Robust Stability in Multivariable Neural Network Control using Harmonic Analysis

J. Fernandez de Canete, S. Gonzalez-Perez, P. del Saz-Orozco, and I. Garcia-Moral

Abstract—Robust stability and performance are the two most basic features of feedback control systems. The harmonic balance analysis technique enables to analyze the stability of limit cycles arising from a neural network control based system operating over nonlinear plants. In this work a robust stability analysis based on the harmonic balance is presented and applied to a neural based control of a non-linear binary distillation column with unstructured uncertainty. We develop ways to describe uncertainty in the form of neglected nonlinear dynamics and high harmonics for the plant and controller respectively. Finally, conclusions about the performance of the neural control system are discussed using the Nyquist stability margin together with the structured singular values of the uncertainty as a robustness measure.

Keywords—Robust stability, neural network control, unstructured uncertainty, singular values, distillation column.

I. INTRODUCTION

A key reason for using feedback in model based control systems is to reduce the effects of uncertainty which appear in different forms as disturbances or as other imperfections in the model assumed during the control law design process.

The design of feedback compensators in general has to satisfy the four objectives of nominal closed loop stability, nominal performance, robust stability and robust performance [1]. Robust stability implies stability not only for the nominal model of a process but also for the family of models of the process under the uncertainty assumptions made. To deal with the robust stability of linear systems with uncertainty some well known controller design methods has been applied like the LQG, H_∞ and QFT [2] and several results have been reported to quantify robustness measures [3],[4].

Besides, the most of plants under control are nonlinear in nature, showing multiple steady regimes and complex behaviours such as limit cycles. The harmonic balance analysis approach is an approximate technique for analysing nonlinear system oscillations, based on the calculation of the sinusoidal input describing function (SIDF) of a nonlinear system, representing a static nonlinearity by a variable gain [5].

Neural networks offer a promising approach to model based control of multivariable (MIMO) complex nonlinear plants, and several neural control schemes such as predictive control, inverse model control and adaptive control has been reported [6].

In order to be able to design a robust controller for a given process, it is necessary not only to derive a nominal model of the process but also the model uncertainty to which the control system has to be robust. The uncertainty can be used to represent the difference between the real open loop dynamics and the nominal open loop model at any time, or to represent the change of plant behaviour with time. Methods for identifying uncertainty models have been developed for time and frequency domain data [7], [8].

In this paper we analyze the robustness of neural based multivariable control systems with uncertainty, provided that both the neural controller and plant are represented by its SIDF approximation and linearized model respectively. For this purpose we make use of the generalized Nyquist stability criterion applied to analyze nominal stability. The plant uncertainty model is derived using an identification procedure applied to a binary distillation column under feedback neurocontrol, while controller higher harmonics stands for the controller uncertainty, both treated as additive input perturbations. The approach here developed gives alternatives to computing stability margins fro the real closed loop when multivariable uncertainty is defined by its singular value decomposition.

II. THE HARMONIC BALANCE METHOD

The basic idea of the describing function DF approach for analysing nonlinear system behaviour is to replace nonlinear elements with (quasi)linear descriptors whose gains are a function of input amplitude. These descriptors are governed

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by the form of input signal, which is assumed in advance. The sinusoidal input describing function SIDF approach here used can be used for two primary purposes, limit cycle analysis and characterizing the input/output (I/O) behaviour of a nonlinear plant in the frequency domain. [9]. For obtaining an SIDF I/O model of a nonlinear plant, each static nonlinearity in the plant's differential equation set is replaced with the corresponding describing function in analytic form and then set up to solve the equations of harmonic balance.

Considering the MIMO system of Fig. 1 composed by a nonlinear system approximated around an operation point by a linear transfer matrix $G(s)$ and a neural based controller $NN_c(e)$, the open loop system can be approximated in two parts in series, a linear approximation of the system $G(s) = \{G_{ij}(s)\}$ and a nonlinear part (the neural controller) with a describing function matrix $N^1(a,\omega) = \{N_{kl}^1(a,\omega)\}$, where a is a vector inputs amplitude and ω is its common frequency vector.
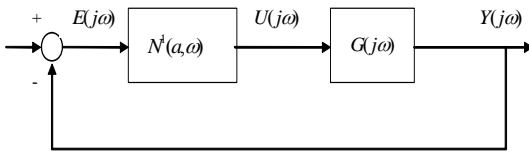

Fig. 1 Structure of the approximated closed loop neural control with SIDF

The describing function is merely the fundamental Fourier coefficient complex representation of the output divided by the amplitude of the assumed sinusoidal input. It is reasoned that any nonlinearity generated higher harmonic components are filtered out in traversing the feedback loop, which classically is composed of low-frequency band components. Therefore, it is assumed that the nonlinearity generates no sub-harmonics, and only a first harmonic analysis is accomplished for the nominal open loop system.

Under such considerations, if we apply a sinusoidal input signal $e_k(t) = \mathbf{Re}(a_k \cdot e^{j\omega t})$, $\omega > 0$ for $k = 1 \dots n$, at the output of the nonlinear neural controller $NN(e)$ we obtain the Fourier series expansion

$$u_i(t) = \sum_{l=1}^{n} \mathrm{Re}(N_{il}^1(a_l,\omega) \cdot a_l \cdot e^{j\omega t}) + \dots \qquad (1)$$

where N_{il}^1 is the first harmonic gain form the *lth* input of the nonlinear controller to its *ith* output and depends on the amplitude a_i, computed as

$$N_{il}^1(a_l,\omega) = \frac{1}{\pi \cdot a_l} \int_{-\pi}^{\pi} u_i(t) \cdot e^{-j\omega t} \cdot d(\omega t) \qquad (2)$$

assuming that $e_k(t) = 0 \quad k \neq l$. Therefore, the plant output frequency response is given by

$$Y(j\omega) = G(j\omega) \cdot N^1(a,\omega) \cdot E(j\omega) \qquad (3)$$

and for null reference command signal we have $Y(j\omega) = -E(j\omega)$ yielding the harmonic balance matrix equation as

$$I + G(j\omega) \cdot N^1(a,\omega) = 0 \qquad (4)$$

Consequently the condition for fulfillment of the harmonic balance equation is given by

$$\det(I + G(j\omega) \cdot N^1(a,\omega)) = 0 \qquad (5)$$

The existence of limit cycles for the nominal closed loop control system of Fig. 1 can be determined through the use of the generalized Nyquist stability criterion [10] where avoidance of the critical point 1+*j*0 at the *s* complex plane is at the centre of interest from the point of view of absolute stability assessment. This criterion establishes that the number of unstable closed-loop poles of the former control configuration is equal to the sum of the number of times the Nyquist plot Γ of the open loop transfer matrix $G(j\omega) \cdot N^1(a,\omega)$ encircles the crtical point in a clockwise direction plus the number of unstable open-loop poles.

III. FREQUENCY DOMAIN APPROACH TO ROBUST STABILITY ANALYSIS

Robust stability means that the system is stable not only for the nominal plant $G(s)$ and SIDF approximate controller, but also for a family of models containing all the possible models of the plant under uncertainty, considering also the uncertainty due to the higher harmonics effects in the controller dynamics. Therefore, the robustness of the closed loop system is guaranteed taken into account both sources of uncertainty.

The drawback of the previous MIMO harmonic balance method for stability analysis is that, in spite of its graphical character, it does not supply robustness measurements. These measurements can be achieved by means of a different method based on the structured singular values.

A convenient way to formulate the problem is in terms of the M-Δ standard configuration comprising the total uncertainty Δ and the sensitivity transfer function matrix M as is depicted in Fig. 2. For the case of input additive uncertainties the M matrix is given by the control sensitivity matrix while in case of multiplicative uncertainty M stands for the complementary sensitivity matrix.
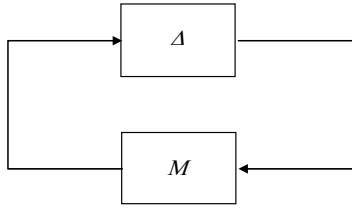
Fig. 2 Robustness analysis with the Δ-M configuration

The open loop system comprises a linear nominal model transfer function matrix $G(s)$ with additive uncertainty plant matrix $\Delta_G(s)$ and a nonlinear SIDF matrix $N^1(a,\omega)$ with high order neglected harmonics $\Delta_N(a,\omega)$ as controller uncertainty (Fig. 3).

The nominal model $G(s)$ has been determined by linearization of a nonlinear plant model at a nominal operation point, while we have used the theory of estimation in the form of unfalsified linear uncertainty models [11] for the plant uncertainty determination.

On the other hand, the controller uncertainty $\Delta_N(a,\omega)$ is given by the summation of the rest of harmonics

$$\Delta_N(a,\omega) = \left\{ \sum_{j=2}^{\infty} N_{kl}^j(a,\omega) \right\} \qquad (6)$$
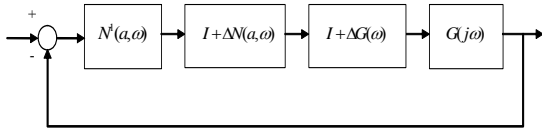

Fig. 3 Neural control closed loop with multiplicative plant and controller uncertainties

Transforming the additive uncertainties to input multiplicative ones, and considering both $G(s)$ and $N^1(a,\omega)$ square matrices for inversion (this constraint will be later relaxed), then the real open loop (Fig. 3) is given by

$$L_R(a,\omega) = G(j\omega)(I + \Delta G(\omega)) \cdot$$
$$(I + \Delta N(a,\omega))N^1(a,\omega) \qquad (7)$$

where the matrices $\Delta G(\omega) = inv(G(j\omega))\Delta_G(\omega)$ and $\Delta N(\omega) = \Delta_N(\omega))inv(N^1(\omega))$ define the input multiplicative uncertainties. In this way, the feedback system of Fig. 3 should be robust against the controller uncertainty introduced by the SIDF approximation in addition to the robustness requirement with respect to the uncertainty in the plant.

IV. ROBUST ANALYSIS OF LIMIT CYCLES

Considering the open loop nominal transfer matrix function $L(a,\omega) = G(j\omega) \cdot N^1(a,\omega)$, the singular values of $L(a,\omega)$

are given by

$$\sigma_i(L(a,\omega)) = \sqrt{\lambda_i(L^*(a,\omega)L(a,\omega))} \qquad (7)$$

being $\overline{\sigma}(L)$ and $\underline{\sigma}(L)$ the maximum and minimum singular values respectively, and λ_i the ith eigenvalue.

It can be easily demonstrated that the α-norm $\|L\| = \sup_a(\sup_\omega \overline{\sigma}(L(a,\omega)))$ fulfills the norm properties [12] and can be taken as the MIMO gain of $L(a,\omega)$.

The small-gain theorem [7] establishes that if both the nominal system $M(s)$ and the uncertainty system $\Delta(s)$ of Fig. 2 are stable and have bounded gains then, the closed loop is stable if $\|\Delta(\omega)\| \cdot \|M(j\omega)\| < 1 \quad \forall \omega$, that is, $\overline{\sigma}(\Delta(\omega)) \cdot \overline{\sigma}(M(j\omega)) < 1$.

Applying block reduction techniques, the closed loop structure of Fig. 3 can be matched to the M-Δ standard configuration with total uncertainty $\Delta(a,\omega)$ given by

$$\Delta(a,\omega) = \Delta N(a,\omega) + \Delta G(\omega) + \Delta G(\omega)\Delta N(a,\omega)) \qquad (8)$$

and the sensitivity transfer function matrix M corresponds to the complementary sensitivity function matrix $T(a,\omega)$

$$T(a,\omega) = inv(I + N(a,\omega)G(\omega)) \cdot N(a,\omega)G(\omega) \qquad (9)$$

The small gain theorem then gives the sufficient condition for stability of the closed loop with respect to the input multiplicative uncertainty verifying that

$$\overline{\sigma}(\Delta(a,\omega)) < \frac{1}{\overline{\sigma}(T(a,\omega))} \qquad (10)$$

In order to find the maximum singular value of $\overline{\sigma}(\Delta(a,\omega))$ we apply the α-norm properties to eq. (8) yielding

$$\overline{\sigma}(\Delta) \le \overline{\sigma}(\Delta N) + \overline{\sigma}(\Delta G) + \overline{\sigma}(\Delta N)\overline{\sigma}(\Delta G) \qquad (11)$$

and substituting in (10) we derive an upper bound on the plant uncertainties for the robust stability and therefore absence of limit cycle oscillations when

$$\overline{\sigma}(\Delta N) + \overline{\sigma}(\Delta G) + \overline{\sigma}(\Delta N)\overline{\sigma}(\Delta G) < \frac{1}{\overline{\sigma}(T)} \qquad (12)$$

$$\overline{\sigma}(\Delta G) < \frac{\dfrac{1}{\overline{\sigma}(T)} - \overline{\sigma}(\Delta N)}{1 + \overline{\sigma}(\Delta N)} \qquad (13)$$

Then, the $inv(\overline{\sigma}(T)) - \overline{\sigma}(\Delta N))$ difference relative to $1 + \overline{\sigma}(\Delta N)$ can be considered as a measure of robustness.

V. APPLICATION

The proposed robust stability analysis was tested in a simulation study of neural based control of a nonlinear binary distillation column developed by [13], composed by the mass, component mass and enthalpy balance equations implemented in SIMULINK [14] and designed to separate a mixture of methanol and water, with 9 bubble caps trays provided with heated electrically reboiler and water refrigerated tubular condenser (Fig. 5).

The manipulated variables are the heat input to the reboiler Q and the reflux flowrate L, while distillate and bottom composition are the controlled variables.

The training set for the neural identification comprised 200 data points belonging to the open loop operating range for plant inputs reflux flowrate L (0-5E-06 m3/h) and heat flow Q (0-2000 J/s) for fixed feed rate conditions $F = 1$ E-06 m3/h, $X_F = 0.3$, and $q = 1$. An additional data set consisting of 150 data points was used to test the neural network model afterwards. For training pattern generation we assume an initial steady state for the column after a start-up process. We obtained an optimum 2-10-2 network SIMULINK neural identification block.
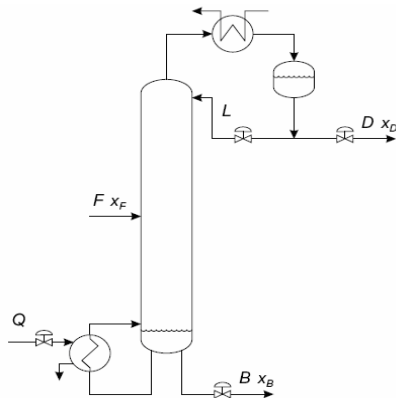

Fig. 5 Schematic of the distillation column

The training set for neural control comprised 150 data points as inputs and reflux rate and heat flow as outputs belonging to the closed loop operating range for desired and actual top and bottom compositions values x_D (0.0-1.0) and x_B (0.0-1.0). An additional data set consisting of 120 data points was used to test the neural network controller also. The control task was made using the neural identification network previously trained, obtaining an optimum 2-12-2 network SIMULINK block for the neural controller, with top and bottom composition errors (Fig. 6).

To deal with this problem the singular value approach has been used to derive an upper bound on the plant uncertainties for the robust stability and therefore absence of limit cycle oscillations for the real closed loop system.

The model uncertainty has been identified through the application of the l1-identification toolbox [15] using the
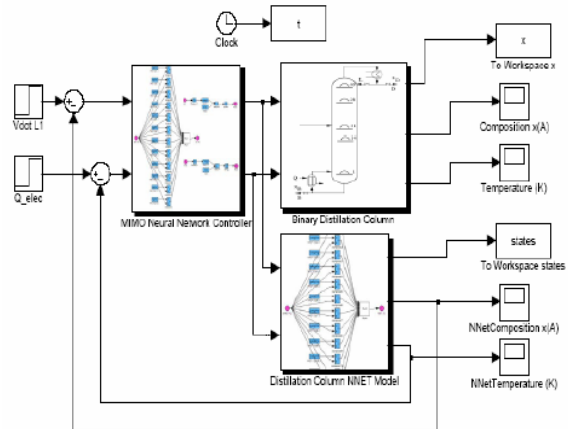

Fig. 6 SIMULINK structure for neural distillation control

discrete version of the linearized plant model as nominal model, obtaining the maximum and minimum values for the coefficients of the discrete uncertainty model whose continuous version defines $\Delta_G(s)$. A maximum bound of $\overline{\sigma}(\Delta G(\omega)) < 0.15$ is obtained.

The $inv(\overline{\sigma}(M))$ for controlling sensitivity function has been calculated with the $\overline{\sigma}(\Delta N)$ controller uncertainty, and an upper bound for plant uncertainties for the absence of limit cycle oscillations is calculated by applying (13). It can be shown that a minimum robustness margin of 0.22 is obtained, therefore the closed loop is stable since $\overline{\sigma}(\Delta G) < 0.22$ condition is fulfilled.

The response of the neural controller for pulse changes in both distillate and bottom composition for original system (Fig. 7-a) shows adequate tracking performance, while induced changes in plant parameters (change in 20% tray efficiencies) out of the enabled uncertainty range causes limit cycle behavior or even unstability (Fig. 7-b).

VI. CONCLUSIONS AND FUTURE WORK

We have analyzed the robustness of neural based multivariable control systems with uncertainty, provided that both the neural controller and plant are represented by its SIDF approximation and linearized model respectively. For this purpose we make use of the generalized Nyquist stability criterion applied to analyze nominal stability. The plant uncertainty model is derived using an identification procedure applied to a binary distillation column under feedback neurocontrol, while controller higher harmonics stands for the controller uncertainty, both treated as additive input perturbations. The approach here developed gives alternatives to computing stability margins for the real closed loop when multivariable uncertainty is defined by its singular value decomposition.

Future works are directed toward the development of a design method for training the neural controller in order to satisfy not only performance requirements but robustness conditions.
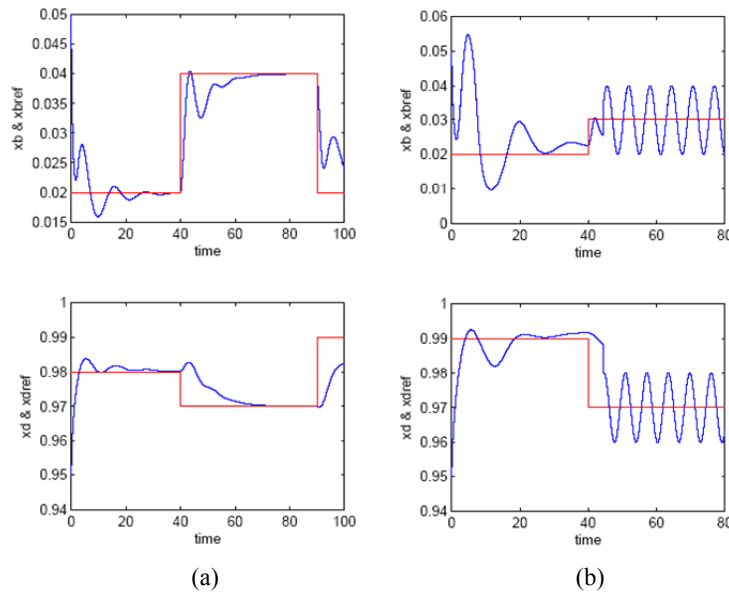
Fig. 7 Response for pulse change in set-point distillate and bottom composition for original system (a) and perturbed system (b)

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