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Analysis Fraction Flow of Water versus Cumulative Oil Recoveries Using Buckley Leverett Method

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***Abstract*—**To derive the fractional flow equation oil displacement will be assumed to take place under the so-called diffusive flow condition. The constraints are that fluid saturations at any point in the linear displacement path are uniformly distributed with respect to thickness; this allows the displacement to be described mathematically in one dimension. The simultaneous flow of oil and water can be modeled using thickness averaged relative permeability, along the centerline of the reservoir. The condition for fluid potential equilibrium is simply that of hydrostatic equilibrium for which the saturation distribution can be determined as a function of capillary pressure and therefore, height. That is the fluids are distributed in accordance with capillary-gravity equilibrium.

This paper focused on the fraction flow of water versus cumulative oil recoveries using Buckley Leverett method. Several field cases have been developed to aid in analysis. Producing water-cut (at surface conditions) will be compared with the cumulative oil recovery at breakthrough for the flowing fluid.

***Keywords*—**Fractional Flow, Fluid Saturations, Permeability, Cumulative Oil Recoveries, Buckley Leverett Method.

I. Introduction

CONSIDERING how important it is for reservoir engineer to properly estimate water flooding project parameters such as: the volume of oil displaced at any time, the rate of oil production, and the volume of water that must be handled per volume of oil once water production begins; models and aids predicting displacement performance have been presented in literature [1]-[6]. Some of the prediction tools used by engineers consist of graphical aids.

Graphical methods are still useful to petroleum engineers because in some cases it is possible to obtain comparable accuracies in a shorter time as compared to computer methods. On the other hand, due to the increased knowledge in fluid mechanics through porous media, advent of high-speed computer and better simulation software, it is now possible to use fewer and better assumptions to adjust models to real word scenarios. For many years, consultants, professors, students and engineers have used the intersection of the tangent to the fractional-flow water saturation curve, f_w, vs. S_w, to calculate the average water saturation after breakthrough. In recent years, new technology has been used to expand the scale of this f_w, vs. S_w, plot after breakthrough, for better visualization and achievement of more accurate results. From field technique it is clear that the technique is subject to errors, since it is difficult to determine the exact point at which the tangent to the fractional-flow curve intersects the curve [7]-[9].

II. Fractional Flow Theory

To derive the fractional flow equation oil displacement will be assumed to take place under the so-called diffusive flow condition [1], [4]. The constraints are that fluid saturations at any point in the linear displacement path are uniformly distributed with respect to thickness; this allows the displacement to be described mathematically in one dimension. The simultaneous flow of oil and water can be modeled using thickness averaged relative permeability, along the centerline of the reservoir. The condition for fluid potential equilibrium is simply that of hydrostatic equilibrium for which the saturation distribution can be determined as a function of capillary pressure and therefore, height. That is the fluids are distributed in accordance with capillary-gravity equilibrium [7].

The condition of vertical equilibrium will be favored by:

1. A large vertical permeability K_V
2. Small reservoir thickness
3. Large density difference between the fluids
4. High capillary forces meaning large capillary transition zone (H)
5. Low fluid viscosities low injection rates
6. Low injection rates

The diffuse flow condition occurs when:

1. The displacement occurs at very high injection rates so that the effects of capillary and gravity forces are negligible. The vertical equilibrium condition is not satisfied.
2. The displacement is at low injection rates in reservoirs for which the measured capillary transition zone greatly exceeds the reservoir thickness and the vertical equilibrium condition applies.

The second condition visualized by observing Fig. 1. Since the transition zone (H) is much larger than the reservoir thickness, the water saturation can be considered uniformly distributed with respect to the reservoir thickness. Fig. 2 represents a small transition curve [8], [10].

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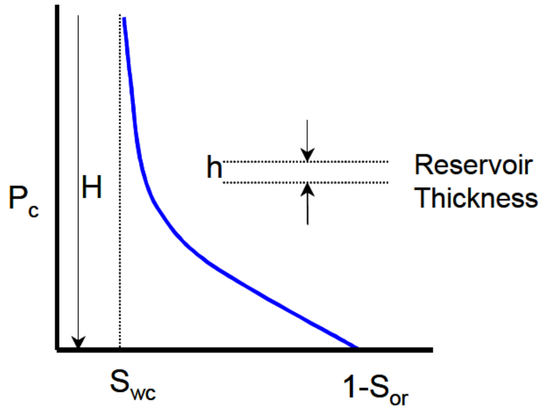

Fig. 1 Approximation to the diffuse flow condition for H >>h

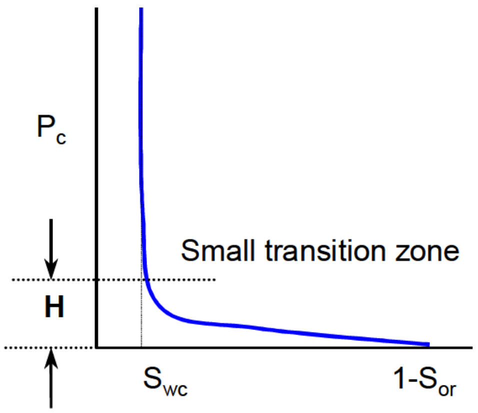

Fig. 2 An example of a capillary curve with a small transition zone

Fig. 3 shows schematics of the top view of a linear reservoir which has uniform cross sectional area A. Displacement will be considered in this prototype reservoir model which can be tilted as indicated in Fig. 4. Both injection and production wells are considered to be perforated across the entire, formation thickness, in the dip-normal direction [1].

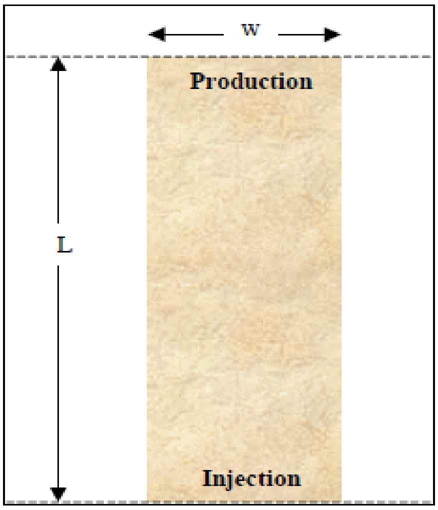

Fig. 3 Linear prototype reservoir model 1-D

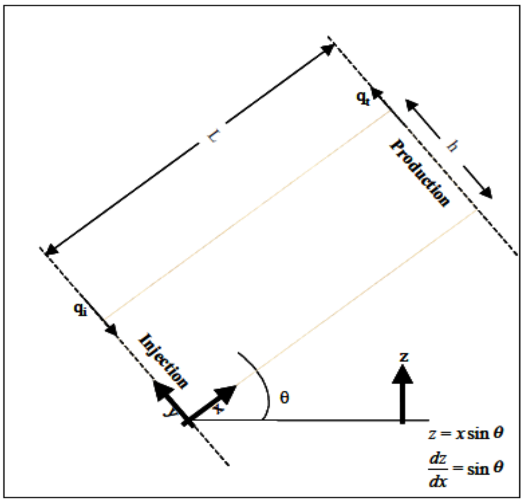

Fig. 4 Linear prototype reservoir mode 1-D displacement. Cross section

The objective in this study is to describe the fluid saturation distributions in the y-direction as the fluid moves through the x-direction.

Considering oil displacement in the tilted reservoir of Fig. 4, Darcy's equation, is applied for linear flow. The 1-D equations for the simultaneous flow of oil and water are:

$$q_o = \frac{-KK_{ro}A\rho_o}{\mu_o}\left(\frac{\partial\varphi_o}{\partial x}\right) \quad (1)$$

$$q_w = \frac{-KK_{rw}A\rho_w}{\mu_w}\left(\frac{\partial\varphi_w}{\partial x}\right) \quad (2)$$

where $\varphi$the potential is defined as

$$\varphi = \frac{P}{\rho} + \text{gz} = \frac{P}{\rho} + \text{gxsin}\,\theta \quad (3)$$

Thus, the flow rate for oil is

$$q_o = \frac{-KK_{ro}}{\mu_o}\left\{\frac{\partial P_o}{\partial x} + \frac{\rho_o g \sin\theta}{1.0133\times10^6}\right\} \quad (4)$$

and the flow rate of water is

$$q_w = \frac{-KK_{rw}}{\mu_w}\left\{\frac{\partial P_w}{\partial x} + \frac{\rho_w g \sin\theta}{1.0133\times10^6}\right\} \quad (5)$$

Subtract Equation (4) from Equation (5) and recall

$$P_c = P_o - P_w \rightarrow capillarypressure$$

$$q_t = q_o + q_w \rightarrow totalflowrate$$

$$-\frac{q_o\mu_o}{KK_{ro}A}+\frac{q_w\mu_w}{KK_{rw}A}=\frac{\partial P_c}{\partial x}+\frac{g(\rho_o-\rho_w)\sin\theta}{1.0133\times10^6} \qquad (6)$$

If in (Equation 6), the oil flow rate was substituted in terms of water and total flow rate, therefore [8],

$$q_w\times\left\{\frac{\mu_w}{KK_{rw}}+\frac{\mu_o}{kK_{ro}}\right\}=\frac{q_t\mu_o}{KK_{ro}}+A\left\{\frac{\partial P_c}{\partial x}-\frac{g\Delta\rho\sin\theta}{1.0133\times10^6}\right\} (7)$$

Define fractional water flow f_w in the reservoir as

$$f_w=\frac{q_w}{q_w+q_o}=\frac{q_w}{q_t} \qquad (8)$$

Substitute Equation (8) in Equation (7),

$$f_w=\frac{1+\frac{KK_{ro}A}{q_t\mu_o}\left(\frac{\partial P_c}{\partial x}-\frac{\Delta\rho g\sin\theta}{1.0133\times10^6}\right)}{1+\frac{\mu_w K_{ro}}{\mu_o K_{rw}}} \qquad (9)$$

Or in field units

$$f_w=\frac{1+1.127\times10^{-3}\frac{KK_{ro}A}{q_t\mu_o}\left(\frac{\partial P_c}{\partial x}-0.4335\,\Delta\gamma\sin\theta\right)}{1+\frac{\mu_w K_{ro}}{\mu_o K_{rw}}} \qquad (10)$$

For horizontal flow and neglecting the capillary pressure gradient we have:

$$f_{wh}=\frac{1}{1+\frac{K_{ro}\mu_o}{K_{rw}\mu_w}} \qquad (11)$$

Then expressed Equation (10) as

$$f_w = f_{wh}\left\{1+\frac{K_o 1.127\times10^{-3}A}{\mu_o q_t}\frac{\partial P_c}{\partial x}-\frac{4.8855\times10^{-4}A\Delta\gamma\sin\theta}{\mu_o q_t}\right\} (12)$$

Or

$$f_w=f_{wh}\{1+N_c+N_g\} \qquad (13)$$

N_c= Capillary number, dimensionless
N_g= Gravity number, dimensionless

III. Application of Fractional Flow Theory in Oil Recovery Calculations

There are different methods for calculating the oil recovery depending on the type of reservoir, either homogeneous or layered [2, 3].

A. Homogeneous Reservoirs - Buckley-Leverett Method

Before water breakthrough it is easily obtained the saturation profiles and the oil recovery is equal to the water injected (a trivial result). But there is a need to evaluate the oil recovery after breakthrough as well.

After breakthrough at producing well X_2=L

$W_{id}=\frac{W_i}{LA\emptyset}$= dimensionless number of pore volumes of injected water

1PV= $LA\emptyset$

Fig. 5 shows water saturation distributions at two different times; one is at breakthrough and the other at a later time in a linear waterflood [1].

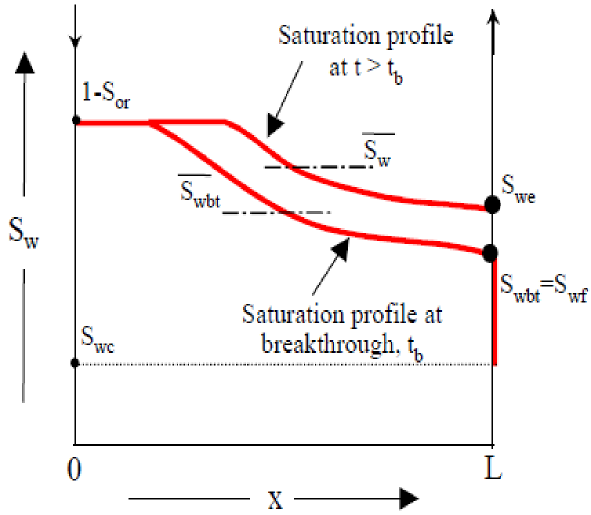

Fig. 5 Water saturation distributions at breakthrough and subsequently, in a linear water flood

At breakthrough, S_{wbt} = water saturation at breakthrough = S_{wf} front reaches production well. And the reservoir water production increases suddenly from zero to f_{wbt}. This confirms existence of shock [8], [10].

$$q_{id}=\frac{q_i}{LA\emptyset} \qquad (14)$$

Dimensionless oil production at breakthrough

$$N_{pd_{bt}}=W_{id_{bt}}=q_{id}.t_{bt}=(\bar{S}_{wbt}-S_{wc})=\frac{1}{\left(\frac{df_w}{dS_w}\right)\Big|_{S_{wbt}}} \qquad (15)$$

After breakthrough, both oil and water will be produced

$$\frac{W_i}{LA\emptyset}=\frac{1}{\left(\frac{df_w}{dS_w}\right)\Big|_{S_{we}}}=W_{id} \qquad (16)$$

At this stage to evaluate oil recoveries

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$$\bar{S}_w = S_{we} + (1 - f_{we}) \frac{1}{\left(\frac{df_w}{dS_w}\right)\Big|_{S_{we}}} \quad (17)$$

Or

$$\bar{S}_w = S_{we} + (1 - f_{we}).W_{id} \quad (18)$$

Subtract S_{wc} from both sides of the equation

$$N_{pd} = \bar{S}_w - S_{wc} = S_{we} - S_{wc} + (1 - f_{we})W_{id} \quad (19)$$

IV. Results and Discussions

Oil is being displaced by water in a horizontal, direct line drive under the diffuse flow condition. The rock relative permeability functions for water and oil are listed in Table I.

TABLE I
RELATIVE PERMEABILITY SATURATION DATA FOR BUCKLEY- LEVERET METHOD (FROM DAKE 1988)

S_w	K_{rw}	K_{ro}
0.20	0	0.800
0.25	0.002	0.610
0.30	0.009	0.470
0.35	0.020	0.370
0.40	0.033	0.285
0.45	0.051	0.220
0.50	0.075	0.163
0.55	0.100	0.120
0.60	0.132	0.081
0.65	0.170	0.050
0.70	0.208	0.027
0.75	0.251	0.010
0.80	0.300	0

TABLE II
CASES TO ANALYZE THE DIFFERENT FRACTIONAL FLOW RESULTS FOR BUCKLEY- LEVERETT METHOD

Case	Oil viscosity	Water viscosity
1	**50 cp**	**0.5 cp**
2	**5 cp**	**0.5 cp**
3	**0.4 cp**	**1.0 cp**

Pressure is being maintained at its initial value for which,

$B_o = 1.3\ {}^{rb}/_{stb}$ and B_w=1.0 ${}^{rb}/_{stb}$

The producing water-cut (at surface conditions) was compared with the cumulative oil recovery at breakthrough. Assume that the relative permeability and PVT data are relevant for all three cases.

For horizontal flow the fractional flow in the reservoir is:

$$f_w = \frac{1}{1 + \frac{\mu_w}{K_{rw}} \cdot \frac{K_{ro}}{\mu_o}} \quad (20)$$

while the producing water-cut at the surface, f_{ws} , is:

$$f_{ws} = \frac{q_w/B_w}{q_w/B_w + q_o/B_o} \quad (21)$$

Combining the above two Equations leads to an expression for the surface water-cut as:

$$f_{ws} = \frac{1}{1 + \frac{B_w}{B_o}\left(\frac{1}{f_w} - 1\right)} \quad (22)$$

The fractional flow in the reservoir for the three cases can be calculated as follows:

Case 1 is $\frac{\mu_w}{\mu_o} = 0.01$

Case 2 is $\frac{\mu_w}{\mu_o} = 0.1$

Case 3 is $\frac{\mu_w}{\mu_o} = 2.5$

TABLE III
EVALUATION OF FRACTIONAL FLOW EQUATION FOR CASES 1 TO 3

S_w	K_{rw}	K_{ro}	$\frac{K_{ro}}{K_{rw}}$	Fractional Flow (f_w) Case 1	Case 2	Case 3
0.20	**0**	**0.800**	**∞**	**0**	**0**	**0**
0.25	**0.002**	**0.610**	**305.000**	**0.247**	**0.032**	**0.001**
0.30	**0.009**	**0.470**	**52.222**	**0.657**	**0.161**	**0.008**
0.35	**0.020**	**0.370**	**18.500**	**0.844**	**0.354**	**0.021**
0.40	**0.033**	**0.285**	**8.636**	**0.921**	**0.537**	**0.044**
0.45	**0.051**	**0.220**	**4.314**	**0.959**	**0.699**	**0.085**
0.50	**0.075**	**0.163**	**2.173**	**0.979**	**0.821**	**0.155**
0.55	**0.100**	**0.120**	**1.200**	**0.988**	**0.893**	**0.250**
0.60	**0.132**	**0.081**	**0.614**	**0.994**	**0.942**	**0.394**
0.65	**0.170**	**0.050**	**0.294**	**0.997**	**0.971**	**0.576**
0.70	**0.208**	**0.027**	**0.130**	**0.999**	**0.987**	**0.755**
0.75	**0.251**	**0.010**	**0.040**	**0.999**	**0.996**	**0.909**
0.80	**0.300**	**0**	**0**	**1.000**	**1.000**	**1.000**

Fractional flow plots for the three cases are shown in Fig. 6 and the results obtained by applying Welge's graphical technique, at breakthrough, are listed below:

TABLE IV
OIL RECOVERIES AND SATURATION AT BREAKTHROUGH FOR BUCKLEY- LEVERETT METHOD

CASE	$S_{w_{bt}}$	$f_{w_{bt}}$ (RESERVOIR)	$f_{ws_{bt}}$ (SURFACE)	$\bar{S}_{w_{bt}}$	$N_{pd_{bt}}$ (PV)
1	0.28	0.55	0.61	0.34	0.14
2	0.45	0.70	0.75	0.55	0.35
3	0.80	1.00	1.00	0.80	0.60

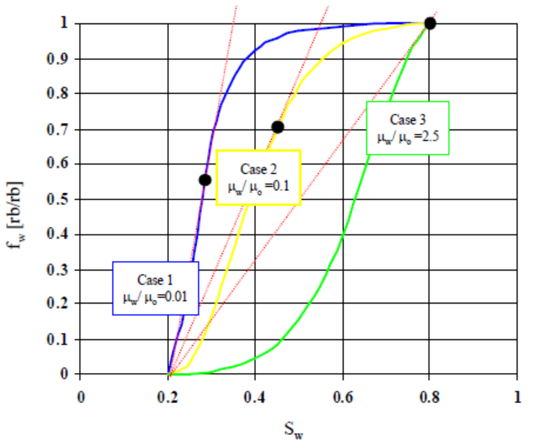

Fig. 6 Fractional flow plots for different oil-water viscosity ratios

An important parameter in determining the effectiveness of a waterflood is the end point mobility ratio defined as:

$$M = \frac{K_{rw}/\mu_w}{K_{ro}/\mu_o} \tag{23}$$

And, for horizontal flow, stable, piston-like displacement will occur for M≤ 1. An even more significant parameter for characterizing the stability of Buckley-Leverett displacement is the shock front mobility ratio, M_s , defined as:

$$M_s = \frac{K_{ro}(S_{wf})/\mu_o + K_{rw}(S_{wf})/\mu_w}{K_{ro}/\mu_o} \tag{24}$$

In which the relative permeabilities in the numerator are evaluated for the shock front water saturation, S_{wf}. Hagoort has shown, using a theoretical argument backed by experiment, that Buckley-Leverett displacement could be regarded as stable for the less restrictive condition that $M_s < 1$. If this condition is not satisfied, there will be severe viscous channeling of water through the oil and breakthrough will occur even earlier than predicted using the Welge technique. Values of M and M_s for the three cases defined in Table V.

TABLE V
VALUES OF THE SHOCK FRONT AND END POINT RELATIVE PERMEABILITIES CALCULATED USING THE DATA (FRACTIONAL FLOW)

CASE NO.	$\frac{\mu_o}{\mu_w}$	S_{wf}	$K_{rw}(S_{wf})$	$K_{ro}(S_{wf})$	M_s	M
1	100	0.28	0.006	0.520	1.40	37.40
2	10	0.45	0.051	0.220	0.91	3.75
3	0.4	0.80	0.300	0	0.15	0.15

Using these data the results of the study can be analyzed as follows:

Case 1 - this displacement is unstable due to the very high value of the oil/water viscosity ratio. This results in the by-passing of oil and consequently the premature breakthrough of water. The oil recovery at breakthrough is very small and many pore volumes of water will have to be injected to recover all the movable oil. Under these circumstances oil recovery by water injection is hardly feasible and consideration should be given to the application of thermal recovery methods with the aim of reducing the viscosity ratio.

Case 2 - the oil/water viscosity ratio is an order of magnitude lower than in Case 1, which leads to a stable and much more favorable type of displacement $(M_s < 1)$

Case 3 - for the displacement of this very low viscosity oil $(\mu_o = 0.4cp)$both the end point and shock front mobility ratios are less than unity and piston like displacement occurs The tangent to the fractional flow curve, from$S_w = S_{wc}$, $f_w = 0$ meets the curve at the point$S_{w_{bt}} = 1 - S_{or}$, $f_{w_{bt}}$=1 and therefore$S_{w_{bt}} = \bar{\bar{S}}_{w_{bt}} = S_{or}$. The total oil recovery at breakthrough is$\bar{\bar{S}}_{w_{bt}} - S_{wc} = 1 - S_{or} - S_{wc}$, which the total movable oil volume is.

V. Conclusion

Following conclusions can be drawn from this work: The values of the producing water- cut (at surface conditions) and the cumulative oil recovery at breakthrough using Buckley-Leverett method are compared. Buckley- Leverett method shows that Buckley- Leverett displacement could be regarded as stable for the less restrictive condition that $M_s < 1$. If this condition is not satisfied, there will be severe viscous channeling of water through the oil and breakthrough. When the mobility ratio is unfavorable, the Buckley- Leverett method is no longer applied. The relative permeability and fractional flow profiles are controlled by rock type, and it's appropriate description helps engineers to make confident predictions from waterflooding project calculation. The new technique, based on this general function, allows the determination of average water saturation after breakthrough and provides more accurate estimation of waterflood performance parameters.

Nomenclature

A	: Cross- sectional area available for flow, sq ft $[m^2]$
$A\emptyset dx$	: Pore volume
df_w/ds_w	: Slope of the curve of fractional flow
f_w	: Fractional flow
f_{wbt}	: Producing water cut at producing well breakthrough, fraction
$\emptyset$	: Porosity, fraction
K_{ro}	: Relative Permeability to oil phase
K_{rw}	: Relative Permeability to water phase
μ_w	: Water Viscosity, cp

μ_o : Oil Viscosity, cp
Q_i : Pore volume of cumulative injected fluid
S_{wi} : Initial water saturation, fraction
S_w : Water saturation, fraction
S_{wbt} : Water saturation at breakthrough, fraction
W_i : Cumulative injected water volume, bbl $[m^3]$
B_o : Oil formation volume factor
B_w : Water formation volume factor
M : Mobility ratio
N_p : Cumulative barrels of oil produced
PV : Pore volume
rb/STB : Reservoir barrels per stock tank barrel
STB : Stock tank barrels of oil
S_o : Oil saturation
$\overline{S_w}$: Average water saturation
$\overline{S_{wbt}}$: Average water saturation behind the waterflood front
S_{wc} : Connate water saturation
S_{wi} : Initial water saturation

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