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The Global Stability Using Lyapunov Function

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***Abstract*—**An important technique in stability theory for differential equations is known as the direct method of Lyapunov. In this work we deal global stability properties of Leptospirosis transmission model by age group in Thailand. First we consider the data from Division of Epidemiology Ministry of Public Health, Thailand between 1997-2011. Then we construct the mathematical model for leptospirosis transmission by eight age groups. The Lyapunov functions are used for our model which takes the forms of an Ordinary Differential Equation system. The globally asymptotically for equilibrium states are analyzed.

***Keywords*—**Age Group, Leptospirosis, Lyapunov Function, Ordinary Differential Equation.

I. Introduction

A function with particular properties known as a Lyapunov function is constructed to prove stability or asymptotic stability of an equilibrium in a given region. In this study we first analyze the data Leptospirosis cases in Thailand between 1997 and 2011 [1-15]. Then we constructed the mathematical model to study the Leptospirosis transmission by age group and analyzed the globally asymptotically by using the Lyapunov function for the human populations which incorporate the structure of the population imposed by the characteristics of the leptospirosis disease by age group.

Leptospirosis has been recognized as a major public health problem. Since the pathogenic members of the genus *Leptospira* have as their hosts a variety of animals, such as the dog, pig, cow, rat and mouse [16-17]. In humans, it can cause a wide range of symptoms, some of which may be mistaken for other diseases. Some infected persons, however, may have no symptoms at all. Without treatment, Leptospiros is can lead to kidney damage, meningitis, liver failure, respiratory distress and and even death [18]. The incubation period, estimated from experimental infection, is between 3 and 7 days [19-22] but periods of 10-30 days have also been reported.

Leptospirosis was first reported in Thailand in 1942 [23]. In 1972, leptospirosis was included as one of the 58 reportable infectious diseases under the National Passive Surveillance System. Between 1997 and 2011, the number of cases ranged from 2,226 to 14,285 cases per year (Table I) and represented an annual the average mean of the incidence rate of approximately 8.41/100,000 populations [1-15].

TABLE I
Report Cases of Leptospirosis by year in Thailand from 1997 to 2011 [1-15]

Year	Number of Cases	Incidences Rate / 100,000 pop.
1997	2334	3.84
1998	2226	3.62
1999	6080	9.87
2000	14285	23.13
2001	10217	16.45
2002	6864	10.97
2003	4962	7.88
2004	3199	5.12
2005	2868	4.61
2006	3941	6.29
2007	3279	5.21
2008	4210	6.66
2009	5439	8.57
2010	4193	7.76
2011	3972	6.22

The disease showed a seasonal fluctuation with most of the cases occurring between June and December. The peak incidence was observed in October. Most leptospirosis cases were seen among farmers aged 25-44 years [1-15].

TABLE II
Report Percentage of Leptospirosis by Age Group In Thailand from 1997 to 2011 [1-15]

Year	<6 year	6 – 14 year	15-24 year	25-34 year	35-44 year	45-54 year	55-64 year	65+ and un known
1997	0.51	5.10	18.08	**26.44**	20.95	15.85	8.70	4.37
1998	0.27	3.32	17.66	**29.61**	22.24	14.42	8.90	3.60
1999	0.18	3.55	17.47	**29.08**	22.63	15.23	8.39	3.47
2000	0.37	4.54	14.70	**26.26**	24.56	16.83	8.69	4.05
2001	0.56	5.15	14.75	**26.08**	23.75	15.94	8.89	4.86
2002	0.70	6.51	13.37	**23.99**	22.58	17.56	10.43	4.85
2003	1.21	7.28	13.60	21.18	**23.84**	9.81	5.90	17.53
2004	2.31	8.82	12.10	20.32	**23.29**	9.94	6.38	16.85
2005	1.85	9.59	12.90	17.85	**22.45**	11.12	6.17	18.06
2006	1.01	7.82	13.68	17.96	**23.01**	11.27	6.42	18.83
2007	1.01	6.98	12.53	17.53	**23.36**	12.26	6.98	19.34
2008	0.64	5.25	11.92	17.46	**23.71**	13.66	6.67	20.69
2009	0.51	5.48	12.34	16.34	**23.59**	12.80	8.51	20.43
2010	0.49	5.78	12.40	16.32	**21.58**	13.71	8.27	21.44
2011	0.53	4.41	10.85	15.79	21.17	14.78	9.47	**23.01**

Mathematical models used here, it is the deterministic model which consists of a set of differential equations have a long tradition in the study of wildlife diseases. For the

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majority of models, the primary motivation is to predict the impact of intervention by either culling or vaccination and a universal motivation is a better understanding of the dynamics of infection [24]. For the mathematical modeling of leptospirosis transmission, in 2006 [25], W. Triampo presented a simple deterministic model for the transmission of leptospirosis by using reported case are investigated in Thailand between 1999 to 2001. Then, J. Holt [26] studied the dynamics of infection in an African rodent that is thought to be the principal source of infection in parts of Tanzania. Their results indicates that removal of animals by trapping rather than reducing the suitability of the environment for rodents will have the greater impact on reducing human cases of leptospirosis. In 2008, our previous research [27] presented age structural transmission model for leptospirosis. We analyzed the locally asymptotically and in 2011 [28] we analyzed the stability and lyapunov functions for the dynamics of leptospirosis by use the systems of non-linear ordinary differential equations which incorporated the structure of the population imposed by the characteristics of the leptospirosis disease, Susceptible, Exposed, Infectious and recovered for human populations. Our previous paper presented a SEIR model for leptospirosis transmission in Thailand. We studied the global stability of the disease. We proved our systems were globally asymptotically stable by using the Lyapunov function. Depending on the basic reproductive number.

In this research, we analyze the global stability of Leptospirosis model by age group using Lyapunov function. The paper is organized as follows. Section II, is presented the mathematical formulation for Leptospirosis transmission by age group. Then we use the systems of non-linear ordinary differential equations to study the globally asymptotically by using Lyapunov function.

II. Mathematical Model

In this section, we first mathematical model for evolution of Leptospirosis transmission by age group in the population is formulated. We construct the model by using the basic ideas and structure of mathematical modeling in epidemiology, the model for the disease will be developed under the next basic hypotheses [29-31]. In the model, we assume that the human population is constant and separated into four classes, susceptible (S_H), exposed (E_H), infectious(I_H) and recovered (R_H). For infectious class was separated to eight subclasses $I_{Hi}, i = 1,2,...,8$, following age group.

I_{H1} means the number of human populations who have age less than 6 years,

I_{H2} means the number of human populations who have 6-14 years,

I_{H3} means the number of human populations who have 15-24 years,

I_{H4} means the number of human populations who have 25-34 years,

I_{H5} means the number of human populations who have 35-44 years,

I_{H6} means the number of human populations who have 45-54 years,

I_{H7} means the number of human populations who have 55-64 years,

I_{H8} means the number of human populations who have age more than 65 years and unknown cases group

where $I_H = \sum_{i=1}^{8} I_{Hi}$.

The mouse population, we divide into three classes, susceptible (S_M), exposed (E_M) and infectious (I_M).Then the model is given by the following equations

$$\frac{dS_H}{dt} = \lambda_H N_H - \mu_H S_H - \frac{\varepsilon_{HE} b_M}{N_H} S_H I_M, \quad (1)$$

$$\frac{dE_H}{dt} = \frac{\varepsilon_{HE} b_M}{N_H} S_H I_M - \mu_H E_H - \alpha_H E_H, \quad (2)$$

$$\frac{dI_{Hi}}{dt} = \alpha_{Hi} E_H - (\mu_H + d_H + r_H) I_{Hi}, \quad i = 1,2,...,8 \quad (3)$$

$$\frac{dR_H}{dt} = r_H I_H - \mu_H R_H, \quad (4)$$

$$\frac{dS_M}{dt} = \lambda_M N_M - \mu_M S_M - \frac{\varepsilon_{ME} b_M}{N_H} S_M I_H, \quad (5)$$

$$\frac{dE_M}{dt} = \frac{\varepsilon_{ME} b_M}{N_H} S_M I_H - \mu_M E_M - \phi_M E_M, \quad (6)$$

$$\frac{dI_M}{dt} = \phi_M E_M - \mu_M I_M. \quad (7)$$

which $\sigma_{MH} = \frac{\varepsilon_{HE} b_M}{N_H}$, $\alpha_H = \sum_{i=1}^{8} \alpha_{Hi}$, $\sigma_{HM} = \frac{\varepsilon_{ME} b_M}{N_H}$

N_H is represented the total number of human populations with constant,

N_M is represented the total number of mouse populations with constant ,

μ_H is represented the natural death rate of human,

λ_H is represented the birth rate of human,

μ_M is represented the natural death rate of mouse,

d_H is represented the death rate of human from infection,

b_M is the number of bites per unit of time of mouse,

ε_{HE} is represented the transmission probability from infectious mouse to susceptible human and human becomes to exposed class,

ε_{ME} is represented the transmission probability from infectious human to susceptible mouse and mouse become to exposed mouse,

α_{Hi} is represented the proportions of exposed class becomes to infectious i class on human population,

ϕ_M is represented the proportions of exposed class becomes to infectious class on mouse population,

r_H is the recovery rate in human populations.

In our model, we assume no alternative hosts for the mouse. $N_H = S_H + E_H + \sum_{i=1}^{8} I_{Hi} + R_H$ and $N_M = S_M + E_M + I_M$. It can be seen that the equations (1) to (7), the non-negative octant R_+^{14} is positively invariant. Next, we will show the basic reproductive number of our model proposition for analyzing. From equations (1) to (7), we can rewrite the equations to:

$$\frac{dS_H}{dt} = \lambda_H N_H - (\mu_H + \sigma_{MH} I_M) S_H, \quad (8)$$

$$\frac{dE_H}{dt} = \sigma_{MH} I_M S_H - (\mu_H + \alpha_H) E_H, \quad (9)$$

$$\frac{dI_{Hi}}{dt} = \alpha_{Hi} E_H - (\mu_H + d_H + r_H) I_{Hi}, \ i = 1,2,...,8 \quad (10)$$

$$\frac{dR_H}{dt} = r_H I_H - \mu_H R_H, \quad (11)$$

$$\frac{dS_M}{dt} = \lambda_M N_M - (\mu_M + \sigma_{HM} I_H) S_M, \quad (12)$$

$$\frac{dE_M}{dt} = \sigma_{HM} I_H S_M - (\mu_M + \phi_M) E_M, \quad (13)$$

$$\frac{dI_M}{dt} = \phi_M E_M - \mu_M I_M. \quad (14)$$

III. Analysis the Model

The equations (8) to (14) have two equilibrium states: for the basic reproductive number is less than or equal to one, the disease free equilibrium state:

$$B^* = (S_H^*, E_H^*, I_{Hi}^*, R_H^*, S_M^*, E_M^*, I_M^*) \quad (15)$$

which

$$B^* = (\frac{\lambda_H N_H}{\mu_H}, 0,0,0,0,0,0,0,0,0,0, \frac{\lambda_M N_M}{\mu_M}, 0, 0). \quad (16)$$

For the basic reproductive number is more than one, the endemic equilibrium state:

$$\overline{B} = (\overline{S_H}, \overline{E_H}, \overline{I_{Hi}}, \overline{R_H}, \overline{S_M}, \overline{E_M}, \overline{I_M}) \quad (17)$$

which satisfies:

$$\overline{S_H} = \frac{\lambda_H N_H}{\mu_H + \sigma_{MH} \overline{I_M}}, \quad (18)$$

$$\overline{E_H} = \frac{\theta_1 \overline{I_M}}{(\mu_H + \sigma_{MH} \overline{I_M})}, \quad (19)$$

$$\overline{I_{Hi}} = \frac{\theta_2 \overline{I_M}}{(\mu_H + \sigma_{MH} \overline{I_M})} \sum_{i=1}^{8} \alpha_{Hi}, \quad (20)$$

$$\overline{R_H} = \frac{\theta_3 \overline{I_M}}{(\mu_H + \sigma_{MH} \overline{I_M})} \sum_{i=1}^{8} \alpha_{Hi}, \quad (21)$$

$$\overline{S_M} = \frac{\lambda_M N_M (\mu_H + \sigma_{MH} \overline{I_M})}{\mu_M (\mu_H + \sigma_{MH} \overline{I_M}) + \sigma_{HM} \theta_2 \overline{I_M} \sum_{i=1}^{8} \alpha_{Hi}}, \quad (22)$$

$$\overline{E_M} = \frac{\theta_4 \overline{I_M}}{\mu_M (\mu_H + \sigma_{MH} \overline{I_M}) + \sigma_{HM} \theta_2 \overline{I_M} \sum_{i=1}^{8} \alpha_{Hi}}, \quad (23)$$

$$\overline{I_M} = \frac{(\theta_5 - \mu_M \mu_H)}{(\mu_M \sigma_{MH} + \sigma_{HM} \theta_2 \sum_{i=1}^{8} \alpha_{Hi})}, \quad (24)$$

when $\frac{\lambda_M \phi_M N_M \sigma_{HM} \sigma_{MH} \lambda_H N_H \sum_{i=1}^{8} \alpha_{Hi}}{\mu_M (\mu_M + \phi_M)(\mu_H + \alpha_H)(\mu_H + d_H + r_H)} > \mu_M \mu_H$ and

$\theta_1 = \frac{\sigma_{MH} \lambda_H N_H}{(\mu_H + \alpha_H)}$, $\theta_2 = \frac{\theta_1}{(\mu_H + d_H + r_H)}$, $\theta_3 = \frac{r_H \theta_2}{\mu_H}$,

$\theta_4 = \frac{\lambda_M N_M \sigma_{HM} \theta_2 \sum_{i=1}^{8} \alpha_{Hi}}{(\mu_M + \phi_M)}$, $\theta_5 = \frac{\phi_M}{\mu_M} \theta_4$.

Then, the basic reproductive number for equations (8) to (14) is:

$$B_r = \sqrt{\frac{\lambda_M \phi_M N_M \sigma_{HM} \sigma_{MH} \lambda_H N_H \sum_{i=1}^{8} \alpha_{Hi}}{\mu_M^2 \mu_H (\mu_M + \phi_M)(\mu_H + \alpha_H)(\mu_H + d_H + r_H)}}. \quad (25)$$

A. Global Stability of the Diesase Free Equilibrium State

We now study the global behavior of the disease free equilibrium state for equations (8) to (14) by applying the direct Lyapunov method.

If $0 < B_r < 1$ holds, then each of these equations (8) to (14) has two equilibrium states: an infection-free equilibrium B^* with the coordinates $S_H^* = \frac{\lambda_H N_H}{\mu_H}$, $E_H^* = I_{Hi}^* = R_H^* = E_M^* = I_M^* = 0$, $S_M^* = \frac{\lambda_M N_M}{\mu_M}$ and an endemic equilibrium state $\overline{B} = (\overline{S_H}, \overline{E_H}, \overline{I_{Hi}}, \overline{R_H}, \overline{S_M}, \overline{E_M}, \overline{I_M})$ which defined in (18) to (24). The following Theorem provides global properties of the equations (8) to (14).

Theorem 1 The disease free equilibrium B^*,

$$B^* = (\frac{\lambda_H N_H}{\mu_H}, 0,0,0,0,0,0,0,0,0,0, \frac{\lambda_M N_M}{\mu_M}, 0, 0)$$

is globally asymptotically stable when $B_r \le 1$ which

$$\left.\begin{aligned} \mu_H &= \sigma_{HM} S_M^* - d_H \\ \mu_M &= \sigma_{MH} S_H^* \end{aligned}\right\} \quad (26)$$

Proof The Lyapunov function candidate:

$$L(t) = (S_H - S_H^* \ln S_H) + E_H + \sum_{i=1}^{8} I_{Hi} + R_H + (S_M - S_M^* \ln S_M) + E_M + I_M. \quad (27)$$

Then, we have the derivative satisfies:

$$L^{'}(t) = S^{'}_{H} - S^{*}_{H}\frac{d}{dt}\ln S_{H} + E^{'}_{H} + \sum_{i=1}^{8} I^{'}_{Hi} + R^{'}_{H} + S^{'}_{M} - S^{*}_{M}\frac{d}{dt}\ln S_{M}) + E^{'}_{M} + I^{'}_{M}$$

$$L^{'}(t) = S^{'}_{H}(1-\frac{S^{*}_{H}}{S_{H}}) + E^{'}_{H} + \sum_{i=1}^{8} I^{'}_{Hi} + R^{'}_{H} + S^{'}_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + E^{'}_{M} + I^{'}_{M}$$

$$L^{'}(t) = \lambda_{H}N_{H}(1-\frac{S^{*}_{H}}{S_{H}}) + \mu_{H}S^{*}_{H}(1-\frac{S_{H}}{S^{*}_{H}}) + \lambda_{M}N_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + \lambda_{M}N_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + \mu_{M}S^{*}_{M}(1-\frac{S_{M}}{S^{*}_{M}}) - \mu_{H}E_{H} - \mu_{H}R_{H} + \sum_{i=1}^{8} I^{'}_{Hi}(\sigma_{HM}S^{*}_{M} - \mu_{H} - d_{H}) + I_{M}(\sigma_{MH}S^{*}_{H} - \mu_{M}). \quad (28)$$

Using the conditions (26), substitute these conditions into the eighth and ninth terms of (28), then (28) becomes into:

$$L^{'}(t) = \lambda_{H}N_{H}(1-\frac{S^{*}_{H}}{S_{H}}) + \mu_{H}S^{*}_{H}(1-\frac{S_{H}}{S^{*}_{H}}) + \lambda_{M}N_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + \lambda_{M}N_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + \mu_{M}S^{*}_{M}(1-\frac{S_{M}}{S^{*}_{M}}) - \mu_{H}E_{H} - \mu_{H}R_{H}. \quad (29)$$

For the disease free equilibrium state $S^{*}_{H} = \frac{\lambda_{H}N_{H}}{\mu_{H}}$ and $S^{*}_{M} = \frac{\lambda_{M}N_{M}}{\mu_{M}}$. Substituting these relations into the second term and fourth term of (24), respectively, equation (28) become to below equation:

$$L^{'}(t) = \lambda_{H}N_{H}(1-\frac{S^{*}_{H}}{S_{H}}) + \lambda_{H}N_{H}(1-\frac{S_{H}}{S^{*}_{H}}) + \lambda_{M}N_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + \lambda_{M}N_{M}(1-\frac{S^{*}_{M}}{S_{M}}) + \lambda_{M}N_{M}(1-\frac{S_{M}}{S^{*}_{M}}) - \mu_{H}E_{H} - \mu_{H}R_{H}$$

$$L^{'}(t) = \lambda_{H}N_{H}(2-\frac{S^{*}_{H}}{S_{H}}-\frac{S_{H}}{S^{*}_{H}}) + \lambda_{M}N_{M}(2-\frac{S^{*}_{M}}{S_{M}}-\frac{S_{M}}{S^{*}_{M}}) - \mu_{H}E_{H} - \mu_{H}R_{H}$$

$$L^{'}(t) = -\lambda_{H}N_{H}\frac{(S^{*}_{H}-S_{H})^{2}}{S_{H}S^{*}_{H}} - \lambda_{M}N_{M}\frac{(S^{*}_{M}-S_{M})^{2}}{S_{M}S^{*}_{M}} - \mu_{H}E_{H} - \mu_{H}R_{H} \quad (30)$$

So, $L^{'}(t) \le 0$. Using LaSalle's extension to Lyapunov's method [32] the limit set of each solution is contained in the largest invariant set for which $S_{H} = S^{*}_{H}$, $E_{H} = 0$, $I_{Hi} = 0, \forall i,\ i = 1,2,...,8$, $R_{H} = 0$, $S_{M} = S^{*}_{M}$, $E_{M} = 0$, $I_{M} = 0$ which is the singleton $\{B^{*}\}$. This means that the disease free equilibrium B^{*},

$$B^{*} = (\frac{\lambda_{H}N_{H}}{\mu_{H}}, 0,0,0,0,0,0,0,0,0, \frac{\lambda_{M}N_{M}}{\mu_{M}}, 0, 0)$$

is globally asymptotically stable. This achieves the proof.

B. Global Stability of the Endemic Equilibrium State

Theorem 2. The endemic equilibrium state $\overline{B}$,

$$\overline{B} = (\overline{S_{H}}, \overline{E_{H}}, \overline{I_{Hi}}, \overline{R_{H}}, \overline{S_{M}}, \overline{E_{M}}, \overline{I_{M}})$$

of equations (8) to (14) exists and is globally asymptotically stable if

$$\left.\begin{aligned} a &= \frac{S_{M}}{S_{M} - \overline{S_{M}}} \\ b &= \frac{\sigma_{MH}\overline{S_{H}}}{\mu_{M}} \end{aligned}\right\} \quad (31)$$

with these always positive since $S_{M} > \overline{S_{M}}$.

Proof A Lyapunov function candidate:

$$\xi(t) = (S_{H} - \overline{S_{H}}\ln S_{H}) + E_{H} + \sum_{i=1}^{8} I_{Hi} + R_{H} + a(S_{M} - \overline{S_{M}}\ln S_{M}) + E_{M} + bI_{M}. \quad (32)$$

The derivative satisfies of $\xi(t)$ is:

$$\xi^{'}(t) = S^{'}_{H} - \frac{\overline{S_{H}}}{S_{H}}S^{'}_{H} + E^{'}_{H} + \sum_{i=1}^{8} I^{'}_{Hi} + R^{'}_{H} + a(S^{'}_{M} - \frac{\overline{S_{M}}}{S_{M}}S^{'}_{M}) + E^{'}_{M} + bI^{'}_{M}.$$

$$\xi^{'}(t) = S^{'}_{H}(1-\frac{\overline{S_{H}}}{S_{H}}) + E^{'}_{H} + \sum_{i=1}^{8} I^{'}_{Hi} + R^{'}_{H} + aS^{'}_{M}(1-\frac{\overline{S_{M}}}{S_{M}}) + E^{'}_{M} + bI^{'}_{M}. \quad (33)$$

$$\xi^{'}(t) = \lambda_{H}N_{H}(1-\frac{\overline{S_{H}}}{S_{H}}) + \mu_{H}\overline{S_{H}}(1-\frac{S_{H}}{\overline{S_{H}}}) + \sigma_{MH}I_{M}\overline{S_{H}} - \mu_{H}E_{H} - \mu_{H}\sum_{i=1}^{8} I_{Hi} - d_{H}\sum_{i=1}^{8} I_{Hi} - \mu_{H}R_{H} + a\lambda_{M}N_{M}(1-\frac{\overline{S_{M}}}{S_{M}}) + a\mu_{M}\overline{S_{M}}(1-\frac{S_{M}}{\overline{S_{M}}}) - a\sigma_{HM}I_{H}S_{M} + a\sigma_{HM}I_{H}\overline{S_{M}} + \sigma_{HM}I_{H}S_{M} - \mu_{M}E_{M} - b\mu_{M}I_{M}. \quad (34)$$

Substituting the relations $\overline{S_{H}} = \frac{\lambda_{H}N_{H}}{\mu_{H}}$ and $\mu_{M} = \frac{\lambda_{M}N_{M}}{\overline{S_{M}}}$ in the second and ninth terms of equation (34), respectively, equation (34) become to below equation:

$$\xi^{'}(t) = \lambda_{H}N_{H}(1-\frac{\overline{S_{H}}}{S_{H}}) + \lambda_{H}N_{H}(1-\frac{S_{H}}{\overline{S_{H}}}) + a\lambda_{M}N_{M}(1-\frac{\overline{S_{M}}}{S_{M}}) +$$

$$a\lambda_M N_M(1-\frac{S_M}{\overline{S_M}})-\mu_M E_M+I_M(\sigma_{MH}\overline{S_H}-b\mu_M)+$$
$$\sum_{i=1}^{8} I_{Hi}(\sigma_{HM}S_M+a\sigma_{HM}\overline{S_M}-a\sigma_{HM}S_M)-\mu_H E_H-$$
$$(\mu_H+d_H)\sum_{i=1}^{8} I_{Hi}-\mu_H R_H. \quad (35)$$

Substituting the relations in equation (31) into equation (35) then we have:

$$\xi'(t)=\lambda_H N_H(2-\frac{\overline{S_H}}{S_H}-\frac{S_H}{\overline{S_H}})+a\lambda_M N_M(2-\frac{\overline{S_M}}{S_M}-\frac{S_M}{\overline{S_M}})-\mu_M E_M$$
$$-\mu_H E_H-(\mu_H+d_H)\sum_{i=1}^{8} I_{Hi}-\mu_H R_H$$
$$\xi'(t)=-\lambda_H N_H\frac{(\overline{S_H}-S_H)^2}{S_H\overline{S_H}}-a\lambda_M N_M\frac{(\overline{S_M}-S_M)^2}{S_M\overline{S_M}}-\mu_M E_M$$
$$-\mu_H E_H-(\mu_H+d_H)\sum_{i=1}^{8} I_{Hi}-\mu_H R_H. \quad (36)$$

Thus, the conditions in (36) ensures that $\xi'(t)\le 0$ for all $\overline{S_H}, \overline{E_H}, \overline{I_{Hi}}, \overline{R_H}, \overline{S_M}, \overline{E_M}, \overline{I_M}$. Then the endemic equilibrium state $\overline{B}$ is globally asymptotically stable. This achieves the proof.

Applying the direct Lyapunov method to study global properties of mathematical model for Leptospirosis transmission by age groups separated following Leptospirosis case in Thailand, are shown above theorems.

IV. Conclusion

In this paper, we prove global stability of the mathematical model with Leptospirosis transmission by age group in Thailand. This conclusion is valid under the constant population size assumption and under the condition:

$$B_r=\sqrt{\frac{\lambda_M\phi_M N_M\sigma_{HM}\sigma_{MH}\lambda_H N_H\sum_{i=1}^{8}\alpha_{Hi}}{\mu_M^2\mu_H(\mu_M+\phi_M)(\mu_H+\alpha_H)(\mu_H+d_H+r_H)}}. \quad (37)$$

The Lyapunov functions are constructed for establish the global stability of the equilibrium states.

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