Fuzzy Inference Systems

Erika Martinez Ramirez and Rene V. Mayorga

Abstract**—**Nowadays, more engineering systems are using some kind of Artificial Intelligence (AI) for the development of their processes. Some well-known AI techniques include artificial neural nets, fuzzy inference systems, and neuro-fuzzy inference systems among others. Furthermore, many decision-making applications base their intelligent processes on Fuzzy Logic; due to the Fuzzy Inference Systems (FIS) capability to deal with problems that are based on user knowledge and experience. Also, knowing that users have a wide variety of distinctiveness, and generally, provide uncertain data, this information can be used and properly processed by a FIS. To properly consider uncertainty and inexact system input values, FIS normally use Membership Functions (MF) that represent a degree of user satisfaction on certain conditions and/or constraints. In order to define the parameters of the MFs, the knowledge from experts in the field is very important. This knowledge defines the MF shape to process the user inputs and through fuzzy reasoning and inference mechanisms, the FIS can provide an "appropriate" output. However an important issue immediately arises: How can it be assured that the obtained output is the optimum solution? How can it be guaranteed that each MF has an optimum shape? A viable solution to these questions is through the MFs parameter optimization. In this Paper a novel parameter optimization process is presented. The process for FIS parameter optimization consists of the five simple steps that can be easily realized off-line. Here the proposed process of FIS parameter optimization it is demonstrated by its implementation on an Intelligent Interface section dealing with the on-line customization / personalization of internet portals applied to E-commerce.

Keywords**—**Artificial Intelligence, Fuzzy Logic, Fuzzy Inference Systems, Nonlinear Optimization.

I. INTRODUCTION

DEVELOPING intelligent applications is not a new subject. However, developing intelligent applications that ensure an "optimum" solution is of great interest to Artificial Intelligent (AI) researchers. Intelligent applications normally include a proper methodology among a wide variety of AI techniques such as, Artificial Neural Nets, Fuzzy Inference Systems (FIS), Neuro-Fuzzy Inference Systems, [1]; and Rough Sets implementations [8]. In Fuzzy Set Theory [1], ambiguous data can be represented by Membership Functions (MFs). An MF normally maps a smooth transition from "belonging to a set" to "not belonging to a set".

Parameterized functions, commonly used to define MFs of one dimension, include non-continuous MFs (*Triangular MF, Trapezoidal MF)* over the entire input domain; and continuous MFs *(Gaussian MF*, *Generalized bell MF,* or *bell MF*, and *Sigmoidal MF* among others) over the entire input domain.

Fuzzy inference systems (FIS) are based on *fuzzy rules* (IF-Then) and *fuzzy reasoning* [1]. The extension principle provides a general procedure for extending crisp domains of mathematical expressions to fuzzy domains. Normally, the human terms or linguistic values are defined by fuzzy sets on two universes of discourse: one named as the *antecedent* or *premise*, whereas the other is called the *consequence* or *conclusion* [1]**.**

Fuzzy reasoning is an inference procedure that obtains conclusions from a set of fuzzy IF-THEN rules and known facts; and is also known as *approximate reasoning*. The compositional rule of inference plays a key role in fuzzy reasoning. A special case of this rule is the *extension principle*. In *approximate reasoning*, there are two important inference rules: a *generalized modus ponens* (GMP) and a *generalized modus tollens* (GMT).

The concepts of fuzzy set theory, fuzzy IF-THEN rules, and fuzzy reasoning are the basis of the framework called fuzzy inference system. Thus, three conceptual components form the basic structure of a fuzzy inference system: rule base (fuzzy rules selection); database (membership functions used in the fuzzy rules), and reason mechanism (inference procedure).

Moreover, a representative (crisp) value can be extracted from a fuzzy set, called defuzzification. The process of defuzzification transforms a fuzzy output of a fuzzy inference system into a crisp output. Jang et al (1997) mention five methods for defuzzifying a fuzzy set of a universe of discourse, such as: *centroid* of area method (CENTROID), *bisector* of area method (BISECTOR), *mean of maximum* method (MOM), *smallest of maximum* method (SOM), and *largest of maximum* method (LOM) [1].

Many AI applications are based on FIS due to their ability to process imprecise data and ambiguous concepts. Here following [3], a methodology for FIS parameters optimization is presented. By optimizing the FIS Membership Functions

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(MFs) with respect to a performance criterion, the resulting FIS can lead to an optimal solution with respect to that criterion. The proposed FIS parameter optimization methodology consists of a set of simple steps, which includes:

1. Conversion of non-continuous MFs to continuous MFs.

If the entire original set of MFs consists of continuous functions of the corresponding inputs in their entire domain; then this step is skipped. In the case that the entire set or any of the original MFs (i.e. *triangular, trapezoidal* MFs, others) is not continuous over the entire inputs domain; then each MF is converted into a continuous form (i.e. a g*aussians* MF) over the corresponding entire input domain. In order to perform properly this conversion, the parameters of the discontinuous MFs can be considered.

2. Selection of the appropriate Fuzzy Reasoning and Defuzzification Method.

At this step the FIS reasoning mechanisms are set by properly selecting different implication (minimum –MIN- and product –PROD-) and aggregation (maximum –MAX- and summation –SUM-) operators, and the Defuzzification methods (CENTROID, MOM, and BISECTOR).

3. Implementation of the selected Fuzzy Reasoning and Defuzzification Method.

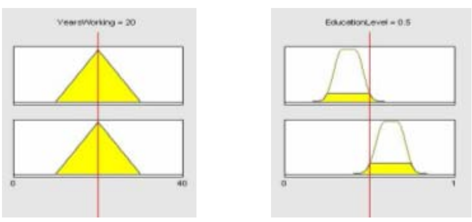
The third step implements the selected FIS reasoning and defuzzification method. The figures to be shown next and in subsequent sections, subsequently have been produced with the use of the Matlab Toolbox [7].

These results from this step can be evaluated considering two cases:

(1) predefined '*safe*' input, Fig. 1-(a), (where the input value –represented by the vertical line- is located in the middle of MF, giving more degree of belief to that MF. The shade area (weight) covers almost all the MF; and

(2) arbitrary *'risk'* input, Fig. 1-(b); where the input value is intersecting two MFs, creating similar shade area in the MFs involved; making difficult to select either of the MFs.

As exemplified in a subsequent section, using predefined ("safe") inputs values can generate very similar crisp outputs values, in all combinations of FISs. Whereas, selecting arbitrary ("risky") input values, can generate a different solution in some of their outputs. In addition, the chosen T-norm operators in the fuzzy reasoning mechanism can generate outputs with different shapes.



(a) (b)

Fig 1 Illustration of a predefined 'safe' input (a) and arbitrary 'risk' input (b).

4. Optimization of the FIS.

Once all the MFs have been properly defined and the FIS reasoning and defuzzification method are selected, the process to optimize the FIS parameters can begin. Here, this process centers on the optimization of a performance criterion defined in terms of a Frobenious norm, as suggested in [3], that can be explicitly expressed in terms of the FIS MFs (antecedent, consequent) parameters.

5. Evaluation of results from the optimization process.

In this step, the results from the optimization process are properly analyzed and evaluated. The steps for optimization methodology have been defined and are explained in more detail in the following sections as well as the illustration of the results.

II. An Initial Working FIS Model

Here, we exemplify the proposed parameter optimization by considering as an initial working FIS model a previous module system presented in [2]. In reference [2], the authors present a Paradigm for the proper matching of an Intelligent Human-Computer Interface with an Intelligent System. An instance of this Paradigm it is the development of a suitable Intelligent Interface for on-line (e-commerce) Internet Portal Customizations. The Intelligent Interface consists of an intelligent engine (*i-engine*) that receives information (characteristics and areas of interest) from an internet web user, and processes this information to generate User Ascribed Qualities (UAQs). In [2], the information from the user is referred to as 'user profiles' and the UAQs interpret and categorize these profiles to identify the user. The i-engine is based on a Fuzzy Inference System (FIS) module that contains containing two FISs sub-modules in cascade called "Main" and "Preference Link". The "Main" FIS receives the characteristics (fuzzy inputs: *gender, age, marital status*, etc.) from the web user; while the "Preference Link" receives preferences/interests (fuzzy inputs: *news, health, entertainment,* etc.) from the user. The "Main" FIS generates categories and characteristics associated with the user such as *purchaser link, purchaser capacity, purchaser free time,* among others (fuzzy outputs); whereas the "Preference" FIS generates the particular user's interests (fuzzy outputs: *preferences, health, news,* and *other links).* Each fuzzy input of the two FISs modules is represented through Membership Functions (MF) and the MFs chosen are considered *triangular, trapezoidal, gaussian,* and *gaussian-2.* These two FIS are in cascade; that is, an input (*age*) and an output (*purchaser capacity*) from the "Main" FIS are inputs for the "Preference" FIS.

The FIS considered here as an initial working model to exemplify the proposed FIS parameter optimization corresponds to the first Fuzzy Agent "Main" from the cascade FIS in [2]. In the optimization process, the non-continuous MFs (*triangular, trapezoidal* and *gaussian-2*) present in this initial working model are converted into continuous (*gaussian*) MFs as described in the next section.

III. Conversion of non-continuous MFs to continuous MFs

Some FIS can contain non-continuous MFs (*Triangular MF, Trapezoidal MF)* and/or continuous MFs *(Gaussian MF, Generalized bell MF* -or bell MF- and *Sigmoidal MF*) over their entire inputs domains. A graphic representation of several continuous and discontinuous MFs over their entire inputs domains is illustrated in detail in Jang et al in [1].

In the conversion of non-continuous MFs to continuous MFs process, it is important to consider continuous MFs over the entire input domains. This facilitates the definition of the objective function to be optimized, and some derivatives needed by the optimization process; as exemplified in a subsequent section. Some suitable Available continuous MFs include Gaussian and Bell MFs [1]. In particular, the Gaussian MF can closely represent the original non-continuous MFs over the entire inputs domains.

A desirable feature (but not a necessary one) of the MFs discontinuous-continuous conversion process, in the entire inputs domains, is that the outputs of the modified FIS (for a wide range of inputs) resemble as much as possible the outputs (for the same inputs) of the initial working FIS model, prior to the parameter optimization.

In order to choose a "method" to convert the original non-continuous MFs into continuous MF over a large input domain, some approaches can be explored. One method is to convert the original MFs "manually"; i.e. one can easily convert triangular, trapezoidal, bell, functions to Gaussian functions; however, the outputs yielded by the corresponding modified FIS can be different than the outputs produced by the original FIS.

For example, let's consider the *trapezoidal* MF [1]. This trapezoidal MF has the parameters x, a, b, c, and d, and the Gaussian MF has parameters: center (c) and sigma (σ) where $(\sigma) = \dfrac{c-b}{2}$, $(c) = b + \dfrac{c-b}{2}$. After modifying the original MF parameters to obtain the corresponding parameters for the Gaussian MF, the resultant MF had a different shape and meaning regarding the original MF. However, it is necessary to do this with caution; since the outputs of the FIS with these continuous functions may not be the same as the outputs of the original FIS.

Another approach is to consider the original (i.e. bell, gauss-2) MFs and convert them into gaussian MFs; but in this case approximating the cross points of the converted MFs with the original MFs, respectively. This process of converting original-to-gaussian MFs can be applied to the original inputs variables of the "Main" FIS [2]. The following figure illustrates this conversion for this FIS model using the input 'Age' from the Main FIS. The dashed lines (a), (b), and (c) are positioned in the intersections (cross-point) between the MFs.

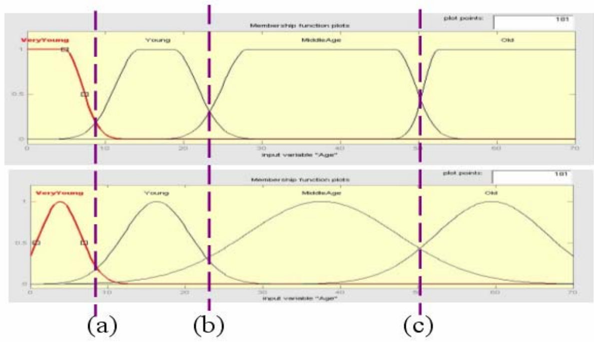


Fig. 2 Conversion the Original input to Gaussian MFs. (The dashed lines on (a), (b) and (c) show the cross points.)

Fig. 2 shows that the new Gaussian MFs maintains a relationship with the original, since the x and y cross-points are closely related to the originals, and therefore maintain the same meaning. Consequently, this conversion process it is applied here to convert all the input MFs of the original working model 'Main' FIS to Gaussian MFs.

The characteristics and details of the inputs illustrated in the Fig. 2 are included in Table I.

TABLE I
CHARACTERISTICS OF INPUT VARIABLE: AGE FOR THE 'MAIN' FIS

	Original MF properties	**Gaussian MF properties**
Name	'Age'	'Age'
Range	[0 70]	[0 70]
NumMFs	4	4
MF1 ='VeryYoung'	'gauss2mf',[8.12 -1.6 2.18 4.77]	'gaussmf',[2.62 3.84]
MF2= 'Young'	'gauss2mf',[3.09 14.4 3.07 18.6]	'gaussmf',[4.26 16.3]
MF3= 'MiddleAge'	'gauss2mf',[3.18 28.2 2.55 47]	'gaussmf',[9.73 37.6]
MF4='Old'	'gauss2mf',[1.74 52.5 7.93 74.9]	'gaussmf',[7.14 59.5]

The characteristics presented in Table I for the original working model 'Main' MFs and modified MFs are: Name, Range, Number of Membership Functions (NumMFs), and the properties of each Membership Functions (MF1=' '). This input variable has four (4) MFs.

Furthermore, the rules considered in the modified FIS with gaussian MFs are the same as the original working model FIS. Once all the new MFs have been converted to continuous MFs, it is required to select an appropriate Fuzzy Reasoning and Defuzzification Method as shown in the next section.

IV. Selection of the Appropriate Fuzzy Reasoning and Defuzzification Method

This section presents a procedure for the proper selection of the FIS fuzzy reasoning mechanism. In this case, the Implication Operator (T-norm) and Aggregation Operator (T-conorm) are determined by a trial and error combination process using the operators MIN, MAX, SUM and PRODUCT. The types of Defuzzification methods considered here are Mean of Maximum (MOM), Centroid of Area (COA or Centroid), and Bisector of Area (BOA or Bisector) [1].

In order to assign the different types of T-norm and T-conorm Operators and Defuzzification Methods and select the proper FIS fuzzy reasoning; here first a general system of two inputs, one output, and two general rules is considered as shown in Fig. 3. The inputs represent two possible extreme cases. First, they have associated Gaussian MFs and a Triangular MF; second, in their weight or measure of degree of belief: (a) an input value intersects two MFs generating almost equal shade area –weight- in both MFs; and (b) an input value in the center of MF that shades all its area facilitating the selection of that MF.

Fig. 3 illustrates the Fuzzy Reasoning, considering the Implication Operator as MIN, the Aggregation Operator as MAX, and the Defuzzification Method as MOM.

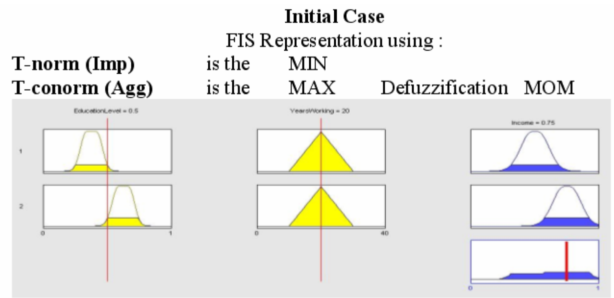


Fig. 3 FIS with Implication (T-norm) of MIN and Aggregator (T-conorm) MAX and the Defuzzification Method of MOM.

After assigning a different operator to T-norm and T-conorm and a different type to defuzzification method to the system in Fig. 3; four combinations of operators are considered. Such combinations are: MIN MAX, PROD MAX, MIN SUM, and PROD SUM. The behaviours of the FIS are also compared after assigning the combinations of operators for each type of defuzzification method, illustrated in Figures 4-1, 4-2, and 4-3. In Fig. 4-1, the “MIN MAX” square represents the results obtained when the first operator MIN corresponds to Implication and MAX corresponds to Aggregation. The same explanation is given to “PROD MAX”, “MIN SUM” and “PROD SUM” squares. Also, the type of defuzzification method chosen for Fig. 4-1 is MOM. Fig. 4-2 represents the results obtained for a defuzzification method Centroid and Fig. 4-3, for a Bisector.

The Fig. 4-1 illustrates that the final output of the considered FIS is directly related to the output MF that contains more weight in its area. For the output ‘MIN SUM,’ the sum of the two MF areas indicates an output that intersects in the two MFs, in comparison with ‘PROD SUM’ in which the output considers the MF with more weight in its area. This defuzzification method selects the appropriate MF output.

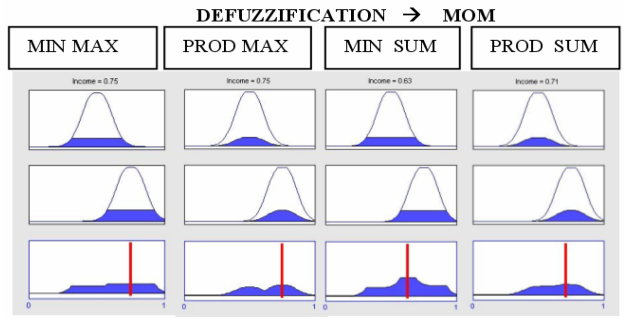


Fig. 4-1 FIS outputs from the different combinations of operators, with Implication (T-norm) of MIN, Aggregator (T-conorm) MAX , and the Defuzzification Method of MOM.

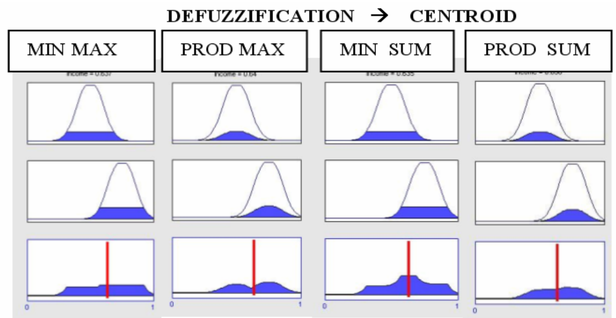


Fig. 4-2 FIS outputs from the different combinations of operators, with Implication (T-norm) of MIN, Aggregator (T-conorm) MAX, and the Defuzzification Method of CENTROID.

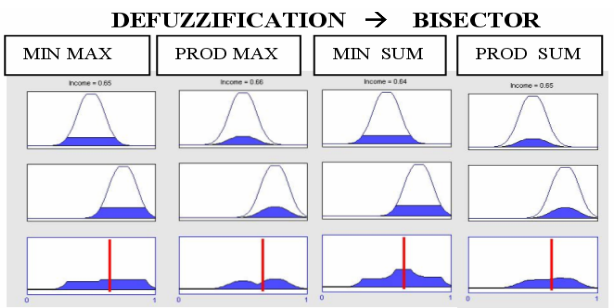


Fig. 4-3 FIS outputs from the different combinations of operators. with Implication (T-norm) of MIN, Aggregator (T-conorm) MAX, and the Defuzzification Method of BISECTOR.

Fig. 4-2 illustrates that the final output for the Centroid, is the center of the entire area (in the horizontal axis) of the outputs MFs. By choosing the centroid in the considered general FIS, the output might yield other MF for the output than the expected one.

Fig. 4-3 includes the obtained outputs for the Bisector. Here, the MF corresponding to the output is similar to the Centroid. That is, the output in one side has the same area of weight (shade) to the other side. These methods also yield to other MF for the general FIS.

As a result, the fuzzy reasoning selected from this analysis has the *Product* in the Implication Operator (T-norm) and *Sum* in the Aggregation Operator (T-conorm). Also, the defuzzification method selected is Mean of Maximum, MOM. This fuzzy reasoning was assigned to the modified FIS. The combination used in the original working model FIS and the combination used in the modified FIS yield similar outputs for the same inputs.

Even though, this section may indicate that the appropriate fuzzy reasoning should be PROD SUM and MOM; the parameter optimization described in the next section needs to consider PROD SUM and centroid. Due to the fact that Centroid defuzzification method facilitates to consider an objective function in terms of a fobrenious norm representing a performance criterion. This is not detrimental to the goal of constructing an improved FIS; rather, as shown in section VI, using the proposed parameter optimization this goal can still be attained.

In the next section is exemplified in detail the selection of PROD SUM combination for the modified FIS.

V. Application of Selected Fuzzy Reasoning and Defuzzification Method; and Evaluation of the obtained Results

After selecting a fuzzy reasoning for the general FIS, these combinations of operators and type of defuzzification methods are assigned to the modified "Main" FIS. This section explains the results obtained from the original "Main" FIS and the modified "Main" FIS in the presented tables.

In order to compare and analyze the results of the "Main" FIS using different operators in their fuzzy reasoning, two input types are:

Case 1. FIS comparison results of the outputs' values obtained from the user interface 'safety input';

Case 2. FIS comparison results of the outputs' values obtained arbitrarily 'risky input'.

Here, the headings of the tables are fare explained for both cases. The results are illustrated in two types of tables: in values crisp, and in 'linguistic' interpretation of those values crisp.The following illustration presents the organization of the results for the first type of table.

Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition		Gaussian MF (Variations in the Imp and Agg composition)		
	Original	Gauss - MOM	Gauss-Centroid	Prod – Max	Min - Sum	Prod - Sum
Imp	MIN	MIN	MIN	PROD	MIN	PROD
Agg	MAX	MAX	MAX	MAX	SUM	SUM
Deffuz	MOM	MOM	CENTROID	CENTROID	CENTROID	CENTROID

There are three sets of outputs, resulting from: FIS with the original MFs (Original MF (trap,bell,etc)), FIS with gaussian MFs with MIN-MAX composition (Gaussian MF and MIN-MAX Composition) and FIS with gaussian MFs with different combinations (Gaussian MF (Variations in the Imp and Agg composition).

For each of these sets of outputs, it was specified the Definition of Method, Implication Operator (Imp), Aggregation Operator (Agg), Defuzzification Method (Defuzz). The tables, furthermore, concentrate the results obtained after creating a FIS with the same MFs properties but with modified Implication and Aggregation operators for each case, and also the Deffuzzification method is changed from Mean of Maximum (MOM) to Centroid of Area (CENTROID).

The second type of tables provide the results for the different combinations of the T-norm operator, as presented in the illustration, for the three defuzzification methods: MOM, CENTROID, and Bisector. Moreover, the results included in this type of table are from the original FIS; the Gaussian converted FIS using MIN and MAX as implication and Aggregation operators, respectively; and the Gaussian converted FIS using PROD and SUM as implication and Aggregation operators, respectively.

Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition			Gaussian MF and RODUCT – SUM Composition		
	Original	Gauss - MOM	Gauss-Centroid	Gauss-Bisector	Prod - Sum MOM	Prod - Sum Centroid	Prod - Sum Bisector
Imp	MIN	MIN	MIN	MIN	PROD	PROD	PROD
Agg	MAX	MAX	MAX	MAX	SUM	SUM	SUM
Deffuz	MOM	MOM	CENTROID	BISECTOR	MOM	CENTROID	BISECTOR

CASE 1. FIS comparison results of the outputs values obtained from the user interface 'safety input'

The input values, and their interpretation, obtained from the Interface are included in Table II. The inputs are: gender (input 1) is **Male**, age (input 2) is **Middle Age**, level of education (studies – input 3) is **University**, where the years that he has been working (Yearswork – input 4) are **Several**, his marital status (maritalSta – Input 5) is **Single**, his children age (Childage – Input 6) is in the area of **Kids**, and his occupation (Occupation – Input 7) is in **Engineering.**

TABLE II
"Safety Input" values

INPUTS	Crisp Values	Fuzzy Interpretation
Input 1 (gender)	***0***	***Male***
Input 2 (age)	***36.68***	***MiddleAge***
Input 3 (studies)	***2.5***	***University***
Input 4 (Yearswork)	***34.92***	***Several***
Input 5 (maritalSta)	***1***	***Single***
Input 6 (Childage)	***8.892***	***Kids***
Input 7 (Occupation)	***0.08***	***Engineering***

Tables III and IV (located in Appendix A) include the crisp outputs values corresponding to the considered 'safety' input. The difference from the original output value to any of the outputs corresponding to the T-norm T-conorms combinations is less than 0.03; then, the interpretation of these values is the same for all the considered combinations. These outputs are presented in the Table V:

TABLE V
INTERPRETATION OF THE RESULTS OBTAINED

Output	Interpretation
Out 1 (PurchCap)	*Very Good*
Out 2 (PurchLink)	*Men*
Out 3 (FreeTime)	*Moderate*
Out 4 (ExpLevel)	*Experts*
Out 5 (Occupation)	*(Technical-Engineering)*

According to the "safety" input, the outputs obtained are: Purchaser Capacity (PurchCap – Out 1) is **Very Good,** Purchaser Type (PurchLink – Out 2) is **Men**, estimated Availability Time for leisure activates (FreeTime – Out 4) is **Moderate**, Level of experience in the field (ExpLevel – Out 4) is **Experts**, Occupation (Occupation – out 5) is **Technical-Engineering**.

CASE 2. FIS comparison results of the outputs values obtained arbitrarily 'risky input'.

For this case, the x values for this input are positioned close to the intersection of two MFs; therefore, the output in this area can have more than one output option. As a result, the output for the defuzzification method **centroid** and **bisector** method has different output values.

The input values and their interpretation obtained from the Interface are included in Table V. The inputs are: gender (input 1) is **Male**, age (input 1) is **Middle Age**, level of education (studies – input 3) is **University**, where the years that he has been working (Yearswork –input 4) are **Several,** his marital status (maritalSta – Input 5) is **Single**, his children age (Childage – Input 6) is in the area of **Kids**, and his occupation (Occupation – Input 7) is in **Business.** This Table VIII includes both the obtained 'Crisp Values' and the 'Fuzzy Interpretation' using a "Risky Input".

TABLE VIII
"RISKY INPUT" VALUES

INPUTS	Crisp Values	Fuzzy Interpretation
Input 1 (gender)	*0.47*	*Male*
Input 2 (age)	*24.5*	*MiddleAge*
Input 3 (studies)	*3.0*	*University*
Input 4 (Yearswork)	*28.4*	*Several*
Input 5 (maritalSta)	*0.51*	*Single*
Input 6 (Childage)	*8.892*	*Kids*
Input 7 (Occupation)	*0.49*	*Bussiness*

Tables VI-I and VI-II (located in Appendix A) summarize the results obtained for the risky input presenting the values and the interpretation of these outputs values, respectively. Tables VII-I and VII-II (located in Appendix A) compare the results for the different defuzzification methods, also the values and interpretation of the obtained outputs.

After comparing the results from Tables VI-I and VI-II, VII-I and VII-II, the output values corresponding to the T-norm T-conorm combinations with a difference greater than 1 to the output of the original FIS provide an altered output. For example, on Tables VII-I and VII-II, *Purchaser Capacity* (Out 2 (PurchLink)) the output from the original working model FIS is **Women;** however, the output from the modified FIS with the MIM-MAX-MOM composition is **Men** and PROD-SUM-MOM composition is **Teenagers**.

The modified FIS for the same inputs, using a particular T-norm, T-conorm, and Defuzzification method can yield outputs that do not resemble the outputs of the original working model FIS. That is, why it may be necessary to resort to parameter optimization.

The next step, "Parameter Optimization of the FIS," optimizes the actual MFs parameters and implements the new optimized MFs parameters in the Main FISs to generate optimum parameter values and also solutions.

VI. OPTIMIZATION OF THE FIS PARAMETERS

Once the MFs are converted to be continuous Gaussians functions over the entire input domain; each Gaussian sigma parameter and center parameter are some x values as exemplified in Table IX. The Gaussian Membership Function [1] is:

$$gaussian\ (x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2} \qquad (1)$$

The "Main" FIS contains seven inputs, giving a total of 24 Gaussian MFs with 24 centers and 24 sigma parameters represented as follows:

x → u1 ... u7
c → c11 ... c74 → x(2), x(4)…x(48)
sigma → s11 ... s74 → x(1), x(3)…x(47)

To represent these variables, the same input variable is chosen: *Age*, as in Section II. The range, number of MFs (numMFs), the MFs, and their sigma and center parameters, respectively, are presented in the Table IX.

TABLE IX
CHARACTERISTICS OF INPUT VARIABLE: AGE

	Gaussian MF properties		
Name	'Age'		
Range	[0 70]		
NumMFs	4	Sigma	center
MF1 ='VeryYoung'	'gaussmf',[2.62 3.84]	2.62	3.84
MF2='Young'	'gaussmf',[4.26 16.3]	4.26	16.3
MF3='MiddleAge'	'gaussmf',[9.73 37.6]	9.73	37.6
MF4='Old'	'gaussmf',[7.14 59.5]	7.14	59.5

The equivalent optimization variables are:

TABLE X
MF PARAMETER VARIABLES FOR THE INPUT VARIABLE: AGE

for Optimization		'Age'
Parameter Name	Var Name	[0 70]
x5	s21	2.62
x6	c21	3.84
x7	s22	4.26
x8	c22	16.3
x9	s23	9.73
x10	c23	37.6
x11	s24	7.14
x12	c24	59.5

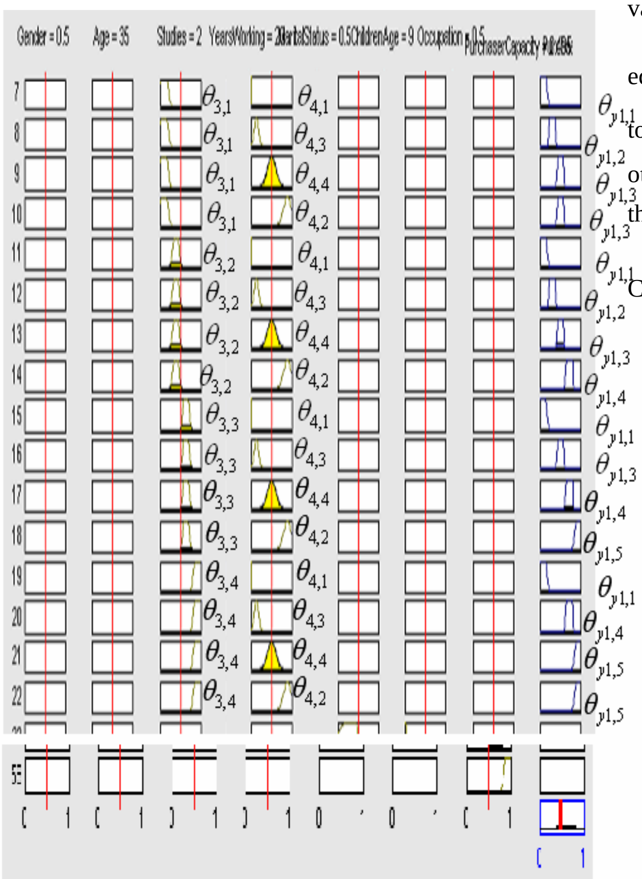


Fig. 5 Specification of MFs parameter for each rule (in total 12) of the output F1.

A similar analysis is applied to the rest of the inputs: the variables x1 to x4 correspond to the variable "**Gender**"; x13 to x20 to "**studies**" input; x21 to x28 to "**years working**" input; x29 to x32 to "**marital status**" input; x33 to x40 to "**ChildrenAge**" input; and x41 to x48 to "**Occupation**" input.

Once the input MFs parameters have been defined, the F outputs equation of the "Main" FIS were obtained. Also, considering the sum-product composition, ***Theorem 4.1*** [1], is applied to MAIN FIS. The first step is to obtain the firing strength as follow:

$$\mu_{Yicj}(z) = w_1 \int_z \mu_{C_i}(z)dz + w_2 \int_z \mu_{C_i}(z)dz \qquad (2)$$

For example, to obtain the F1 (for the first output: Purchaser Capacity), the firing strength is the product of (input 3-"*Studies*") * (input 4-"*Years Working*") * (output 1 – "*Purchaser Capacity*") and sum the next rule, successively until the end, illustrated in Fig. 5. The product of the inputs represents the Firing Strength. Second, after analyzing the output of the system and considering defuzzification method as Mean of Maximum, each membership function receives the value of 1.

The formulas for each output are: where gMFu34 is equivalent to $\mu_{\theta_{3,4}}(u_3)$ and gMFu41 is also equivalent to $\mu_{\theta_{4,1}}(u_4)$. The same interpretation is applied to all the others MFs, respectively. And the output tMFy11w1_1 means that $\mu_{\theta_{y1,1}}(z)$.

Therefore, the output equation for the output one (Purchaser Capacity) is:

$$\begin{aligned} F1 = {} & gMFu34 * gMFu41 * tMFy11w1\_1 + \ldots \\ & gMFu34 * gMFu43 * tMFy12w1\_2 + \ldots \\ & gMFu34 * gMFu44 * tMFy13w1\_3 + \ldots \\ & gMFu34 * gMFu42 * tMFy13w1\_4 + \ldots \\ & gMFu31 * gMFu41 * tMFy11w1\_5 + \ldots \\ & gMFu31 * gMFu43 * tMFy12w1\_6 + \ldots \\ & gMFu31 * gMFu44 * tMFy13w1\_7 + \ldots \\ & gMFu31 * gMFu42 * tMFy14w1\_8 + \ldots \\ & gMFu32 * gMFu41 * tMFy11w1\_9 + \ldots \\ & gMFu32 * gMFu43 * tMFy13w1\_10 + \ldots \\ & gMFu32 * gMFu44 * tMFy14w1\_11 + \ldots \\ & gMFu32 * gMFu42 * tMFy15w1\_12 + \ldots \\ & gMFu33 * gMFu41 * tMFy11w1\_13 + \ldots \\ & gMFu33 * gMFu43 * tMFy14w1\_14 + \ldots \\ & gMFu33 * gMFu44 * tMFy15w1\_15 + \ldots \\ & gMFu33 * gMFu42 * tMFy15w1\_16 \end{aligned} \qquad (3)$$

Following the same procedure, the equations for the outputs 2 to 5 are obtained. Where each input has:

Input → u_1 has 2 MFs $\mu(u_1)\theta_{1,i}$, where $i = 1 \ldots 2$;

Input → u_2 has 4 MFs $\mu(u_2)\theta_{2,i}$, where $i = 1 \ldots 4$;

Input → u_3 has 4 MFs $\mu(u_3)\theta_{3,i}$, where $i = 1 \ldots 4$;

Input → u_4 has 4 MFs $\mu(u_4)\theta_{4,i}$, where $i = 1 \ldots 4$;

Input → u_5 has 2 MFs $\mu(u_5)\theta_{5,i}$, where $i = 1 \ldots 2$;

Input → u_6 has 4 MFs $\mu(u_6)\theta_{6,i}$, where $i = 1 \ldots 4$;

Input → u_7 has 4 MFs $\mu(u_7)\theta_{7,i}$, where $i = 1 \ldots 4$;

Subsequently, a Jacobian matrix (Mayorga 2002), is created using the functions (F1, F2, F3, F4, F5), as follows:

$$\begin{array}{ccccccc} u1 & u2 & u3 & u4 & u5 & u6 & u7 \end{array}$$
$$\begin{bmatrix} 0 & 0 & \frac{\partial F1}{\partial u3} & \frac{\partial F1}{\partial u4} & 0 & 0 & 0 \\ \frac{\partial F2}{\partial u1} & \frac{\partial F2}{\partial u2} & 0 & 0 & \frac{\partial F2}{\partial u5} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F3}{\partial u5} & \frac{\partial F3}{\partial u6} & 0 \\ 0 & 0 & \frac{\partial F4}{\partial u3} & \frac{\partial F4}{\partial u4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial F5}{\partial u7} \end{bmatrix} = J \tag{4}$$

Here, the objective function to be optimized (Mayorga, 2002) is selected to be

$$\left\| \Psi\ (u\ ,a\ ) \right\|_F^2 \tag{5}$$

This objective function represents a suitable performance criterion in terms of the Frobenius norm, (Mayorga 2002). This criterion ensures that for small changes in the outputs there should be small changes in the outputs, (Mayorga 2002)

After applying the Frobenius norm to the Jacobian matrix, the following objective function is obtained:

$$\|J\|_F^2 = \left[\sqrt{ \begin{array}{l} \left|\frac{\partial F1}{\partial u3}\right|^2 + \left|\frac{\partial F1}{\partial u3}\right|^2 + \left|\frac{\partial F1}{\partial u4}\right|^2 + \left|\frac{\partial F2}{\partial u1}\right|^2 + \left|\frac{\partial F1}{\partial u3}\right|^2 + \\ \left|\frac{\partial F1}{\partial u4}\right|^2 + \left|\frac{\partial F2}{\partial u1}\right|^2 + \left|\frac{\partial F2}{\partial u2}\right|^2 + \left|\frac{\partial F2}{\partial u5}\right|^2 + \left|\frac{\partial F2}{\partial u5}\right|^2 + \\ \left|\frac{\partial F3}{\partial u5}\right|^2 + \left|\frac{\partial F3}{\partial u6}\right|^2 + \left|\frac{\partial F4}{\partial u3}\right|^2 + \left|\frac{\partial F4}{\partial u4}\right|^2 + \left|\frac{\partial F5}{\partial u7}\right|^2 \end{array} } \right]^2 \tag{6}$$

subject to

s11_LowB	<=	x1	(s11)	<=	s11_UppB
c11_LowB	<=	x2	(c11)	<=	c11_UppB
s12_LowB	<=	x3	(s12)	<=	s12_UppB
c12_LowB	<=	x4	(c12)	<=	c12_UppB
		...			
s74_LowB	<=	x47	(s74)	<=	s74_UppB
c74_LowB	<=	x48	(c74)	<=	c74_UppB

Specification of the Intervals for the MFs Parameters

To specify the range values for the MFs parameters, it is important to mention that the size of the range does affect the optimization process. If a large range is indicated for the Upper (UppB) and Lower (LowB) boundaries, the optimized resultant parameters might alter the membership function shape, which will provide a new MF in a different position and also altering the output of the system. The range selected was 2%, and the following Fig. 6 illustrates this range for the input *Age*. Also, for the purpose to show the interval for each MF parameter, the x-axis is increased. The next section contains the results obtained.

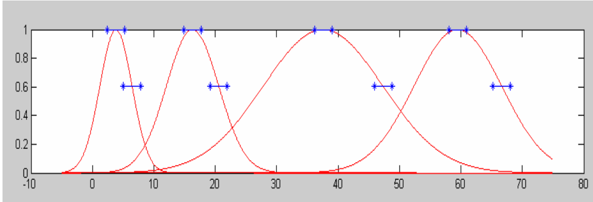


Fig. 6 Ranges specification for the *Age* Input.

VII. Evaluation of Optimized Results

In order to evaluate the output of the system, different ranges were considered, such as:

- Original ranges proposed determined subjectively
- Reduced and verified ranges of 2% for both Sigma and Center
- Reduced range of 5% Sigma and 2% Center

Table XI shows that the solution from 'Reduce and Verified ranges of 2%' generated an optimized Frobenious Output of 21.5035, and the others solutions generated outputs less than 1. The reason is that, in the optimization process, the membership function is adjusted to a shape that generates a reduced output.

TABLE XI
Comparison of the Obtained Results from the Different Set of Defined Ranges

	Original ranges proposed	Reduce and Verified ranges of 2%	Range of Sig 5% and Center 2%
Selected Output	Solution 65	Solution 65	Solution 65
u1	0.707	0.707	0.707
u2	69.5302	69.5302	69.5302
u3	1.4255	1.4255	1.4255
u4	8.7348	8.7348	8.7348
u5	0.0413	0.0413	0.0413
u6	9.6231	9.6231	9.6231
u7	0.8784	0.8784	0.8784
Frobenius Output	65.034	65.034	65.034
Optimized Frobenius Output	*7.00E-04*	21.5035	0.3192
iterations:	16	16	20
funcCount:	809	801	1001

The MF parameters are valid because they are in the indicated range. However, the mean of the MF in relation to the system has changed. The following screenshots illustrate this situation. The Input 6 corresponds to the **'ChildrenAge'**

S6i refers to Sigma of the MFs (*i*) of the Input 6

C6i refers to Center of the MFs (*i*) of the Input 6

Range %	2%		5%	2%
Parameter	S6i	C6i	S6i	C6i
Range Value	0.36	0.36	0.36	0.9

Table XII shows the values for each parameter, considering "Boundaries reduced to 2%" and the "Boundaries Reduced to 2% center and 5% sigma".

TABLE XII
COMPARISON OF DIFFERENT SETS OF DEFINED RANGES

			(1)	'ChildrenAge'			Range	[0 18]
5% & 2%	2%		Parameter		Original Value		2%	5% & 2%
-0.088	0.452	<=	x(33)	s61	0.812	<=	1.172	1.712
2	2	<=	x(34)	c61	2.36	<=	2.72	2.72
0.47	1.01	<=	x(35)	s62	1.37	<=	1.73	2.27
7.28	7.28	<=	x(36)	c62	7.64	<=	8	8
1.42	1.96	<=	x(37)	s63	2.32	<=	2.68	3.22
15.24	15.24	<=	x(38)	c63	15.6	<=	15.96	15.96
0.098	0.098	<=	x(39)	s64	0.348	<=	0.598	1.248
-0.432	-0.432	<=	x(40)	c64	-0.182	<=	0.068	0.178

The Fig. 7 shows the original 'Kids' MF is in the interval from 5 to 10 over the X axis. However, the 'Large Boundaries (Proposed)' is positioned in the range from approximately 11 to 16. Even though the shape is similar to the original, the position altered the output of the system; while it formerly referred to kids in the range of 5 to 10, it now refers to kids between the ages of 11 to 16. Also, when the Original MF has a z value of 1, the 'Large Boundaries (Proposed)' MF has a value close to 0. Supporting the above statement, the 'New' MF has a different meaning to the system and consequently the output is altered.

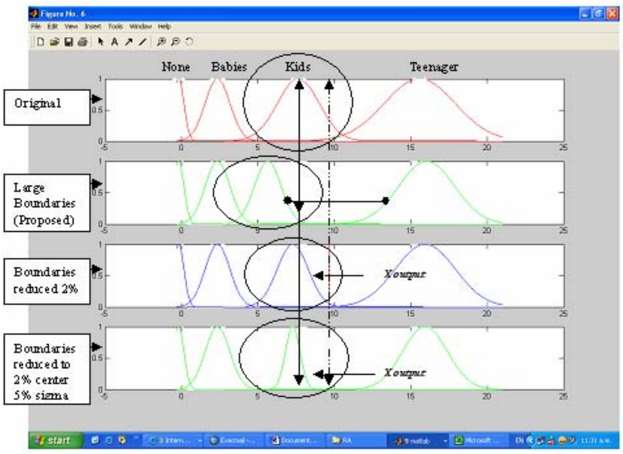


Fig. 7 Representation of MFs affected for the range definition.

The "Boundaries reduced 2%" MF has a significant value close to one and is similar to the original. On the other hand, the last "Boundaries reduced to 2% center 5% sigma" MF keeps the center between the small range (2%), but the shape is smaller. This shape is affected, for example, when the x input is around 9, the output z has a value of 0, and while the 'Large Boundaries (Proposed)' MF has a value greater than zero. The above Fig. 8 included the "Lower Bound" and "Upper Bound" for each MF function for the boundaries of 2% and 2% center and 5% sigma. Focusing our attention on the "Lower Bound", we can see that the "lower bound" and the "Output" are overlapped. Therefore, the MF with the smaller value range has reduced the shape of the MF, while the MF with a lower boundary of 2% keeps a configuration similar to the original.

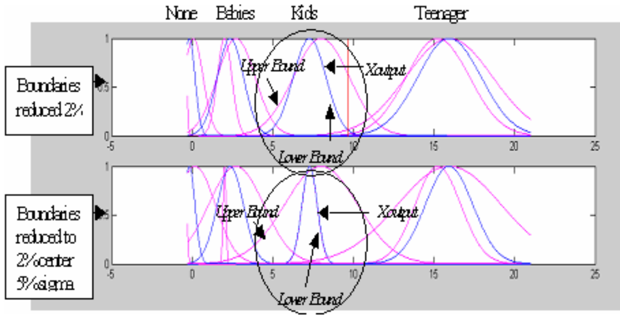


Fig. 8 Illustration of the Lower and Upper boundary for the MFs

The same analysis used for the MFs of the input 6 "Children Age" have been applied to the rest of the inputs. The following screenshots summarize the results for each input of the "MAIN" FIS.

Final Obtained Results

After the ranges of the parameter have been defined, the optimization process is performed. The Matlab function employed, *fmincon*, finds the minimum of a constrained nonlinear multivariable function. In order to identify the set of parameters to be optimized, a program was created to generate arbitrary inputs and to find the correspondent output of the Frobenious equation.

From those outputs, different input values were selected within the higher output range. Those values were 65, 70, 75, 80, and 85. The Frobenious Output and the resultant Optimized Frobenious Outputs are included in Table XIII.

The results for the solution 65 comparing the MF parameters with the original values and including the *lower and upper* boundaries are illustrated in the Table XIV.

TABLE XIII
COMPARISON OF THE OBTAINED RESULTS FROM THE REDUCE AND VERIFIED RANGES PROPOSED 2%

Reduce and Verified ranges proposed					
Selected Output	Solution 65	Solution 70	Solution 75	Solution 80	Solution 85
u1	0.707	0.3751	0.8401	0.8857	0.0256
u2	69.5302	48.9619	3.8248	63.639	51.4067
u3	1.4255	2.9579	0.3522	3.9705	1.4128
u4	8.7348	14.0302	11.0727	12.6583	30.2419
u5	0.0413	0.7922	0.1474	0.0845	0.0367
u6	9.6231	2.2396	15.7534	0.1019	16.0441
u7	0.8784	0.8977	0.9009	0.713	0.094
Frobenius Output	65.034	70.0243	75.0421	80.2468	85.1705
Optimized Frobenius Output	21.5035	12.5217	13.5123	14.5269	20.4439
iterations:	16	19	28	16	19
funcCount:	801	955	1401	801	951

TABLE XIV
COMPARISON OF MFS PARAMETERS FOR SOLUTION 65.

Var. Name Gauss Eq.	Var. Name Opt-Eq.	LowB	UppB	Original	Optimized Parameter
s11	***x(1)***	0.23	0.27	0.25	0.27
c11	***x(2)***	-0.02	0.02	0	0.02
s12	***x(3)***	0.23	0.27	0.25	0.23
c12	***x(4)***	0.98	1.02	1	1.02
s21	***x(5)***	1.22	4.02	2.62	2.62
c21	***x(6)***	2.44	5.24	3.84	3.84
s22	***x(7)***	2.86	5.66	4.26	4.26
c22	***x(8)***	14.9	17.7	16.3	16.3
s23	***x(9)***	8.33	11.13	9.73	8.33
c23	***x(10)***	36.2	39	37.6	37.1755
s24	***x(11)***	5.74	8.54	7.14	5.74
c24	***x(12)***	58.1	60.9	59.5	58.1
s31	***x(13)***	0.16	0.32	0.24	0.3131
c31	***x(14)***	1.43	1.59	1.51	1.4368
s32	***x(15)***	0.176	0.336	0.256	0.1892
c32	***x(16)***	2.42	2.58	2.5	2.5211
s33	***x(17)***	0.345	0.505	0.425	0.425
c33	***x(18)***	3.82	3.98	3.9	3.9
s34	***x(19)***	0.196	0.356	0.276	0.347
c34	***x(20)***	0.268	0.428	0.348	0.4005
s41	***x(21)***	-0.04	0.76	0.36	0.36
c41	***x(22)***	-0.3325	0.4675	0.0675	0.0675
s42	***x(23)***	2.98	4.58	3.78	3.78
c42	***x(24)***	34.7	36.3	35.5	35.5
s43	***x(25)***	1.323	2.923	2.123	1.3823
c43	***x(26)***	4.2	5.8	5	4.3497
s44	***x(27)***	3.447	5.047	4.247	4.1136
c44	***x(28)***	19.2	20.8	20	20.0487
s51	***x(29)***	0.165	0.205	0.185	0.205
c51	***x(30)***	0.203	0.243	0.223	0.243
s52	***x(31)***	0.165	0.205	0.185	0.165
c52	***x(32)***	0.754	0.794	0.774	0.794
s61	***x(33)***	0.452	1.172	0.812	0.812
c61	***x(34)***	2	2.72	2.36	2.36
s62	***x(35)***	1.01	1.73	1.37	1.01
c62	***x(36)***	7.28	8	7.64	7.28
s63	***x(37)***	1.96	2.68	2.32	1.96
c63	***x(38)***	15.24	15.96	15.6	15.96
s64	***x(39)***	0.098	0.598	0.348	0.348
c64	***x(40)***	-0.432	0.068	-0.182	-0.182
s71	***x(41)***	0.0527	0.0927	0.0727	0.0727
c71	***x(42)***	0.0107	0.0507	0.307	0.0507
s72	***x(43)***	0.0537	0.0937	0.0737	0.0762
c72	***x(44)***	0.329	0.369	0.349	0.3491
s73	***x(45)***	0.0525	0.0925	0.0725	0.0925
c73	***x(46)***	0.628	0.668	0.648	0.668
s74	***x(47)***	0.052	0.092	0.072	0.092
c74	***x(48)***	0.943	0.983	0.963	0.983

VIII. CONCLUSIONS

This work presents a simple procedure to optimize the Membership Functions (MF) parameters of a general FIS. First, it is shown the importance of appropriately converting the MFs to continuous MFs. Also, it is demonstrated the relevance of the selection of an appropriate Fuzzy Reasoning and Defuzzification Method; to define the right ranges/ intervals for the MFs parameters, and to analyze all possible solutions. Assigning an appropriate range (lower and upper boundaries) to the antecedent parameters modifies the MF shape during the Optimization process to an "optimal" shape and location. However, a larger range might alter the shape of the inputs MF of the system and also the generated outputs.

In this work, it was determined that, when considering parameter variations in the range of up to 2%; the shape of the membership functions have been optimized without altering the shape and meaning of the MF from the original MF meaning. It was also demonstrated that increasing the range of sigma affected the shape and position of the MF to a smaller degree than increasing the center parameter, since this center range allows the MF to be positioned in an area that might change the meaning.

The information presented in the tables and the figures provided the following conclusions:

- A small variation in the parameter ranges considered reduces the optimized output of the Frobenious Equation to approximately zero. This means that the optimization process might adjust some MFs to a position where the output value is close to zero as well. As a result, the shape and/or position of the MF function might be altered from its original parameters and generate a different system output.
- The variation in the MF center range affects the meaning of the MF more than increasing its sigma range does. This is due to the fact that the MF's position remains within a small range and the sigma is changed in a larger range. Consequently, the original output "meaning" is also changed.

In summary, this Paper has shown that the application of the proposed optimization steps can conduce to a simple parameter optimization, with respect to a performance criterion, of a general FIS system.

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APPENDIX A: TABLES PRESENTING THE RESULTS AFTER COMPARISONS

TABLE III
COMPARING THE OUTPUTS (CRISP VALUES) OF "MAIN" FIS RESULTS WITH THE VARIATIONS IN THE IMPLICATION AND AGGREGATION COMPOSITIONS AND THE CENTROID DEFFUZZIFICATION METHOD. "SAFETY "CRISP VALUES OBTAINED FROM THE INTERFACE

INPUTS Crisp Values obtained from the Interface		OUTPUTS							
		Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition		Gaussian MF (Variations in the Imp and Agg composition)			
			Original	Gauss - MOM	Gauss- Centroid	Prod – Max	Min - Sum	Prod - Sum	
		Imp	MIN	MIN	MIN	PROD	MIN	PROD	
Input 1 (gender)	0	Agg	MAX	MAX	MAX	MAX	SUM	SUM	
Input 2 (age)	36.68	Deffuz	MOM	MOM	CENTROID	CENTROID	CENTROID	CENTROID	
Input 3 (studies)	2.5	Out 1 (PurchCap)	0.9450	0.9450	0.9195	0.9198	0.9192	0.9197	
Input 4 (Yearswork)	34.92	Out 2 (PurchLink)	5.5200	5.5200	5.5348	5.5339	5.5367	5.5357	
Input 5 (maritalSta)	1	Out 3 (FreeTime)	0.5350	0.5450	0.5554	0.5524	0.5557	0.5525	
Input 6 (Childage)	8.892	Out 4 (ExpLevel)	0.8800	0.8800	0.8719	0.8720	0.8713	0.8719	
Input 7 (Occupation)	0.08	Out 5 (Occupation)	0.1500	0.1500	0.1353	0.1357	0.1353	0.1357	

TABLE IV
COMPARING THE OUTPUTS (CRISP VALUES) OF "MAIN" FIS RESULTS WITH THE MIN-MAX AND PROD-SUM COMPOSITIONS FOR THE DEFUZZIFICATIONS METHODS: MOM, CENTROID AND BISECTOR. CRISP VALUES OBTAINED FROM THE INTERFACE

INPUTS Crisp Values obtained from the Interface		OUTPUTS							
		Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition			Gaussian MF and PRODUCT – SUM Composition		
			Original	Gauss - MOM	Gauss- Centroid	Gauss- Bisector	Prod - Sum MOM	Prod - Sum Centroid	Prod - Sum Bisector
		Imp	MIN	MIN	MIN	MIN	PROD	PROD	PROD
Input 1 (gender)	0	Agg	MAX	MAX	MAX	MAX	SUM	SUM	SUM
Input 2 (age)	36.68	Deffuz	MOM	MOM	CENTROID	BISECTOR	MOM	CENTROID	BISECTOR
Input 3 (studies)	2.5	Out 1 (PurchCap)	0.9450	0.9450	0.9195	0.9200	0.9450	0.9197	0.9200
Input 4 (Yearswork)	34.92	Out 2 (PurchLink)	5.5200	5.5200	5.5348	5.5200	5.4800	5.5357	5.5200
Input 5 (maritalSta)	1	Out 3 (FreeTime)	0.5350	0.5450	0.5554	0.5500	0.5250	0.5525	0.5400
Input 6 (Childage)	8.892	Out 4 (ExpLevel)	0.8800	0.8800	0.8719	0.8700	0.7900	0.8719	0.8700
Input 7 (Occupation)	0.08	Out 5 (Occupation)	0.1500	0.1500	0.1353	0.1400	0.1500	0.1357	0.1400

TABLE VI-I.
COMPARING THE OUTPUTS (CRISP VALUES) OF "MAIN" FIS RESULTS WITH THE VARIATIONS IN THE IMPLICATION AND AGGREGATION COMPOSITIONS AND THE CENTROID DEFFUZZIFICATION METHOD. USING ARBITRARY "RISKY " INPUT VALUES

INPUTS Arbitrary values		1) OUTPUTS						
		Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition		Gaussian MF (Variations in the Imp and Agg composition)		
			Original	Gauss - MOM	Gauss-Centroid	Prod – Max	Min - Sum	Prod - Sum
		Imp	MIN	MIN	MIN	PROD	MIN	PROD
Input 1 (gender)	0.47	Agg	MAX	MAX	MAX	MAX	SUM	SUM
Input 2 (age)	24.5	Deffuz	MOM	MOM	CENTROID	CENTROID	CENTROID	CENTROID
Input 3 (studies)	3.0	Out 1 (PurchCap)	0.8000	0.9050	0.8025	0.8111	0.8443	0.8554
Input 4 (Yearswork)	28.4	Out 2 (PurchLink)	3.8400	5.0000	3.6362	3.6685	3.3551	3.3929
Input 5 (maritalSta)	0.51	Out 3 (FreeTime)	0.5400	0.5450	0.4193	0.4128	0.4228	0.4156
Input 6 (Childage)	8.892	Out 4 (ExpLevel)	0.8600	0.8600	0.8528	0.8721	0.8515	0.8721
Input 7 (Occupation)	0.49	Out 5 (Occupation)	0.3800	0.3750	0.4717	0.4707	0.4717	0.4707

TABLE VI-II
COMPARING THE OUTPUTS (INTERPRETATION VALUES) OF "MAIN" FIS RESULTS WITH THE VARIATIONS IN THE IMPLICATION AND AGGREGATION COMPOSITION AND THE CENTROID DEFFUZZIFICATION METHOD. USING ARBITRARY "RISKY " INPUT VALUES

INPUTS Arbitrary values		3) OUTPUTS						
		Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition		Gaussian MF (Variations in the Imp and Agg composition)		
			Original	Gauss - MOM	Gauss-Centroid	Prod – Max	Min - Sum	Prod - Sum
		Imp	MIN	MIN	MIN	PROD	MIN	PROD
Input 1 (gender)	0.47	Agg	MAX	MAX	MAX	MAX	SUM	SUM
Input 2 (age)	24.5	Deffuz	MOM	MOM	CENTROID	CENTROID	CENTROID	CENTROID
Input 3 (studies)	3.0	Out 1 (PurchCap)	*Very Good*	*Very Good*	*Very Good*	*Very Good*	*Very Good*	*Very Good*
Input 4 (Yearswork)	28.4	Out 2 (PurchLink)	*Women*	*Men*	*Women*	*Women*	*Women*	*Women*
Input 5 (maritalSta)	0.51	Out 3 (FreeTime)	*Moderate*	*Moderate*	*Moderate*	*Moderate*	*Moderate*	*Moderate*
Input 6 (Childage)	8.892	Out 4 (ExpLevel)	*Experts*	*Experts*	*Experts*	*Experts*	*Experts*	*Experts*
Input 7 (Occupation)	0.49	Out 5 (Occupation)	*(Business-Administration -Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration -Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration-Marketing)*

TABLE VII-I
COMPARING THE OUTPUTS (CRISP VALUES) OF "MAIN" FIS RESULTS WITH THE MIN-MAX AND PROD-SUM COMPOSITIONS FOR THE DEFUZZIFICATIONS METHODS: MOM, CENTROID AND BISECTOR. USING ARBITRARY INPUT VALUES

INPUTS Arbitrary values		OUTPUTS							
		Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition			Gaussian MF and PRODUCT – SUM Composition		
			Original	Gauss - MOM	Gauss-Centroid	Gauss-Bisector	Prod - Sum MOM	Prod - Sum Centroid	Prod - Sum Bisector
		Imp	MIN	MIN	MIN	MIN	PROD	PROD	PROD
Input 1 (gender)	0.47	Agg	MAX	MAX	MAX	MAX	SUM	SUM	SUM
Input 2 (age)	24.5	Deffuz	MOM	MOM	CENTROID	BISECTOR	MOM	CENTROID	BISECTOR
Input 3 (studies)	3.0	Out 1 (PurchCap)	0.8000	0.9050	0.8025	0.8000	0.9450	0.8554	0.8800
Input 4 (Yearswork)	28.4	Out 2 (PurchLink)	3.8400	5.0000	3.6362	3.8400	1.5200	3.3929	3.4400
Input 5 (maritalSta)	0.51	Out 3 (FreeTime)	0.5400	0.5450	0.4193	0.4300	0.5250	0.4156	0.4500
Input 6 (Childage)	8.892	Out 4 (ExpLevel)	0.8600	0.8600	0.8528	0.8500	0.7900	0.8721	0.8700
Input 7 (Occupation)	0.49	Out 5 (Occupation)	0.3800	0.3750	0.4717	0.4400	0.3800	0.4707	0.4200

TABLE VII-II.
COMPARING THE OUTPUTS (INTERPRETATION) OF "MAIN" FIS RESULTS WITH THE MIN-MAX AND PROD-SUM COMPOSITIONS FOR THE DEFUZZIFICATION METHODS: MOM, CENTROID AND BISECTOR, USING ARBITRARY INPUT VALUES

INPUTS Arbitrary values		OUTPUTS						
		Definition of Method	Original MF (trap,bell, etc)	Gaussian MF and MIN – MAX Composition		Gaussian MF (Variations in the Imp and Agg composition)		
			Original	Gauss - MOM	Gauss-Centroid	Prod – Max	Min - Sum	Prod - Sum
		Imp	MIN	MIN	MIN	PROD	MIN	PROD
Input 1 (gender)	0.47	Agg	MAX	MAX	MAX	MAX	SUM	SUM
Input 2 (age)	24.5	Deffuz	MOM	MOM	CENTROID	CENTROID	CENTROID	CENTROID
Input 3 (studies)	3.0	Out 1 (PurchCap)	*Very Good*	*Very Good*	*Very Good*	*Very Good*	*Very Good*	*Very Good*
Input 4 (Yearswork)	28.4	Out 2 (PurchLink)	*Women*	*Men*	*Women*	*Women*	*Women*	*Women*
Input 5 (maritalSta)	0.51	Out 3 (FreeTime)	*Moderate*	*Moderate*	*Moderate*	*Moderate*	*Moderate*	*Moderate*
Input 6 (Childage)	8.892	Out 4 (ExpLevel)	*Experts*	*Experts*	*Experts*	*Experts*	*Experts*	*Experts*
Input 7 (Occupation)	0.49	Out 5 (Occupation)	*(Business-Administration -Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration-Marketing)*	*(Business-Administration-Marketing)*