International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:6, No:8, 2012

New Delay-Dependent Stability Criteria For Neural Networks With Two Additive Time-varying delay components

Xingyuan Qu and Shouming Zhong

Abstract—In this paper, the problem of stability criteria of neural networks (NNs) with two-additive time-varying delay compenents is investigated. The relationship between the time-varying delay and its lower and upper bounds is taken into account when estimating the upper bound of the derivative of Lyapunov functional. As a result, some improved delay stability criteria for NNs with two-additive time-varying delay components are proposed. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords—Delay-dependent stability; Time-varying delays; Lyapunov functional; Linear matrix inequality(LMI).

I. INTRODUCTION

NEURAL networks(NNs) have been studied over the past few decades extensively and have found many applications in many areas, such as pattern recognition, signal processing, associative memory, static image processing, and combinatorial optimization. And time-delay also often occurs in many industrial and engineeering systems, such as manufacturing systems, telecommunication and economic systems, and is a major cause of instability and poor performance. In recent years, much efforts has been invested in the analysis of time-delay systems, such as delayed stochastic system, delayed stochastic genetic regulatory networks, delayed stochastic complex networks [1]-[4]. Up to now, stability of NNs with time delay has also received attention [5]-[14], since time delay is frequently encountered in NNs, and it is often a source of instability and oscillations in a system. Both delay-independent [8]-[17] and delay-dependent [18]-[27] stability criteria for NNs have been proposed in recent years. Since delay-independent criteria tend to be conservative, especially when the delay is small or it varies in an interval, much attention has been paid to the delay-dependent type. But note that the delay-dependent stability results mentioned above can only provide stability conditions for neural networks with one single delay in the state.

Recently, a new model for neural networks with two additive time-varying delays has been considered in [6], [7]and [26]. By constructing a new Lyapunov functional and using some advanced techniques, a new asymptotic stability criterion for neural networks with two successive delay components is derived in [6]. By choosing a new class of Lyapunov functional, some new delay-dependent asymptotic stability criteria are derived to guaran tee the stability of the delayed neural networks in [26].

In this paper, the problem of stability criteria of neural networks (NNs) with two-additive time-varying delay compenents is investigated. The relationship between the time-varying delay and its lower and upper bounds is taken into account.When estimating the upper bound of the derivative of Lyapunov functional. As a result, some improved delay stability criteria for NNs with two-additive time-varying delay components are proposed. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

II. PROBLEM FORMULATION AND SOME PRELIMINARES

Consider the following delayed neural networks with two-additive time-varying delays:

$$\dot{y}(t) = -Ay(t)+Bg(y(t))+Dg(y(t-d_1(t)-d_2(t)))+u \quad (1)$$

where $y(\cdot) = [y_1(\cdot), y_2(\cdot), ..., y_n(\cdot)]^T \in \mathscr{R}^n$ is the neuron state vector, $g(y(\cdot)) = [g_1(y(t)), g_2(y(t)), ..., g_n(y(t))]^T \in \mathscr{R}^n$ denotes the neuron activation function, and $u = [u_1, u_2, ..., u_n]^T \in \mathscr{R}^n$ is a constant input vector. $B, D \in \mathscr{R}^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. $A = diag(a_1, a_2, ..., a_n)$ with $a_i > 0$, $i = 1, 2, .., n$. $d_1(t)$ and $d_2(t)$ are two time-varying satisfying:

$$0 \le d_{11} \le d_1(t) \le d_{12}, \quad 0 \le d_{21} \le d_2(t) \le d_{22};$$
$$\dot{d_1}(t) \le \mu_1, \quad \dot{d_2}(t) \le \mu_2 \quad (2)$$

where $d_{12} \ge d_{11}$, $d_{22} \ge d_{21}$ and μ_1, μ_2 are constants. Note that d_{11}, d_{21} may not be equal to 0. We denote

$$d(t) = d_1(t) + d_2(t), \quad d_1 = d_{11} + d_{21}, \quad d_2 = d_{12} + d_{22};$$
$$\mu = \mu_1 + \mu_2, \quad h_1 = d_{12} - d_{11}, \quad h_2 = d_{22} - d_{21} \quad (3)$$

In addition, it is assumed that each neuron activation function in system (1), $g_i(\cdot)(i = 1, 2, ..., n)$ is bounded and satisfies the following condition:

$$0 \le \frac{g_i(x) - g_i(y)}{x - y} \le k_i \quad (4)$$

where k_i $(i = 1, 2, ..., n)$ are positive constants, $x, y \in \mathscr{R}$.

Note that by using the Brouwers fixed-point theorem, it can be proven that there at least exists one equilibrium

---

This work was supported by the National Basic Research Program of China(2010CB732501).

Xingyuan Qu and Shouming Zhong are with the School of Mathematics Science, University Electronic Science and Technology of China, Chengdu 611731, PR China.

Shouming Zhong is with Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu 611731, PR China.

Email address: quxingyuan114@163.com.

point for system (1). In the following, the equilibrium point $y^* = [y_1^*, y_2^*, ..., y_n^*]^T$ of system (1) is shifted to the origin by the transformation $z(\cdot) = y(\cdot) - y^*$, which converts the system to the system:

$$\dot{z}(t) = -Az(t) + Bf(z(t)) + Df(z(t - d_1(t) - d_2(t))) \quad (5)$$

where $z(\cdot) = [z_1(\cdot), z_2(\cdot), ..., z_n(\cdot)]^T$ is the state vector of the transformated system $f(z(\cdot)) = [f_1(z(\cdot)), f_2(z(\cdot)), ..., f_n(z(\cdot))]^T$ and $f_i(z_i(\cdot)) = g_i(z_i(\cdot) + y_i^*) - g_i(y_i^*)$,$(i = 1, 2, .., n)$. note that the function $f_i(\cdot)$ $(i = 1, 2, .., n)$ satisfy the following condition:

$$0 \leq \frac{f_i(z_i)}{z_i} \leq k_i, \quad i = 1, 2, ..., n \quad (6)$$

which is equivalent to

$$f_i(z_i)[f_i(z_i) - k_i z_i] \leq 0, \quad i = 1, 2, ..., n \quad (7)$$

In this paper, we will present our practically stability criteria for DNN in (6). Before giving our main result, we present the Lemmas which are employed for future derivations.

Lemma 1. ([26]) For any constant matrix $R \in \mathscr{R}^{n \times n}$,$R = R^T > 0$, scalar $d_2 > d_1 > 0$, such that the following integrations are well defined, then

$(d_2 - d_1) \int_{t-d_2}^{t-d_1} y^T(s)Ry(s)ds \geq \int_{t-d_2}^{t-d_1} y^T(s)dsR \int_{t-d_2}^{t-d_1} y(s)ds$;

$\frac{d_2^2-d_1^2}{2} \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} y^T(s)Ry(s)ds \geq \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} y^T(s)dsR \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} y(s)ds$

Lemma 2. ([27]) For any constant matrix $R \in \mathscr{R}^{n \times n}$, $R = R^T > 0$, scalar $d > 0$ and a vector-valued function $y : [t - d, t] \to \mathscr{R}^n$, the following integrations is well defined:

$-d \int_{t-d}^{t} \dot{y}^T(s)R\dot{y}(s)ds \leq \begin{bmatrix} y(t) \\ y(t-d) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} y(t) \\ y(t-d) \end{bmatrix}$

III. DELAY-DEPENDENT STABILITY CRITERIA

In this section, the following Lyapunov-Krasovskii functional is constructed:

$V(z_t) = \sum_{i=1}^{7} V_i(z_t)$
where

$$V_1(z_t) = z^T(t)Pz(t) + 2\sum_{i=1}^{n} \lambda_i \int_0^{z_i(t)} f_i(s)ds$$

$$\begin{aligned} V_2(z_t) = & \begin{bmatrix} \int_{t-d_2}^{t} z(s)ds + \int_{t-d_2}^{t-d_1} z(s)ds \\ \int_{-d_2}^{0} \int_{t+\theta}^{t} \dot{z}(s)dsd\theta + \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} \dot{z}(s)dsd\theta \end{bmatrix}^T \\ & \times \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} \\ & \times \begin{bmatrix} \int_{t-d_2}^{t} z(s)ds + \int_{t-d_2}^{t-d_1} z(s)ds \\ \int_{-d_2}^{0} \int_{t+\theta}^{t} \dot{z}(s)dsd\theta + \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} \dot{z}(s)dsd\theta \end{bmatrix} \end{aligned}$$

$$\begin{aligned} V_3(z_t) = & \int_{t-d(t)}^{t} [z^T(s)Q_1z(s) + f^T(z(s))Q_2f(z(s))]ds \\ & + \int_{t-d_1(t)}^{t} z^T(s)Q_3z(s)ds + \int_{t-d_2(t)}^{t} z^T(s)Q_4z(s)ds \\ & + \int_{t-d_1}^{t} z^T(s)Q_5z(s)ds + \int_{t-d_2}^{t} z^T(s)Q_6z(s)ds \\ & + \int_{t-d_{11}}^{t} z^T(s)Q_7z(s)ds + \int_{t-d_{12}}^{t} z^T(s)Q_8z(s)ds \\ V_4(z_t) = & \int_{-d_{12}}^{0} \int_{t+\theta}^{t} \dot{z}^T(s)Z_1\dot{z}(s)dsd\theta + \int_{-d_{12}}^{-d_{11}} \int_{t+\theta}^{t} \dot{z}^T(s) \\ & Z_2\dot{z}(s)dsd\theta + \int_{-d_{22}}^{0} \int_{t+\theta}^{t} \dot{z}^T(s)Z_3\dot{z}(s)dsd\theta \\ & + \int_{-d_{22}}^{-d_{21}} \int_{t+\theta}^{t} \dot{z}^T(s)Z_4\dot{z}(s)dsd\theta \\ V_5(z_t) = & \, d_2 \int_{-d_2}^{0} \int_{t+\theta}^{t} \dot{z}^T(s)Z_5\dot{z}(s)dsd\theta \\ & + (d_2 - d_1) \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} \dot{z}^T(s)Z_6\dot{z}(s)dsd\theta \\ V_6(z_t) = & \, d_2 \int_{-d_2}^{0} \int_{t+\theta}^{t} z^T(s)Z_7z(s)dsd\theta \\ & + (d_2 - d_1) \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} z^T(s)Z_8z(s)dsd\theta \\ V_7(z_t) = & \, \frac{d_2^2}{2} \int_{-d_2}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{z}^T(s)Z_9\dot{z}(s)dsd\lambda d\theta \\ & + \frac{d_2^2 - d_1^2}{2} \int_{-d_2}^{-d_1} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{z}^T(s)Z_{10}\dot{z}(s)dsd\lambda d\theta \end{aligned} \quad (8)$$

Where $P = P^T > 0$, $Q_l = Q_l^T > 0$ $(l = 1, 2, ..., 10)$, $Z_i = Z_i^T > 0$ $(i = 1, 2, ..., 10)$, $\begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} > 0$ and $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_n) \geq 0$ are to be determined.

Theorem 1: For given scalar d_{ij} $(i = 1, 2; j = 1, 2)$, and μ_i $(i = 1, 2)$, the system described by (2), (3), (5) and (6) is global asymptotically stable if there exist symmetric positive matrices P, Q_i, Z_i $(i = 1, 2, ..., 10)$ $\begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix}$, positive diagonal matrices, $T_i = diag(t_{1i}, t_{2i}, ..., t_{ni})$, $\Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_n)(i = 1, 2)$ and any matrices $\mathcal{N}$, $\mathcal{L}$, $\mathcal{R}$, $\mathcal{S}$, $\mathcal{T}$, $\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$, P_1, P_2 with appropriate dimensions, such that the following LMIs hold:

$$\hat{\Pi}_{11} = \begin{bmatrix} \Pi & -d_{11}\mathcal{N} & -h_1\mathcal{L} & -d_{21}\mathcal{S} & -h_2\mathcal{T} \\ * & -d_{11}Z_1 & 0 & 0 & 0 \\ * & * & -h_1Z_{12} & 0 & 0 \\ * & * & * & -d_{21}Z_3 & 0 \\ * & * & * & * & -h_2Z_{34} \end{bmatrix} < 0 \quad (9)$$

$$\hat{\Pi}_{12} = \begin{bmatrix} \Pi & -d_{11}\mathcal{N} & -h_1\mathcal{L} & -d_{22}\mathcal{S} & -h_2\mathcal{U} \\ * & -d_{11}Z_1 & 0 & 0 & 0 \\ * & * & -h_1Z_{12} & 0 & 0 \\ * & * & * & -d_{22}Z_3 & 0 \\ * & * & * & * & -h_2Z_4 \end{bmatrix} < 0 \quad (10)$$

$$\hat{\Pi}_{21} = \begin{bmatrix} \Pi & -d_{12}\mathcal{N} & -h_1\mathcal{R} & -d_{21}\mathcal{S} & -h_2\mathcal{T} \\ * & -d_{12}Z_1 & 0 & 0 & 0 \\ * & * & -h_1Z_2 & 0 & 0 \\ * & * & * & -d_{21}Z_3 & 0 \\ * & * & * & * & -h_2Z_{34} \end{bmatrix} < 0 \tag{11}$$

$$\hat{\Pi}_{22} = \begin{bmatrix} \Pi & -d_{12}\mathcal{N} & -h_1\mathcal{R} & -d_{22}\mathcal{S} & -h_2\mathcal{U} \\ * & -d_{12}Z_1 & 0 & 0 & 0 \\ * & * & -h_1Z_2 & 0 & 0 \\ * & * & * & -d_{22}Z_3 & 0 \\ * & * & * & * & -h_2Z_4 \end{bmatrix} < 0 \tag{12}$$

where

$$\Pi = \begin{bmatrix} \Xi_{11} & O & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & O & O \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} + \begin{bmatrix} \Upsilon_1 + \Upsilon 1^T & \Upsilon 2 + \Upsilon 2^T \end{bmatrix}$$

$$\Xi_{11} = \begin{bmatrix} \Pi_{11} & 0 & 0 & 0 & Z_5 \\ * & \Phi_{22} & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & Z_6 \\ * & * & * & \Phi_{44} & 0 \\ ** & * & * & * & \Phi_{55} \end{bmatrix}$$

$$\Xi_{22} = \begin{bmatrix} \Pi_{66} & 0 & 0 & 0 & 0 \\ * & -Q_7 & 0 & 0 & 0 \\ * & * & -Q_8 & 0 & 0 \\ * & * & * & -Q_9 & 0 \\ ** & * & * & * & -Q_{10} \end{bmatrix}$$

$$\Xi_{33} = \begin{bmatrix} \Pi_{11,11} & \Lambda + P_2B & P_2D \\ * & \Omega_{22} & 0 \\ * & * & \Omega_{33} \end{bmatrix}$$

$$\Xi_{44} = \begin{bmatrix} -Z_7 & 0 & 0 & 0 \\ * & -Z_7 & 0 & 0 \\ * & * & -Z_7 & 0 \\ * & * & * & -Z_7 \end{bmatrix}$$

$$\Xi_{13} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & -P_1D \\ 0 & 0 & T_2\Sigma \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{14} = \begin{bmatrix} \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} \\ 0 & 0 & 0 & 0 \\ \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} \\ 0 & 0 & 0 & 0 \\ \Psi_{54} & \Psi_{55} & \Psi_{156} & \Psi_{57} \end{bmatrix}$$

$\Upsilon_1 = \begin{bmatrix} \Phi_1 & 0 & 0 & \Phi_2 & 0 & \Phi_3 & \mathcal{R} & -\mathcal{L} & \mathcal{U} \end{bmatrix}$
$\Upsilon_2 = \begin{bmatrix} -\mathcal{T} & 0 & 0 & 0 & -\mathcal{V} & -\mathcal{W} & -\mathcal{V} & -\mathcal{W} \end{bmatrix}$
$\Pi_{11} = Q_1 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8 + Q_9 + Q_{10} + d_2^2Z_7 + h_{21}^2Z_8 - Z_5 - P_1A - A^TP_1;$
$\Pi_{11,11} = d_{12}Z_1 + h_1Z_2 + d_{22}Z_3 + h_2Z_4 + d_2^2Z_5 + (d_2 - d_1)^2Z_6 + \frac{d_4^2}{4}Z_9 + \frac{(d_2^2-d_1^2)^2}{4}Z_{10} - P_2 - P_2^T;$
$\Phi_{22} = -(1-\mu)Q_1; \Phi_{33} = -Q_5 - Z_6; \Phi_{44} = -(1-\mu_1)Q_3;$
$\Phi_{55} = -Z_5 - Z_6 - Q_6; \Phi_{66} = -(1-\mu_2)Q_4;$
$\Omega_{22} = Q_2 - T_1 - T_1^T; \Omega_{33} = -(1-\mu)Q_2 - T_2 - T_2^T;$
$\Omega_{14} = \Omega_{15} = (d_2 + h_{21})M_{12}; \Omega_{16} = \Omega_{17} = (d_2 + h_{21})M_{22};$
$\Phi_1 = \mathcal{N} + \mathcal{S} + d_2\mathcal{V} + h_{21}\mathcal{W}; \Phi_2 = \mathcal{L} - \mathcal{N} - \mathcal{R};$
$\Phi_3 = \mathcal{T} - \mathcal{S} - \mathcal{U}; \sum = diag(k_1, k_2, ..., k_n);$
$\Psi_{11} = P - P_1 - A^TP_2; \Psi_{12} = P_1B + T_1\Sigma;$
$\Psi_{14} = \Psi_{15} = \Psi_{34} = \Psi_{35} = M_{11} - M_{12};$
$\Psi_{16} = \Psi_{17} = \Psi_{36} = \Psi_{37} = M_{12} - M_{22};$
$\Psi_{54} = \Psi_{55} = -M_{11} - M_{11}^T + M_{12} + M_{12}^T;$
$\Psi_{56} = \Psi_{57} = -M_{12} - M_{12}^T + M_{22} + M_{22}^T;$

Proof:Calculating the derivatives of $V_i(z_t)$ $(i = 1, 2, .., 7)$, along the trajectories of system (5) yields.

$$\dot{V}_1(z_t) = 2z^T(t)P\dot{z}(t) + 2f^T(z(t))\Lambda\dot{z}(t) \tag{13}$$

$$\begin{aligned} \dot{V}_2(z_t) = 2 &\begin{bmatrix} \int_{t-d_2}^{t} z(s)ds + \int_{t-d_2}^{t-d_1} z(s)ds \\ \int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta + \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta \end{bmatrix}^T \\ &\times \begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} \\ &\times \begin{bmatrix} z(t) + z(t-d_1) - 2z(t-d_2) \\ (d_2 + h_{21})\dot{z}(t) - (z(t) + z(t-d_1) - 2z(t-d_2)) \end{bmatrix} \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{V}_3(z_t) \le{}& z^T(t)[Q_1 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8 + Q_9 \\ &+ Q_{10}]z(t) - (1-\mu)z^T(t-d(t))Q_1z(t-d(t)) \\ &- (1-\mu_1)z^T(t-d_1(t))Q_3z(t-d_1(t)) \\ &- (1-\mu_2)z^T(t-d_2(t))Q_4z(t-d_2(t)) \\ &- z^T(t-d_1)Q_5z(t-d_1) - z^T(t-d_2)Q_6z(t-d_2) \\ &- z^T(t-d_{11})Q_7z(t-d_{11}) - z^T(t-d_{12})Q_8z(t-d_{12}) \\ &- z^T(t-d_{21})Q_9z(t-d_{21}) - z^T(t-d_{22})Q_{10}z(t-d_{22}) \\ &- (1-\mu)f^T(z(t-d(t)))Q_2f(z(t-d(t))) \\ &+ f^T(z(t))Q_2f(z(t)) \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{V}_4(z_t) ={}& \dot{z}^T(t)[d_{12}Z_1 + h_1Z_2 + d_{22}Z_3 + h_2Z_4]\dot{z}(t) \\ &- \int_{t-d_{12}}^{t-d_1(t)} \dot{z}^T(s)(Z_{12})\dot{z}(s)ds - \int_{t-d_1(t)}^{t} \dot{z}^T(s)Z_1\dot{z}(s)ds \\ &- \int_{t-d_1(t)}^{t-d_{11}} \dot{z}^T(s)Z_2\dot{z}(s)ds - \int_{t-d_{22}}^{t-d_2(t)} \dot{z}^T(s)(Z_{34})\dot{z}(s)ds \\ &- \int_{t-d_2(t)}^{t} \dot{z}^T(s)Z_3\dot{z}(s)ds - \int_{t-d_2(t)}^{t-d_{21}} \dot{z}^T(s)Z_4\dot{z}(s)ds \end{aligned}$$

Let
$M_1 = \frac{1}{d_1(t)}\int_{t-d_1(t)}^{t} \dot{z}(s)ds, M_2 = \frac{1}{d_{12}-d_1(t)}\int_{t-d_{12}}^{t-d_1(t)} \dot{z}(s)ds,$
$M_3 = \frac{1}{d_1(t)-d_{11}}\int_{t-d_1(t)}^{t-d_{11}} \dot{z}(s)ds, M_4 = \frac{1}{d_2(t)}\int_{t-d_2(t)}^{t} \dot{z}(s)ds,$
$M_5 = \frac{1}{d_{22}-d_2(t)}\int_{t-d_{22}}^{t-d_2(t)} \dot{z}(s)ds, M_6 = \frac{1}{d_2(t)-d_{21}}\int_{t-d_2(t)}^{t-d_{21}} \dot{z}(s)$
ds
Then

$$\begin{aligned} \dot{V}_4(z_t) \le{}& \dot{z}^T(t)[d_{12}Z_1 + h_1Z_2 + d_{22}Z_3 + h_2Z_4]\dot{z}(t) \\ &- (d_{12} - d_1(t))M_2^TZ_{12}M_2 - d_1(t)M_1^TZ_1M_1 \\ &- (d_1(t) - d_{11})M_3^TZ_2M_3 - (d_{22} - d_2(t))M_5^TZ_{34}M_5 \\ &- d_2(t)M_4^TZ_3M_4 - (d_2(t) - d_{21})M_6^TZ_4M_6 \end{aligned} \tag{16}$$

By the Lamma 2, we can obtain

$$\begin{aligned}\dot{V}_5(z_t) &\le d_2^2\dot{z}^T(t)Z_5\dot{z}(t) + (d_2-d_1)^2\dot{z}^T(t)Z_6\dot{z}(t)\\ &+ \begin{bmatrix} z(t) \\ z(t-d_2) \end{bmatrix}^T \begin{bmatrix} -Z_5 & Z_5 \\ * & -Z_5 \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-d_2) \end{bmatrix}\\ &+ \begin{bmatrix} z(t-d_1) \\ z(t-d_2) \end{bmatrix}^T \begin{bmatrix} -Z_6 & Z_6 \\ * & -Z_6 \end{bmatrix} \begin{bmatrix} z(t-d_1) \\ z(t-d_2) \end{bmatrix}\end{aligned} \tag{17}$$

By the Lemma 1, we can obtain

$$\begin{aligned}\dot{V}_6(z_t) &\le d_2^2 z^T(t)Z_7z(t) + (d_2-d_1)^2 z^T(t)Z_8z(t)\\ &- \int_{t-d_2}^{t} z^T(s)dsZ_7\int_{t-d_2}^{t} z(s)ds\\ &- \int_{t-d_2}^{t-d_1} z^T(s)dsZ_8\int_{t-d_2}^{t-d_1} z(s)ds\end{aligned} \tag{18}$$

$$\begin{aligned}\dot{V}_7(z_t) &\le \frac{d_2^4}{4}\dot{z}^T(t)Z_9\dot{z}(t) + \frac{(d_2^2-d_1^2)^2}{4}\dot{z}^T(t)Z_{10}\dot{z}(t)\\ &- \int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta Z_9\int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta\\ &- \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta Z_{10}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta\end{aligned} \tag{19}$$

In addition, using the Leibniz-Newton formula, for any appropriately dimensional matrices $\mathcal{N}, \mathcal{L}, \mathcal{R}, \mathcal{S}, \mathcal{T}, \mathcal{U}, \mathcal{V}, \mathcal{W}$, the following equations are true:

$$2\zeta^T(t)\mathcal{N}[z(t) - z(t-d_1(t)) - d_1(t)M_1] = 0 \tag{20}$$

$$2\zeta^T(t)\mathcal{L}[z(t-d_1(t)) - z(t-d_{12}) - (d_{12}-d_1(t))M_2] = 0 \tag{21}$$

$$2\zeta^T(t)\mathcal{R}[z(t-d_{11}) - z(t-d_1(t)) - (d_1(t)-d_{11})M_3] = 0 \tag{22}$$

$$2\zeta^T(t)\mathcal{S}[z(t) - z(t-d_2(t)) - d_2(t)M_4] = 0 \tag{23}$$

$$2\zeta^T(t)\mathcal{T}[z(t-d_2(t)) - z(t-d_{22}) - (d_{22}-d_2(t))M_5] = 0 \tag{24}$$

$$2\zeta^T(t)\mathcal{U}[z(t-d_{21}) - z(t-d_2(t)) - (d_2(t)-d_{21})M_6] = 0 \tag{25}$$

$$2\zeta^T(t)\mathcal{V}[d_2z(t) - \int_{t-d_2}^{t} z(s)ds - \int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta] = 0 \tag{26}$$

$$2\zeta(t)^T\mathcal{W}[h_{21}z(t) - \int_{t-d_2}^{t-d_1} z(s)ds - \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}(s)ds] = 0 \tag{27}$$

$$\begin{aligned}2[z(t)^TP_1 + \dot{z}^T(t)P_2][-\dot{z}(t) - Az(t) + Bf(z(t))\\ + Df(z(t-d(t)))] = 0\end{aligned} \tag{28}$$

where $\zeta^T(t) = [z^T(t)\ z^T(t-d(t))\ z^T(t-d_1)\ z^T(t-d_1(t))\ z^T(t-d_2)\ z^T(t-d_2(t))\ z^T(t-d_{11})\ z^T(t-d_{12})\ z^T(t-d_{21})\ z^T(t-d_{22})\ \dot{z}^T(t)\ f^T(z(t))\ f^T(z(t-d(t)))\ \int_{t-d_2}^{t} z^T(s)ds\ \int_{t-d_2}^{t-d_1} z^T(s)ds\ \int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta\ \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta]$

Furthermore, there exsits positive diagonal matrices T_1, T_2, such that the following inequalities hold based on (6)

$$0 \le -2f^T(z(t))T_1f(z(t)) + 2z^T(t)T_1\Sigma f(z(t)) \tag{29}$$

$$\begin{aligned}0 \le -2f^T(z(t-d(t)))T_2f(z(t-d(t)))\\ +2z^T(t-d(t))T_2\Sigma f(z(t-d(t)))\end{aligned} \tag{30}$$

Hence, according to (8) and (13)-(30), we can obtain

$$\dot{V}(z_t) \le \xi^T(t)\widetilde{\Pi}\xi(t) \tag{31}$$

where $\xi^T(t) = [\zeta^T(t)\ M_1^T\ M_2^T\ M_3^T\ M_4^T\ M_5^T\ M_6^T]$

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\Pi}_1 & \tilde{\Pi}_2 \\ * & \tilde{\Pi}_3 \end{bmatrix}$$

$$\tilde{\Pi}_1 = \begin{bmatrix} \Pi & -d_1(t)\mathcal{N} & -(d_{12}-d_1(t))\mathcal{L} & -(d_1(t)-d_{11})\mathscr{R} \\ * & -d_1(t)Z_1 & 0 & 0 \\ * & * & -(d_{12}-d_1(t)Z_{12} & 0 \\ * & * & * & -(d_1(t)-d_{11})Z_2 \end{bmatrix}$$

$$\tilde{\Pi}_2 = \begin{bmatrix} -d_2(t)\mathcal{S} & -(d_{22}-d_2(t))\mathcal{T} & -(d_2(t)-d_{21})\mathcal{U} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Pi}_3 = \begin{bmatrix} -d_2(t)Z_3 & 0 & 0 \\ * & -(d_{22}-d_2(t))Z_{34} & 0 \\ * & * & -(d_2(t)-d_{21})Z_4 \end{bmatrix}$$

where $Z_{12} = Z_1 + Z_2$, and $Z_{34} = Z_3 + Z_4$. If $\widetilde{\Pi} < 0$, then there exists a scalar $\varepsilon > 0$, such that

$$\dot{V}(z_t) \le -\varepsilon\xi^T(t)\xi(t) \le -\varepsilon z^T(t)z(t) \tag{32}$$

According to the paper [26], we can know that when $d_1(t) \to d_{11}, d_1(t) \to d_{12}, d_2(t) \to d_{21}$ and $d_2(t) \to d_{22}$, the LMI $\widetilde{\Pi}$ are equal to (9)-(12) which are define in Theorem 1, so we can conclude that the system described by (2), (3), (5) and (6) is asymptotically stable if the LMIs (9)-(12) hold.

Remark 1: It is seen that $d_1(t)$, $d_2(t)$, $d_{12} - d_1(t)$, $d_{22} - d_2(t)$ and $d_1(t) - d_{11}$, $d_2(t) - d_{21}$ are not simple enlarged as d_{12}, d_{22}, $d_{12} - d_{11}$, $d_{22} - d_{21}$, respectively. Instead, the relationship that $d_1(t) + (d_{12} - d_1(t)) = d_{12}$, $d_2(t) + (d_{22} - d_2(t)) = d_{22}$ and $(d_1(t) - d_{11}) + (d_{12} - d_1(t)) = d_{12} - d_{11}$, $(d_2(t) - d_{21}) + (d_{22} - d_2(t)) = d_{22} - d_{21}$ are considered.

Remark 2: A novel term $V_2(z_t)$ which noted in the paper that is included in the Lyapunov functional $V(z_t)$, which plays an important role in reducing conservativeness of our results. In our paper, by taking the states $\int_{t-d_1}^{t} z(s)ds$, $\int_{t-d_2}^{t} z(s)ds$, $\int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta$ and $\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}(s)dsd\theta$ are augmented variables, the atability in Theorem 1 utilizes more information on state variables, which yield less conservation results.

Remark 3: To reduce the conservatism, the lemma 1 is used to deal with the derivative of the $\dot{V}_7(z_t)$, i.e., $-\frac{d_2^2}{2}\int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)Z_9\dot{z}(s)dsd\theta$ and $-\frac{d_2^2-d_1^2}{2}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}^T(s)Z_{10}\dot{z}(s)dsd\theta$ are bounded with $-(\int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta)^T Z_9\int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta$ and $-(\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta)^T Z_{10}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta$ and the $-\int_{-d_2}^{0}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta$, $-\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} \dot{z}^T(s)dsd\theta$ are not retained as augmented variable, not replaced by $d_2z(t)-\int_{t-d_2}^{t} z(s)ds$ and $(d_2-d_1)z(t)-\int_{t-d_2}^{t-d_1} z(s)ds$, which yield less conservative results.

The case in which only two additive time-varying components appear in the state has been considered, and the idea in this paper can be easily extended to the following systems with multiple additive delay components.

IV. EXAMPLES

In the section, a example is given to demonstrate the benefits of the proposed method. Consider the system (5) with parameters [26]:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} 0.88 & 1 \\ 1 & 1 \end{bmatrix}$$

$k_1 = 0.4, k_2 = 0.8$
$f_1(s) = 0.2(|s+1| - |s-1|)$,$f_2(s) = 0.4(|s+1| - |s-1|)$. when $d_{11} = d_{21} = 0$, $d_{12} \leq 0.8$, $d_{22} \leq 2.2236$, The global asymptotic stability of (5) is listed in Table 1. The corresponding upper bounds of d_{22} for various d_{12} derived by Theorem 1 and methods in [6], [7], [25] and [26] are listed in Table 1. It is chear that our results in this paper are significant better than those in [6], [7], [25] and [26]. On the other hand, the previous results cannot handle the case for $2.0164 \leq d_{22} \leq 2.2236$. However, it is seen that we calculated the value of d_{22} for $d_{21} = d_{11} = 0.1$ in this paper.

TABLE I
ALLOWABLE UPPER BOUND OF d_{22} FOR VARIOUS d_{12}

d_{11}	d_{21}	Method	d_{12}	0.8	1.0	1.2
0	0	[25]	d_{22}	0.8831	0.6831	0.4831
0	0	[6]	d_{22}	0.8831	0.6832	0.4843
0	0	[7]	d_{22}	1.5666	1.3668	1.1664
0	0	[26]	d_{22}	2.0164	1.8203	1.6197
0	0	Theorem 1	d_{22}	2.2236	2.0133	1.8894
0.1	0.1	Theorem 1	d_{22}	2.2477	2.1886	2.0172

V. CONCLUTION

This paper has investigated the delay-dependent stability problem for neural networks with two additive time-varying delay components. Some less conservative stability criteria have been obtained by considering the relationship between the time-varying delay and its lower and upper bounds when calculating the upper bound of the derivative of Lyapunov functional. A numerical example has been given to demonstrate the effectiveness of the presented criteria and their improvement over the existing results.

REFERENCES

[1] Z. Wang, Y. Liu, X. Liu, Exponential stabilization of a class of stochastic system with Markovian jump parameters and mode-dependent mixed time-delays, IEEE Trans. Automat. Control, 55 (2010) 1656-1662.
[2] Y. Wang, Z. Wang, J. Liang,On robust stability of stochastic genetic regulatory networks with time-delays :a delay fractioning approach, IEEE Trans.Syst. Man Cybern.Part B 40(2010)729C740.
[3] Z. Wang, Y. Wang, Y. Liu, Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time-delays, IEEE Trans.Neural Networks 21(2010)11C25.
[4] Y. Wang, Z. Wang, J. Liang, Global synchronization for delayed complex networks with randomly occurred nonlinearities and multiple stochastic disturbances, J.Phys.AMath.Theor.42(2009)(art.No.135101).
[5] Y. He, G. P. Liu, D. rees, new delay-dependent stability criteria for neural networks with time-varying delay,IEEE Trans on neural networks, VOL.18, No.1 , 2006.
[6] Y. Zhao, H. Gao, S. Mou, Asymptotic stability analysis of neural networks with successive time delay components , Neurocomputing 71(2008) 2848-2856.
[7] H. Shao, Q. Han, New delay-dependent stability criteria for neural networks with two additive time-varying delay components, IEEE Trans.Neural Networks 22(2011)812C818.
[8] J. Cao, L. Wang, Exponential stability and periodic oscillatory solution in BAM networks withdelays, IEEE Trans.Neural Networks 13(2002)457C463.
[9] S. Arik, Global asymptotic stability of a larger class of neural networks with constant time delay, Phys.Lett.A 311 (2002) 504C511.
[10] J. Cao, J. Wang, Global asymptotic and robust stability of recurrent neural networks with time delays,IEEE Trans.Circuits Syst.I 52(2005) 417C426.
[11] T. L. Liao, F. C. Wang, Global stability for cellular neural networks with time delay,IEEE Trans. Neural Netw., vol. 11, no. 6, pp. 1481C1484, Nov. 2000.
[12] J. D. Cao, L. Wang, Exponential stability and periodic oscillatory solution in BAM networks with delays,IEEE Trans. Neural Netw., vol. 13, no. 2, pp. 457C463, Mar. 2002.
[13] S. Arik, An analysis of global asymptotic stability of delayed cellular neural networks,IEEE Trans. Neural Netw., vol. 13, no. 5, pp. 1239C1242, Sep. 2002.
[14] T. Chen, L. Rong, Delay-independent stability analysis of Cohen-Grossberg neural networks, Phys. Lett. A, vol. 317, pp. 436C449, 2003.
[15] S. Arik, An analysis of exponential stability of delayed neural networks with time varying delays,Neural Netw., vol. 17, pp. 1027C1031, 2004.
[16] V. Singh, A generalized LMI-based approach to the global asymptotic stability of delayed cellular neural networks, IEEE Trans. Neural Netw., vol. 15, no. 1, pp. 223C225, Jan. 2004.
[17] V. Singh, Global robust stability of delayed neural networks: An LMI approach,IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 52, no. 1, pp. 33C36, Jan. 2005.
[18] J. Tian, X. Zhou, Improved asymptotic stability criteria for neural networks with interval time-varying delay,Expert Syst.Appl. 37(2010) 7521C7525.
[19] X. Zhu, Y. Wang, Delay-dependent exponential stability for neural networks with discrete and distributed time-varying delays, Phys.Lett. A 373(2009) 4066C4072.
[20] P. Balasubramaniam, M. SyedAli, Stability analysis of TakagiCSugeno fuzzy CohenCGrossberg BAM neural networks with discrete and distributed time- varying delays, Math.Comput. Modelling 53(2011) 151C160.
[21] C. Zheng, L. Lu, Z. Wang, New LMI based delay-dependent criterion for global asymptotic stability of cellular neural networks, Neurocomputing 72(2009) 3331C3336.
[22] J.K.Tian, X.J.Xie, New asymptotic stability criteria for neural networks with time-varying delay,Phys. Lett. A 374 (2010) 938C943.
[23] H. Li, B. Chen, Q. Zhou, S. Fang, Robust exponential stability for uncertain stochastic neural networks with discrete and distributed time-varying delays, Phys. Lett.A 372(2008) 3385C3394.
[24] L. Ma, F. Da, Mean-square exponential stability of stochastic Hopfield neural networks with time-varying discrete and distributed delays, Phys. Lett.A 373 (2009) 2154C2161.
[25] Y. He, G. P. Liu, D. Rees ,M. Wu, Stability analysis for neural networks with time-varying interval delay, IEEE Trans on neural networks , VOL.18, NO.6, 2007.
[26] J. k. Tian, S. M. Zhong, Improved delay-dependent stability criteria for neural networks with two additive time-varying delay conponents, Neurocomputing 77(2012)114-119.
[27] X. L, Output feedback H_∞ control of systems with parameter uncertainity.int J Control 1996;63:741-50.

Xingyuan Qu was born in Hubei Province, china, in 1988. She received the B.S.degree from Zhengzhou Institute of Aeronautical Industry Management in 2010.She is currently pursuing the M.S.degree with UESTC. Her research interests the stability of neural networks.

ShoumingZhong was born on November 5, 1955. He graduated from University of Electronic Science and Technology of China, majoring applied mathematics on differential equation. He is a professor of School of Mathematical Sciences, University of Electronic Science and Technology of China, on June 1997-present. He is Director of Chinese Mathematical Biology Society, the chair of Biomathematics in Sichuan, Editors of Journal of Biomathematics. He has reviewed for many Journals, such as Journal of theory and application on control, Journal of Automation, Journal of Electronics, Journal of Electronics Science. His research interest is Stability Theorem and its Application research of the Differential System, the Robustness control, Neural network and Biomathematics.