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Intuitionistic Fuzzy Multisets And Its Application in Medical Diagnosis

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Abstract—In this paper a new concept named Intuitionistic Fuzzy Multiset is introduced. The basic operations on Intuitionistic Fuzzy Multisets such as union, intersection, addition, multiplication etc. are discussed. An application of Intuitionistic Fuzzy Multiset in Medical diagnosis problem using a distance function is discussed in detail.

Keywords—Intuitionistic Fuzzy set, Multiset, Intuitionistic Fuzzy Multiset

I. Introduction

MODERN set theory formulated by George Cantor is fundamental for the whole Mathematics. One issue associated with the notion of a set is the concept of vagueness. Mathematics requires that all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a problem for a long time for philosophers, logicians and mathematicians. However, recently it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situations like this, many tools were suggested. They include Fuzzy sets, Multi sets, Rough sets, Soft sets and many more.

Considering the uncertainty factor, Lofti Zadeh [1] introduced Fuzzy sets in 1965, in which a membership function assigns to each element of the universe of discourse, a number from the unit interval $[0, 1]$ to indicate the degree of belongingness to the set under consideration. In 1983, Krassimir. T. Atanassov [2, 3] introduced the concept of Intuitionistic Fuzzy sets (IFS) by introducing a non-membership fuction together with the membership function of the fuzzy set. Among the various notions of higher-order Fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainity and vagueness. IFS reflect better the aspects of human behavior.

A human being who expresses the degree of belongingness of a given element to a set, does not often expresses the corresponding degree of non-belongingness as the complement. This psychological fact states that linguistic negation does not always coincides with logical negation. This idea of Intuitionistic fuzzy sets, which is a natural generalization of a standard Fuzzy set, seems to be useful in modelling many real life situations, like negotiation processes, psychological investigations, reasoning etc. The relation between Intuitionistic Fuzzy sets and other theories modelling imprecision can be seen in [5].

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Many fields of modern mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. Set is a well-defined collection of distinct objects, that is, the elements of a set are pair wise different. If we relax this restriction and allow repeated occurrences of any element, then we can get a mathematical structure that is known as Multisets or Bags. For example, the prime factorization of an integer $n > 0$ is a Multiset whose elements are primes. The number 120 has the prime factorization $120 = 2^3 3^1 5^1$ which gives the Multiset $\{2, 2, 2, 3, 5\}$. A complete account of the development of multiset theory can be seen in [7]. As a generalization of multiset, Yager [8] introduced the concept of Fuzzy Multiset (FMS). An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values.

This paper is an attempt to combine the two concepts: Intuitionistic Fuzzy sets and Fuzzy Multisets together by introducing a new concept called Intuitionistic Fuzzy Multisets.

II. Preliminaries

Definition 2.1: [1] Let X be a nonempty set. A *Fuzzy set* A drawn from X is defined as $A = \{< x : \mu_A(x) >: x \in X\}$. Where : $X \to [0, 1]$ is the membership function of the Fuzzy Set A.

Definition 2.2: [8] Let X be a nonempty set. A *Fuzzy Multiset* (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0, 1]$. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0, 1]$. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\left(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^P(x)\right)$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq, \ldots, \geq \mu_A^P(x)$.

A complete account of the applications of Fuzzy Multisets in various fields can be seen in [9].

Definition 2.3: [3] Let X be a nonempty set. An *Intuitionistic Fuzzy Set* (IFS) A is an object having the form $A = \{< x : \mu_A(x), \nu_A(x) >: x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define respectively the degree of membership and the degree of non membership of the element $x \in X$ to the set A with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Remark 2.4: Every Fuzzy set A on a nonempty set X is obviously an IFS having the form

$$A = \{< x : \mu_A(x), 1 - \mu_A(x) >: x \in X\}.$$

Using the definition of FMS and IFS, a new generalized concept can be defined as follows:

III. Intuitionistic Fuzzy Multiset

Definition 3.1: Let X be a nonempty set. An *Intuitionistic Fuzzy Multiset* A denoted by IFMS drawn from X is characterized by two functions: 'count membership' of $A(CM_A)$ and 'count non membership' of $A(CN_A)$ given respectively by $CM_A : X \rightarrow Q$ and $CN_A : X \rightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval $[0,1]$ such that for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A(x)$ which is denoted by $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^P(x))$ where $(\mu_A^1(x) \geq \mu_A^2(x) \geq, \ldots \geq, \mu_A^P(x)$ and the corresponding non membership sequence will be denoted by $(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^P(x))$ such that $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i = 1, 2, \ldots, p$.

An IFMS A is denoted by

$$\begin{aligned} A \; &= \{< x : \; \left(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^P(x)\right), \\ &\qquad \left(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^P(x)\right) >: x \in X\}. \end{aligned}$$

Remark 3.2: We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

Definition 3.3: Length of an element x in an IFMS A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu_A^j(x) + \nu_A^j(x) \leq 1$ and it is denoted by $L(x : A)$. That is

$$L(x : A) = |CM_A(x)| = |CN_A(x)|\,.$$

Definition 3.4: If A and B are IFMSs drawn from X then $L(x : A, B) = Max\{L(x : A), L(x : B)\}$. Alternatively we use $L(x)$ for $L(x : A, B)$.

Example 3.5: Consider the set $X = \{x, y, x, w\}$ with

$$\begin{aligned} A \;\; &= \;\; \{< x : (0.3, 0.2), (0.4, 0.5) >, \\ &\qquad < y : (1, 0.5, 0.5), (0.0.5, 0.2) >, \\ &\qquad < z : (0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7) >\}, \\ B \;\; &= \;\; \{< x : (0.4), (0.2) >, \\ &\qquad < y : (1, 0.3, 0.2), (0, 0.4, 0.5) >, \\ &\qquad < w : (0.2, 0.1), (0.7, 0.8) >\}. \end{aligned}$$

Here
$L(x : A) = 2$, $L(y : A) = 3$, $L(z : A) = 4$, $L(w : A) = 0$
$L(x : B) = 1$, $L(y : B) = 3$, $L(z : B) = 0$, $L(w : B) = 2$
$L(x : A, B) = 2$, $L(y : A, B) = 3$, $L(x : A, B) = 4$,
$L(w : A, B) = 2$.

Now we define basic operations on IFMS. Note that we can make $L(x : A) = L(x : B)$ by appending sufficient number of 0s and 1s with the membership and non membership values respectively.

Definition 3.6: Let A and B be two IFMS. The distance function is defined as

$$\begin{aligned} d(A, B) \;\; &= \;\; \frac{1}{2}\left(\sum_i \left(\left(\mu_A^i(x) - \mu_B^i(x)\right)^2\right.\right. \\ &\qquad + \left(\left(\nu_A^i(x) - \mu_B^i(x)\right)^2\right. \\ &\qquad \left.\left. + \left(\Pi_A^i(x) - \Pi_B^i(x)\right)^2\right)^{\frac{1}{2}}\right). \end{aligned}$$

where $\Pi_A^i = 1 - \mu_A^i(x) - \nu_A^i(x)$ called the IFMS index or hesitation margin.

Definition 3.7: For any two IFMSs A and B drawn from a set X, the following operations and relations will hold. Let $A = \{< x : \left(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^P(x)\right), \left(\nu_A^1(x), \nu_A^2(x), \ldots, \nu_A^P(x)\right) >: x \in X\}$ and $B = \{< x : \left(\mu_B^1(x), \mu_B^2(x), \ldots, \mu_B^P(x)\right), \left(\nu_B^1(x), \nu_B^2(x), \ldots, \nu_B^P(x)\right) >: x \in X\}$ then

1.

1) Inclusion

   $A \subset B \Leftrightarrow \mu_A^j(x) \leq \mu_A^j(x)$ and $\nu_A^j(x) \leq \nu_B^j(x)$;
   $j = 1, 2, \ldots, L(x)$, $x \in X$
   $A = B \Leftrightarrow A \subset B$ and $B \subset A$

2) Complement

   $A = \{< x : \left(\nu_A^1(x), \ldots, \nu_A^P(x)\right), \left(\mu_A^1(x), \ldots, \mu_A^P(x)\right) >: x \in X\}$.

3) Union $(A \cup B)$
   In $A \cup B$ the membership and non membership values are obtained as follows.

   $\mu_{A\cup B}^j(x) = \mu_A^j(x) \vee \mu_B^j(x)$
   $\nu_{A\cup B}^j(x) = \nu_A^j(x) \wedge \nu_B^j(x)$
   $j = 1, 2, \ldots, L(x)$, $x \in X$.

4) Intersection $(A \cap B)$
   In $A \cap B$ the membership and non membership values are obtained as follows.

   $\mu_{A\cap B}^j(x) = \mu_A^j(x) \vee \mu_B^j(x)$
   $\nu_{A\cap B}^j(x) = \nu_A^j(x) \wedge \nu_B^j(x)$
   $j = 1, 2, \ldots, L(x)$, $x \in X$.

5) Addition $(A \oplus B)$
   In $A \oplus B$ the membership and non membership values are obtained as follows.

   $\mu_{A\oplus B}^j(x) = \mu_A^j(x) + \mu_B^j(x) - \mu_A^j(x) \cdot \mu_B^j(x)$
   $\nu_{A\oplus B}^j(x) = \nu_A^j(x) \cdot \nu_B^j(x)$
   $j = 1, 2, \ldots, L(x), x \in X$.

6) Multiplication $(A \otimes B)$
   In $A \otimes B$ the membership and nonmembership values are obtained as follows.

   $\mu_{A\otimes B}^j(x) = \mu_A^j(x) \cdot \mu_B^j(x)$
   $\nu_{A\otimes B}^j(x) = \nu_A^j(x) + \nu_B^j(x) - \nu_A^j(x) \cdot \nu_B^j(x)$
   $j = 1, 2, \ldots, L(x), x \in X$.

here $\wedge, \cdot, +, -$ denotes maximum, minimum, multiplication, addition, subtraction of real numbers respectively.

IV. Medical Diagnosis via IFMS Theory

Most of human reasoning involves the use of variables whose values are fuzzy sets. This is the basis for the concept of a linguistic variable, that is, a variable whose values are words rather than numbers. But in some situations like decision making problems (such as Medical diagnosis, Sales analysis, Marketing etc.) the description by a linguistic variable in terms of membership function only is not adequate. There is chance of existing a non-null complement. IFS can be used in this context as a proper tool for representing both membership and non-membership of an element to a set. Such situations are explained in [10]. But there are situations that each element has different membership values. In such situations IFMS is more adequate. Here we present IFMS as a tool for reasoning such a situation.

An example of a medical diagnosis will be presented.

Let P = $\{P_1, P_2, P_3, P_4\}$ be a set of patients,
D = {Viral Fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases and S = {Temperature, cough, throat pain, headache, body pain} be a set of symptoms.

One of the question arises is, whether only by taking one time inspection we can arrive a conclusion that a particular person has a particular decease or not? Sometimes it may show symptoms of different diseases also. Then how can we arrive at a proper conclusion? One solution is to examine the patient at different time intervals (it can be two or three times a day). For the analysis purpose we take some main symptoms of each disease given in D.
For this we have to take 3 different samples in 3 different times in a day. The details of a typical example are given below.

In Table I each symptom S_i is described by three numbers: Membership μ, non-membership ν and hesitation margin Π.

TABLE I
Symptoms Vs Diseases

	Viral Fever	Tuberculosis	Typhoid	Throat disease
Temperature	**(0.8,0.1,0.1)**	**(0.2,0.7,0.1)**	**(0.5,0.3,0.2)**	**(0.1,0.7,0.2)**
Cough	**(0.2,0.7,0.1)**	**(0.9,0,0.1)**	**(0.3,0.5,0.2)**	**(0.3,0.6,0.1)**
Throat Pain	**(0.3,0.5,0.2)**	**(0.7,0.2,0.1)**	**(0.2,0.7,0.1)**	**(0.8,0.1,0.1)**
Headache	**(0.5,0.3,0.2)**	**(0.6,0.3,0.1)**	**(0.2,0.6,0.2)**	**(0.1,0.8,0.1)**
Body Pain	**(0.5,0.4,0.1)**	**(0.7,0.2,0.1)**	**(0.4,0.4,0.2)**	**(0.1,0.8,0.1)**

The objective is to make a proper diagnosis for each patient. Here we use Euclidean distance function defined in Definition 3.6.

Let the sample be taken at three different timings in a day; 7 AM, 1 PM and 7 PM.

Here the distance function calculate the distance of each patient P_i from the set of symptoms S_i for each diagnosis $d_k : k = 1, 2, 3, 4$.

Here the first set represents the membership values obtained at 7 AM, 1 PM and 7 PM respectively. The second and third sets represents the corresponding non-membership and hesitation margin.

Table II is constructed by using the Definition 3.1.

Table III gives the distances for each patient from the set of diseases considered.

TABLE II
Patients Vs Symptoms

	Temperature	Cough	Throat Pain	Headache	Body Pain
$\mathbf{P_1}$	(0.6,0.7, 0.5) (0.2, 0.1, 0.4) (0.2, 0.2, 0.1)	(0.4,0.3, 0.4) (0.3, 0.6, 0.4) (0.3, 0.1, 0.2)	(0.1,0.2, 0) (0.7, 0.7, 0.8) (0.2, 0.1, 0.2)	(0.5,0.6, 0.7) (0.4, 0.3, 0.2) (0.1, 0.1, 0.1)	(0.2,0.3, 0.4) (0.6, 0.4, 0.4) (0.2, 0.3, 0.2)
$\mathbf{P_2}$	(0.4,0.3, 0.5) (0.5, 0.4, 0.4) (0.1, 0.3, 0.1)	(0.7,0.6, 0.8) (0.2, 0.2, 0.1) (0.1, 0.2, 0.1)	(0.6,0.5, 0.4) (0.3, 0.3, 0.4) (0.1, 0.2, 0.2)	(0.3,0.6, 0.2) (0.7, 0.3, 0.7) (0, 0.1, 0.1)	(0.8,0.7, 0.5) (0.1, 0.2, 0.3) (0.1, 0.1, 0.2)
$\mathbf{P_3}$	(0.1,0.2, 0.1) (0.7, 0.6, 0.9) (0.2, 0.2, 0.0)	(0.3,0.2, 0.1) (0.6, 0, 0.7) (0.1, 0.8, 0.2)	(0.8,0.7, 0.8) (0, 0.1, 0.1) (0.2, 0.2, 0.1)	(0.3,0.2, 0.2) (0.6, 0.7, 0.6) (0.1, 0.1, 0.2)	(0.4,0.3, 0.2) (0.4, 0.7, 0.7) (0.2, 0, 0.1)
$\mathbf{P_4}$	(0.5,0.4, 0.5) (0.4, 0.4, 0.3) (0.1, 0.2, 0.2)	(0.4,0.3, 0.4) (0.5, 0.3, 0.5) (0.1, 0.4, 0.1)	(0.2,0.1, 0) (0.7, 0.6, 0.7) (0.1, 0.3, 0.3)	(0.5,0.6, 0.3) (0.4, 0.3, 0.6) (0.1, 0.1, 0.1)	(0.4,0.5, 0.4) (0.6, 0.4, 0.3) (0, 0.1, 0.3)

TABLE III
Distance between Patients & Diseases

	Viral Fever	Tuberculosis	Typhoid	Throat disease
$\mathbf{P_1}$	0.49	0.96	**0.45**	1.04
$\mathbf{P_2}$	0.79	**0.50**	0.64	0.90
$\mathbf{P_3}$	0.98	0.89	0.85	**0.49**
$\mathbf{P_4}$	0.51	0.89	**0.36**	0.93

In the above table the lowest distance point gives the proper diagnosis. Patient P_1 suffers from Typhoid, Patient P_2 suffers from Tuberculosis, P_3 suffers from Throat disease and Patient P_4 also from Typhoid.

V. Conclusions

In this paper, we have introduced a new concept called Intuitionistic Fuzzy Multiset and discussed its various basic operations and definitions. Finally an application of IFMS in medical diagnosis is discussed. In the proposed method, we measured the distances of each patient from each disease by considering the symptoms of that particular disease. The concept of multiness is incorporated by taking the samples from the same patient at different times.

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