Thermosolutal MHD Mixed Marangoni Convective Boundary Layers in the Presence of Suction or Injection

Noraini Ahmad, Seripah Awang Kechil and Norma Mohd Basir

Abstract—The steady coupled dissipative layers, called Marangoni mixed convection boundary layers, in the presence of a magnetic field and solute concentration that are formed along the surface of two immiscible fluids with uniform suction or injection effects is examined. The similarity boundary layer equations are solved numerically using the Runge-Kutta Fehlberg with shooting technique. The Marangoni, buoyancy and external pressure gradient effects that are generated in mixed convection boundary layer flow are assessed. The velocity, temperature and concentration boundary layers thickness decrease with the increase of the magnetic field strength and the injection to suction. For buoyancy-opposed flow, the Marangoni mixed convection parameter enhances the velocity boundary layer but decreases the temperature and concentration boundary layers. However, for the buoyancy-assisted flow, the Marangoni mixed convection parameter decelerates the velocity but increases the temperature and concentration boundary layers.

Keywords—Magnetic field, mixed Marangoni convection, similarity boundary layers, solute concentration.

I. INTRODUCTION

THE temperature and surfactant concentration gradients at the interface of two fluid layers give rise to surface tension variations that can induce interfacial flows from region of low surface tension to region of high surface tension. The surface-tension-driven convection or also known as Marangoni convection, is of central importance in industrial, biomedical and daily life applications such as coating flow technology, microfluidics, surfactant replacement therapy for neonatal infants, film drainage in emulsions and foams and drying of semi-conductor wafers in microelectronics [1], [2]. Because of the wide applications, investigations on the effects of Marangoni convection with the presence of the several physical parameters were carried out [3]-[14]. The rectangular double-crucible system was developed to measure the surface velocities of thermal and solutal Marangoni convection in In-Ga-Sb melt [1]. Christopher and Wang [2] examined the effect of Prandtl number to see the relative thickness of momentum and thermal boundary layers. The development of the momentum, thermal and concentration of Marangoni mixed convection boundary

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layers have been considered by Pop et al. [3], Al-Mudhaf and Chamkha [4] and Magyari and Chamkha [5], [6]. Magyari and Chamkha [5], [6] found the exact analytical solutions for the MHD thermosolutal Marangoni convection and thermosolutal Marangoni convection in the presence of heat and mass generation or consumption. It is found that thermosolutal surface tension ratio increases the wall velocity and mass flow rate. Recently, Zueco and Bég [11] studied the influence of applied the magnetic field and free and forced Marangoni convection on the momentum and thermal boundary layers thickness.

The paper aims to extend the work of Zueco and Bég [11] of Marangoni mixed convection boundary layers by including the influence of mass transfer in the presence of suction or injection effects on the thermocapillary and thermosolutal boundary layers. The numerical method of Runge-Kutta Fehlberg with shooting technique is employed to assess the effects of the physical parameters on the momentum, thermal and concentration boundary layers.

II. FORMULATION OF THE PROBLEM

Consider the steady laminar boundary layer flows in electrically-conducting Newtonian fluids with buoyancy effects due to gravity and an external pressure gradient in the presence of a transverse magnetic field. The magnetic Reynolds number is assumed to be small enough to neglect magnetic induction effects. The surface tension, σ at the interface, S between the two fluids is assumed to depend linearly on the temperature and concentration gradients. The interface is assumed to be permeable so as to allow for possible suction or injection. Additionally, we assume the gravity vector, g is aligned with the fluid interface, S and the flow field for two interfacing fluids are uncoupled [11] (see also Fig. 1),
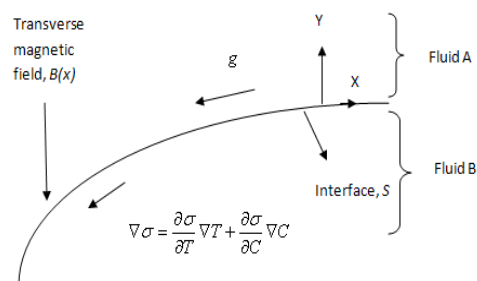

Fig. 1 Flow model in coordinate system with interface condition

The governing equations based on the balance laws of mass, linear momentum, energy and concentration species are,

Mass conservation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum conservation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2} - \tau g\beta(T - T_m)$$
$$- \gamma \frac{B(x)}{\rho}(u - u_e) \tag{2}$$

Energy conservation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Concentration conservation

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where u and v are the velocity components along x and y axes, $B(X)/\rho$ is a density-scaled magnetic field function, $u_e(x)$ is the external velocity, T is the fluid temperature, C is the solutal concentration, T_m is the reference temperature, g is gravitational acceleration, β is the coefficient of thermal expansion, D is the mass diffusivity, v is the kinematic viscosity, α is thermal diffusivity and γ is electrical conductivity of the fluid. The parameter τ specifies the nature of buoyancy forces where $\tau = -1$ is for buoyancy-assisted Marangoni flow and $\tau = +1$ is for buoyancy-opposed Marangoni flow.

The boundary conditions at the interface and free stream are

$$v = -v_0, \quad T = T_s(x), \quad C = C_s(x), \quad y = 0 \tag{5}$$

$$u \to u_e(x), \quad T \to T_m, \quad C \to C_m, \quad y \to \infty \tag{6}$$

and the interface condition is

$$\mu\frac{\partial u}{\partial y} = \sigma_T \frac{\partial T}{\partial x} + \sigma_C \frac{\partial C}{\partial x} \tag{7}$$

where the surface tension σ is defined by the linear expression,

$$\sigma = \sigma_m - \sigma_T(T - T_m) - \sigma_C(C - C_m), \tag{8}$$

where μ is the dynamic viscosity, σ_m and C_m represent the surface tension and concentration at the reference temperature, T_m, respectively, with both quantities having constant values

and $\sigma_T = -\dfrac{\partial \sigma}{\partial T}$ and $\sigma_C = -\dfrac{\partial \sigma}{\partial C}$ are rates of change of surface tension with temperature and solute concentration, respectively.

Following [11] and [12], we introduce the following nondimensional variables,

$$X = \frac{(x - L_0)}{L}, \quad Y = \frac{\text{Re}^{1/3} y}{L}, \quad U = \frac{u}{u_r}, \quad V = \frac{\text{Re}^{1/3} v}{u_r}$$

$$U_e(X) = \frac{u_e(x)}{u_r}, \quad u_m = \frac{\sigma_T \Delta_T}{\mu}, \quad t = \frac{(T - T_m)}{\Delta_T} \tag{9}$$

$$c = \frac{(C - C_m)}{\Delta_C}, \quad M = \frac{\sigma B_0^2 L}{\rho u_r}$$

where u_m is the Marangoni velocity, L is a scale length, t is dimensionless temperature, c is dimensionless solute concentration, U is nondimensional velocity, B_0 is applied magnetic field strength, $\text{Re} = u_m L/v$ is the Reynolds number based on Marangoni velocity, L_0 defines the location of the origin of the curvilinear axis X, Y designates the extension of the interface S and Δ_T and Δ_C are positive increments of temperature and solute concentration linked to the temperature and solute concentration gradient imposed on the interface, respectively.

Substituting the variables (9) into (1)-(7) yield,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = U_e \frac{dU_e}{dX} + \frac{\partial^2 U}{\partial Y^2} - \tau\lambda t - M(U - U_e) \tag{11}$$

$$U\frac{\partial t}{\partial X} + V\frac{\partial t}{\partial Y} = \frac{1}{\text{Pr}}\left(\frac{\partial^2 t}{\partial Y^2}\right) \tag{12}$$

$$U\frac{\partial c}{\partial X} + V\frac{\partial c}{\partial Y} = \frac{1}{Sc}\left(\frac{\partial^2 c}{\partial Y^2}\right) \tag{13}$$

and the boundary conditions (5)–(7) become

$$\frac{\partial U}{\partial Y} = \frac{\partial t}{\partial X} + \varepsilon \frac{\partial c}{\partial X}, \quad V = -V_0, \quad t = T_s(X),$$
$$c = C_s(X), \text{ at } Y = 0$$
$$U \to U_e(X), \, t \to 0, \, c \to 0, \text{ at } Y \to \infty \tag{14}$$

where $\lambda = g\beta\Delta_T L/u_r^2$ is the Marangoni mixed convection parameter, $u_r = \text{Re}^{-1/3} u_m$ is reference velocity, M is the Hartmann magnetohydrodynamic parameter, $\text{Pr} = v/\alpha$ is the Prandtl number and $Sc = v/D$ is the Schmidt number. ε is the Marangoni parameter defined by

$$\varepsilon = \frac{Ma_C}{Ma_T} = \frac{\sigma_C \Delta C}{\sigma_T \Delta T} \qquad (15)$$

where, $Ma_C = \sigma_C \Delta CL / \mu\alpha$ and $Ma_T = \sigma_T \Delta TL / \mu\alpha$ are the solutal Marangoni number and the thermal Marangoni number, respectively.

Following [11], we use the similarity transformations,

$$U = U_0 X^m f'(\eta),$$
$$V = -U_0 \ell_0 X^{m-p-1}[(m-p)f(\eta) + p\eta f'(\eta)],$$
$$t = -t_0 X^n \theta(\eta), \ c = -c_0 X^n \phi(\eta), \ \eta = \frac{X^p Y}{\ell_0},$$
$$U_e(X) = U_0 X^m, \ B(X) = B_0^{m-p} X^{m-p} \qquad (16)$$

where m, n, p are fixed and primes indicate differentiation with respect to η. The similarity solutions exist for $m = 3$, $n = 5$ and $p = 1$. Therefore,

$$U = U_0 X^3 f'(\eta), \ V = -U_0 \ell_0 X[2f(\eta) + \eta f'(\eta)],$$
$$t = -t_0 X^5 \theta(\eta), \ c = -c_0 X^5 \phi(\eta), \ \eta = \frac{XY}{\ell_0},$$
$$U_e(X) = U_0 X^3, \ B(X) = B_0^2 X^2 \qquad (17)$$

where $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ represent the velocity, temperature and concentration profiles in the similarity plane, respectively, and η being the similarity variable. The constant scale factors U_0, t_0, c_0 and ℓ_0 are chosen in order to simplify the equations that satisfy the following conditions,

$$U_0 \ell_0^2 = \frac{1}{2}; \ \frac{t_0 \ell_0^2}{U_0} = 1; \ \frac{t_0 \ell_0}{U_0} = \frac{1}{5}; \ \frac{c_0}{t_0} = 1 \qquad (18)$$

which give $t_s(X) = -t_0 X^5$ and $c_s(X) = -c_0 X^5$ for the non-dimensional interface temperature and concentration distribution, where the minus sign is due to the orientation of the x-axis that is along the direction of decreasing temperature and concentration where Δ_T and σ_T are assumed positive.

Using the similarity transformation (17), (10)–(14) are transformed into the following ordinary differential equations,

$$f''' + ff'' + \frac{3}{2}(1 - f'^2) + \tau\lambda\theta + M(1 - f') = 0 \qquad (19)$$

$$\theta'' + \text{Pr}(f\theta' - \frac{5}{2}f'\theta) = 0 \qquad (20)$$

$$\phi'' + Sc(f\phi' - \frac{5}{2}f'\phi) = 0 \qquad (21)$$

and the boundary conditions (14) become

$$f(0) = f_w, \ f''(0) = -1 - \varepsilon, \ \theta(0) = 1, \ \phi(0) = 1$$
$$f' \to 1, \ \theta \to 0, \ \phi \to 0, \ \text{as} \ \eta \to \infty \qquad (22)$$

where f_w is the suction/injection parameter. $f_w > 0$ is the constant of suction parameter and $f_w < 0$ is the constant of injection parameter.

III. RESULTS AND DISCUSSION

The system of non-linear ordinary differential equations (19)–(21) subject to boundary conditions (22) is solved numerically using the Runge-Kutta Fehlberg with shooting technique. Zueco and Bég [11] solved for the momentum and energy equations problem using the numerical simulation method (NSM) through an electronic circuit simulator Pspice, while Chamkha et al. [12] used the implicit iterative tri-diagonal finite-difference method.

TABLE I
NUMERICAL VALUES OF $f'(0)$ AND $-\theta'(0)$ FOR BOUYANCY-OPPOSED MARANGONI FLOW, $\tau = +1$

	Chamkha et al. [12]		Zueco and Bég [11]		Present	
Pr	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$
0.13	1.2311	0.5371	1.232	0.536	1.2309	0.5371
0.25	1.1973	0.7627	1.197	0.763	1.1973	0.7633
0.50	1.1556	1.1064	1.156	1.106	1.1556	1.1065
0.74	1.1301	1.3636	1.131	1.364	1.1304	1.3638
1.00	1.1109	1.5996	1.112	1.600	1.1110	1.5996
1.50	1.0855	1.9799	1.086	1.980	1.0858	1.9799
2.00	1.0686	2.3002	1.067	2.301	1.0689	2.3002
3.53	1.0386	3.0844	1.038	3.085	1.0389	3.0844
5.00	1.0223	3.6860	1.022	3.688	1.0228	3.6860
8.00	1.0031	4.6805	1.003	4.680	1.0035	4.6808
10.1	0.9947	5.2664	0.995	5.267	0.9953	5.2680

TABLE II
NUMERICAL VALUES OF $f'(0)$ AND $-\theta'(0)$ FOR BOUYANCY-ASSISTED MARANGONI FLOW, $\tau = -1$

	Chamkha et al. [12]		Zueco and Bég [11]		Present	
Pr	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$
2.80	0.6993	2.0900	0.699	2.089	0.6994	2.0902
3.00	0.7122	2.2095	0.712	2.201	0.7122	2.2093
3.53	0.7349	2.4840	0.735	2.483	0.7349	2.4844
5.00	0.7689	3.1119	0.769	3.111	0.7689	3.1118
8.00	0.8012	4.1199	0.801	4.088	0.8015	4.1209
10.1	0.8137	4.7083	0.813	4.708	0.8137	4.7075

Table I and II show the numerical values of the wall velocity $f'(0)$ and heat transfer coefficient $-\theta'(0)$ for various values of Prandtl number when the Marangoni mixed convection parameter $\lambda = 1$, magnetic parameter $M = 0$ and the pressure gradient is absent $(-dp/dx = u_e(x)du_e/dx = 0)$ for buoyancy-opposed Marangoni flow $(\tau = +1)$ and buoyancy-assisted Marangoni flow $(\tau = -1)$. It can be

observed that the numerical values of the Runge-Kutta Fehlberg with shooting technique are in good agreemnent with the results of Zueco and Bég [11] and Chamkha et al. [12].
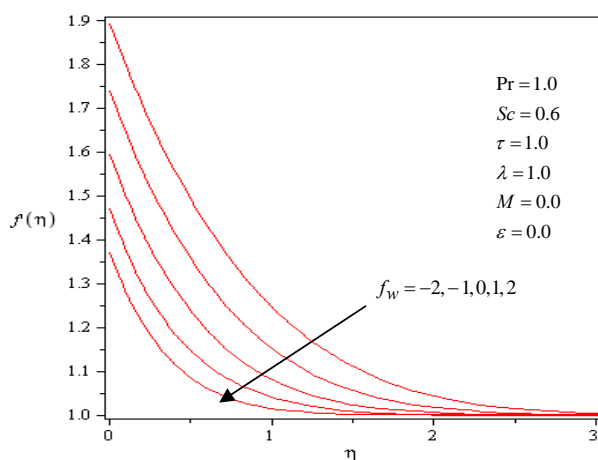

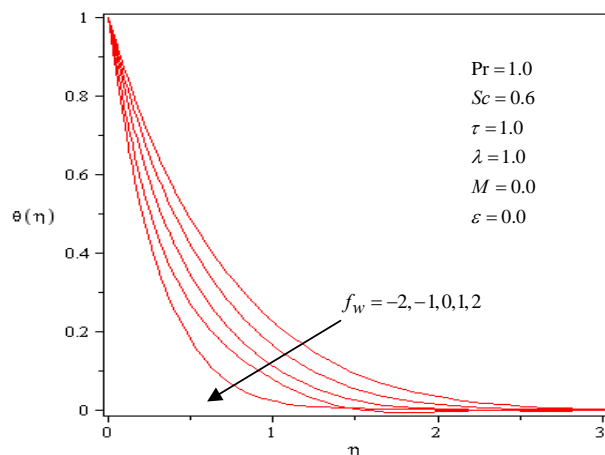Fig.2 Effects of suction or injection on velocity for buoyancy-opposed Marangoni flow.

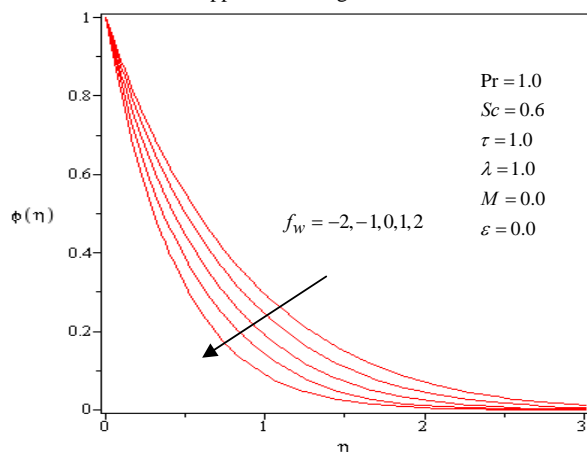Fig. 3 Effects of suction or injection on temperature for buoyancy-opposed Marangoni flow.

Fig. 4 Effects of suction or injection on concentration for buoyancy-opposed Marangoni flow.

The graphical results for the profiles of the momentum, thermal and concentration boundary layers are presented in Figs. 2–13 to show the effects of the physical parameters f_w, λ and M on the boundary layer thickness. Figs. 2–4 present the characteristics of the velocity, temperature and concentration profiles, respectively, for buoyancy-opposed Marangoni flow and various values of the suction/injection paramater, f_w. It can be observed that as f_w changes from injection to suction, the velocity, temperature, and concentration distributions in the flow field decrease. Therefore, the momentum, thermal and concentration boundary layer thickness decrease.
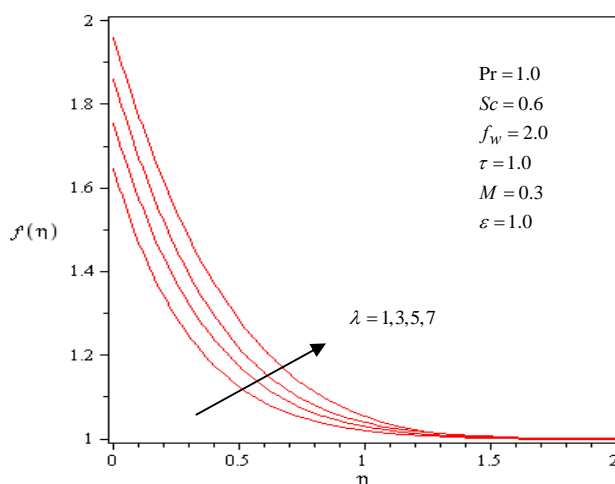

Fig. 5 Effects of λ on velocity in the presence of suction for buoyancy-opposed Marangoni flow.
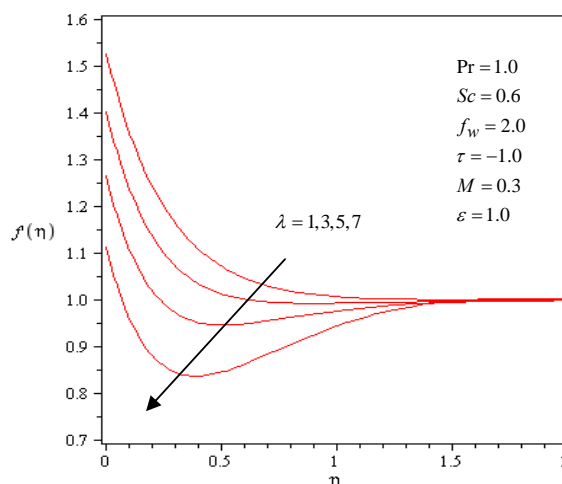

Fig. 6 Effects of λ on velocity in the presence of suction for buoyancy-assisted Marangoni flow.

Figs. 5 and 6 show the effects of Marangoni mixed convection parameter, λ on the velocity in the presence of suction, $f_w = 2.0$ for the buoyancy-opposed and assisted Marangoni flow, respectively. The boundary layer thickness

in the case of buoyancy-opposed Marangoni flow increases but reduces for the case of buoyancy-assisted Marangoni flow.
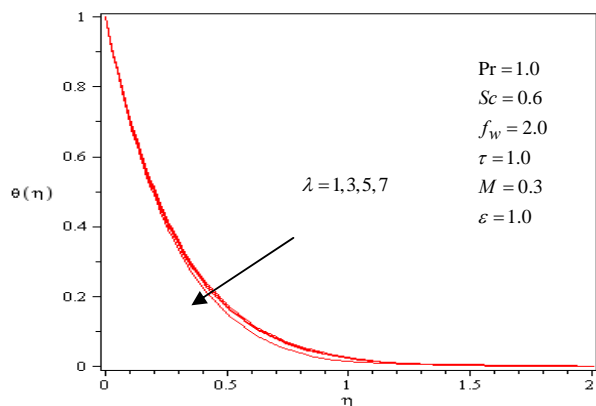

Fig.7 Effects of λ on temperature in the presence of suction for buoyancy-opposed Marangoni flow.
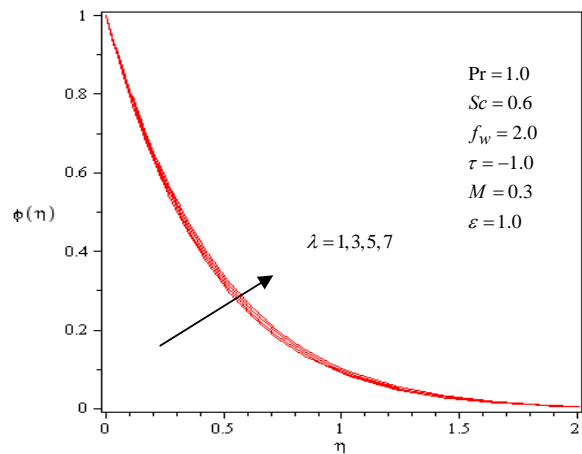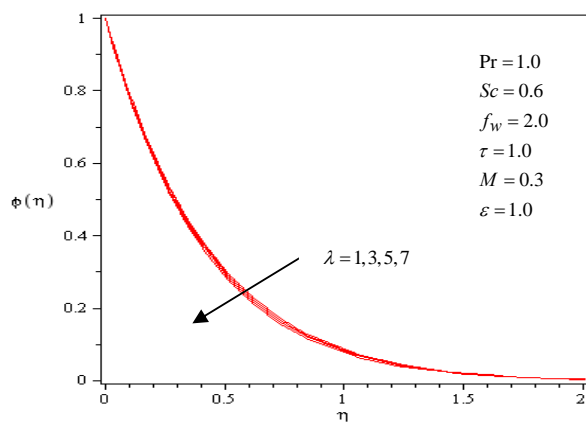

Fig.8 Effects of λ on concentration in the presence of suction for buoyancy-opposed Marangoni flow.
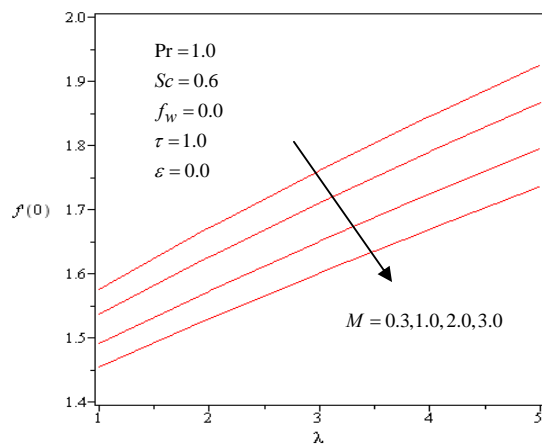

Fig.9 Effects of λ on temperature in the presence of suction for buoyancy-assisted Marangoni flow

Fig. 10 Effects of λ on concentration in the presence of suction for buoyancy-assisted Marangoni flow.

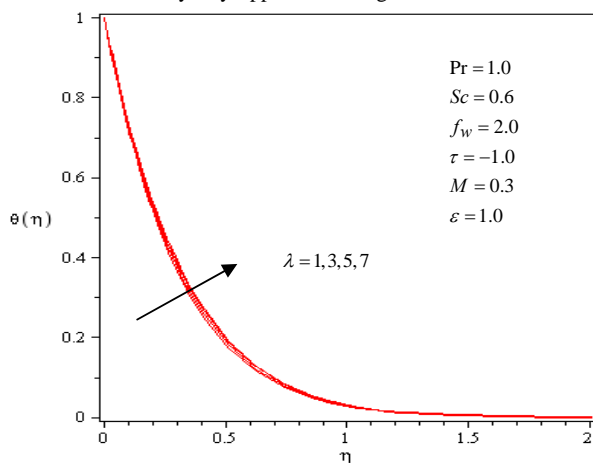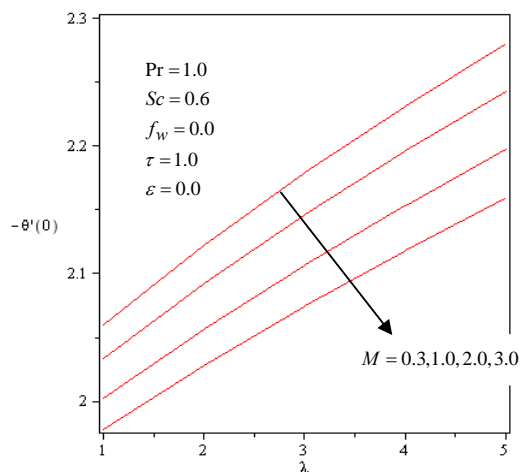Fig. 11 Effects of λ and M on wall velocity $f'(0)$.

Fig. 12 Effects of λ and M on the surface heat transfer coefficient $-\theta'(0)$
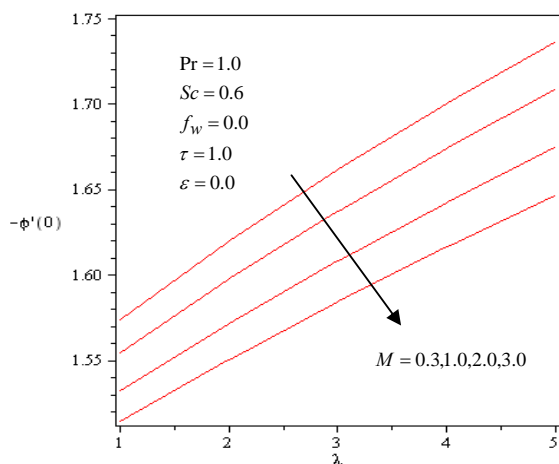
Fig. 13 Effects of λ and M on the surface mass transfer coefficient
$-\phi'(0)$

Figs. 7−10 show that as λ increases, the temperature and concentration boundary layer thickness decrease in the case of buoyancy-opposed Marangoni flow but they increase in the case of buoyancy-assisted flow. Figs. 11-13 show the effects of the mixed Marangoni convection parameter λ and the applied magnetic field M on the wall velocity and surface heat and mass transfer coefficients, respectively. The effect of mixed Marangoni convection parameter is to increase the wall velocity and surface heat and mass transfer coefficients but the magnetic field decelerates the wall velocity and hence reduces the surface heat and mass transfer.

IV. CONCLUSION

The steady coupled dissipative layers of thermosolutal MHD Marangoni mixed convection boundary layer flow in electrically-conducting fluids along in the presence of suction or injection effect has been studied. The boundary layer equations have been derived using similarity transformation and solved numerically using the Runge-Kutta Fehlberg with shooting technique. The velocity, temperature and concentration boundary layers thickness decrease with the increase of the magnetic field strength and the injection to suction. For buoyancy-opposed flow, the Marangoni mixed convection parameter accelerates the velocity but decreases the temperature and concentration boundary layers. However, for the buoyancy-assisted flow, the Marangoni mixed convection parameter decelerates the velocity but increases the temperature and concentration boundary layers.

REFERENCES

[1] K. Arafune, and A. Hirata, "Thermal and solutal Marangoni convection in In-Ga-Sb system," *Journal of Crystal Growth,* vol. 197, pp. 811−817, 1999.
[2] D. M. Christopher, and B. Wang, "Prandtl number effects for Marangoni convection over a flat surface," *International Journal Thermal Sciences,* vol. 40, pp. 564−570, 2001.
[3] I. Pop, A. Postelnicu, and T. Grosan, "Thermosolutal Marangoni forced convection boundary layers," *Meccanica,* vol. 36, pp. 555−571, 2001.
[4] A. Al-Mudhaf, and A. J. Chamkha, "Similarity solutions for MHD thermosolutal Marangoni convection over a flat surface in the presence of heat generation or absorption effects," *Heat and Mass Transfer*, vol. 42, pp. 112−121, 2005.
[5] E. Magyari, and A. J. Chamkha, "Exact analytical solutions for thermosolutal Marangoni convection in the presence of heat and mass generation or consumption," *Heat and Mass Transfer*, vol. 43, pp. 965-975, 2007.
[6] E. Magyari, and A. J. Chamkha, "Exact analytical results for the thermosolutal MHD Marangoni boundary layer," *International Journal of Thermal Sciences*, vol. 47, pp. 848−857, 2008.
[7] T. Watanabe, "Forced and free convection boundary layer flow with uniform suction or injection on a vertical flat plate," *Acta Mechanica*, vol. 89, pp. 123−132, 1991.
[8] C. C. Wang and C. K. Chen, "Mixed convection boundary layer flow on inclined wavy plates including the magnetic field effect," *International Journal of Thermal Sciences,* vol. 44, pp. 577−586, 2005.
[9] D. Pal, " Mixed convection heat transfer in the boundary layers on an exponentially stretching surface with magnetic field," *Applied Mathematics and Computation*, vol. 217, pp. 2356−2369, 2010.
[10] N. Arifin, F. Ali, R. Nazar, and I. Pop, "Thermosolutal Marangoni mixed convection boundary layer," in *Proc. of the 9th WSEAS international conference applications of Computer Engineering*, Netherlands, 2001, pp. 214−218.
[11] J. Zueco, and O. A. Bég, "Network numerical simulation of hydromagnetic Marangoni mixed convection boundary layers," *Chemical Engineering Communications,* vol. 198, pp. 552−571, 2011.
[12] A. J. Chamkha, I. Pop, and H. S. Takhar, " Marangoni mixed convection boundary layer flow," *Meccanica,* vol. 41, pp. 219−232, 2006.
[13] B. Straughan, "Surface tension driven convection in a fluid overlying a porous layer, "*Journal of Computational Physics,* vol. 170, pp. 320−337, 2001.
[14] C. Golia, and A. Viviani, "Non isobaric boundary layers related to Marangoni flows," *Meccanica,* vol. 21, pp. 200−204, 1986.