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Performance Analysis of Multiuser Diversity in Multiuser Two-Hop Decode-and-Forward Cooperative Multi-Relay Wireless Networks

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***Abstract*—**Cooperative diversity (CD) has been adopted in many communication systems because it helps in improving performance of the wireless communication systems with the help of the relays that emulate the multiple antenna terminals. This work aims to provide the derivation of the performance analysis expressions of the multiuser diversity (MUD) in the two-hop cooperative multi-relay wireless networks (TCMRNs). Considering the work analysis, we provide analytically the derivation of a closed form expression of the two most commonly used performance metrics namely, the outage probability and the symbol error probability (SEP) for the fixed decode-and-forward (FDF) protocol with MUD.

***Keywords*—**Cooperative diversity (CD), fixed decode-and-forward (FDF)**,** multiuser diversity (MUD) ,two-hop cooperative multi-relay wireless networks (TCMRN).

I. Introduction

THE wireless channel media has been and always will be suffering from the media impairments, among those, the fading phenomena which in turn, degrades the performance of the wireless communication systems. And hence, there has to be a way to mitigate for the fading effects which led to the recent considerable interest in the cooperative diversity in wireless communication networks [1].

Several schemes have been proposed as a form of the cooperative diversity namely, the amplify-and-forward (AF) and the decode-and-forward (DF). In the former, the relay amplifies the received signal and resends the amplified signal to the destination. Where in the latter the relay decodes what it receives, re-encode it and then send it to the destination. The AF scheme is easy to implement because it has low complexity compared with the DF scheme. These schemes have been extensively addressed out in the literatures [2]-[4].

Multiuser diversity (MUD) is one of the diversity schemes in the multiuser systems which characterizes the case where multiple users have a different independent channel gains from each other's and that there are instances where there is a user who has the largest channel gain among all the other users, this user will be elected to transmit its data to the destination. Recently, this scheme draws the attention of the researchers, for example in [2] the authors addressed out and evaluated the performance metrics for the case where there is one selected source, one destination and one relay. The authors also have addressed the integration between the MIMO (multiple input – multiple output systems) [7]-[8] and the MUD (multiuser diversity) [5]–[6] to exploit the spatial and the multiuser diversities.

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To the best of our knowledge, the case of multiple K accessing users, one destination and multiple relays has not been presented yet.

However, there are frameworks that discussed the integration between the MUD and the cooperative diversity. Among those, [5] presented the capacity study of large networks. Besides that, the authors have addressed the case and evaluated its performance metrics for the AF and DF with the MUD schemes.

In this paper, we extend the performance analysis work of the MUD and the cooperative diversity with single relay node to include multiple M relaying nodes and the scenario being assumed is FDF where again, the relays forwards the received signal from the chosen user to the destination.

The major contributions of this paper can be summarized as follows:

1. A derivation of a closed form formula of the outage probability is provided for TCMRNs, which shows that the outage probability is reduced by a factor $\bar{\gamma}^K$ where K is being the number of the available users.
2. A closed form expression for the SEP is provided, from which, we show that a diversity order of K is achieved for the FDF.

This paper is further organized as follows; Section II illustrates the system and channel model. Section III provides the performance analysis and provides the derivation of the tight closed form expressions of outage probability and SEP. Results and discussion are provided in section IV. Finally we concluded this paper in section V.

II. System And Channel Model

As depicted in Fig. 1, we address the case where there are multiple accessing users, multiple relay nodes and one destination where the M relays will forward the selected user k signal to the destination. The transmitting process is splitted into two time slots. What follows explains how it works:

1. Within the first available time slot, the intermediate M relaying nodes and the destination listen to data being transmitted from the user k.
2. In the second time slot, the M relays decode the received signal, re-encode it and then send the data to the destination.

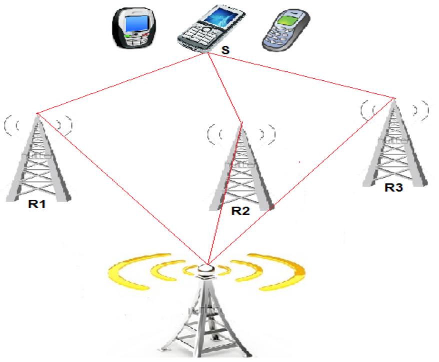

Fig. 1 MUD in TCMRN with K=3 and M=3, the lines between any two nodes represents the communication links

We assume here that the multiple access scheme is TDMA and that the channel state information (CSI) is known at the relay node via feedback channels from the destination which combine the received signals by the maximal ratio combiner (MRC).

At the destination and the m-th intermediate relaying node, the received signal at both of them from the k-th user, respectively, is shown to be:

$$y_d^k = \sqrt{E_k} h_{k,d} x_k + n_{k,d} \quad (1)$$

$$y_m^k = \sqrt{E_k} h_{k,m} x_k + n_{k,m} \quad (2)$$

wherex_k is the signal being transmitted from the k-th user, y_d^k and y_m^k are the transmitted signals from the k-th user received at both the destination and the intermediate m-th relaying node, respectively. $h_{k,d}$and$h_{k,m}$ are the channel gains from the k-th user to the base station (destination) and from the k-th user to the m-th relaying node, respectively. $n_{k,d}$and$n_{k,m}$ are the additive white Gaussian noise (AWGN) with their variances $N_{k,d}$ and $N_{k,m}$, respectively.

In the link between the relaying node and the destination, the signal received from the k-th user at the receiver of the destination is:

$$y_d^m = \sqrt{E_m} h_{m,d}^k x_m^k + n_{m,d}^k \quad (3)$$

where$h_{m,d}^k$ being the channel gain between the intermediate relaying node and the destination when relaying the k-th user's signal, $n_{m,d}^k$ is the AWGN with variance $N_{m,d}^k$, and $\tilde{n}_{m,d}^k$ is the overall AWGN with the variance $\tilde{N}_{m,d}^k$, which is given as $\tilde{N}_{m,d}^k = N_{m,d}^k + \left(E_m \left|h_{m,d}^k\right|^2 N_{k,m} / \left(E_m \left|h_{m,d}^k\right|^2 + N_{k,m}\right)\right)$.

In the maximum ratio combining (MRC) at the destination and for repetition-coded fixed DF, the received signal-to-noise ratio (SNR) at the receiver terminal of the k-th user can be expressed as:

$$\gamma_k^{FDF} = min_k \left\{ \sum_{m=1}^{M} \frac{\left|h_{k,m}\right|^2 E_k}{N_{k,m}}, \frac{\left|h_{k,d}\right|^2 E_k}{N_{k,d}} + \sum_{m=1}^{M} \frac{\left|h_{m,d}^2\right|^2 E_m}{N_{m,d}^k} \right\} \quad (4)$$

where$h_{m,d}$ denote the channel coefficient from the m-th relay to destination, E is the average transmission energy.

In this paper we consider the situation where each user has identical fading statistics, and assume that all the variances of the noise terms are equal, i.e., $N_{k,d} = N_{k,m} = N_{m,d}^k = N_0 = 1/\bar{\gamma}$, where $\bar{\gamma}$is proportional to all the transmitted SNRs. For more simplification, when the channel is exponentially distributed, we can show that at the destination, the SNR of the user-k can be expressed as:

$$\gamma_k^{FDF} = min_k \left\{ \bar{\gamma} \sum_{m=1}^{M} \beta_{km}, \bar{\gamma}\alpha_k + \bar{\gamma} \sum_{m=1}^{M} C_k \right\} \quad (5)$$

where:

$$\alpha_k = \left|h_{k,d}\right|^2 E_k,\ \beta_{km} = \left|h_{k,m}\right|^2 E_k,\ C_{km} = \left|h_{m,d}^k\right|^2 E_m$$

are the signal powers received at the destination from the k-th user, at the m-th relaying node from the k-th user, and from the intermediate relaying node to the destination, respectively.

III. Performance Analysis

In multiuser TCMRNs with MUD, and based on the CSI from the relay node and the source node, the destination Will choose the user who has the largest achievable SNR, i.e., the largest achievable SNR at the destination is $\gamma_s^{FDF} = max_k \gamma_k^{FDF}$.

Thus, the largest achievable channel capacity of MUD for FDF is given as follows [2]:

$$I_s^{FDF} = \frac{1}{2} log_2(1 + \gamma_s^{FDF}) = \frac{1}{2} log_2(1 + max_k \gamma_k^{FDF}) \quad (6)$$

To provide the performance analysis of the proposed work, i.e., the outage probability and SEP, we have to provide the cumulative density function (cdf) of γ_s^{FDF}.

Based on [2], the cdf of γ_s^{FDF} in multiuser TCMRNs with MUD, in a large average SNR systems ($\bar{\gamma} \gg 1$), can be approximated as follows:

$$F_{\gamma_s^{FDF}}(\gamma) = \left(\frac{\gamma}{\bar{\gamma}}\right)^K \prod_{k=1}^{K} \sum_{m=1}^{M} \frac{1}{\bar{\beta}_{km}} \quad (7)$$

Proof:

For the independent identically distributed RV γ_k^{FDF}, the cdf of γ_s^{FDF} can be calculated as:

$$F_{\gamma_s^{FDF}}(\gamma) = \prod_{k=1}^{K} F_{\gamma_k^{FDF}}(\gamma) \quad (8)$$

According to [9], for the fixed DF protocol, the cdf of γ_k^{FDF} can be written as:

$$F_{\gamma_k^{FDF}}(\gamma) = Pr(\gamma_k^{FDF} \leq \gamma)$$

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$$= Pr\left(\sum_{m=1}^{M}\beta_{km} \le \frac{\gamma}{\bar{\gamma}}\right) + Pr\left(\sum_{m=1}^{M}\beta_{km} \ge \frac{\gamma}{\bar{\gamma}}\right) \times Pr\left(\alpha_k + \sum_{m=1}^{M} C_{km}\right) \approx \sum_{m=1}^{M}\frac{\gamma}{\bar{\gamma}\bar{\beta}_{km}} \qquad (9)$$

Thus, taking (9) into (8) and for systems which have a large average SNR $\bar{\gamma}$, the cdf of γ_s^{FDF} for the FDF protocol is written as:

$$F_{\gamma_s^{FDF}}(\gamma) = \left(\frac{\gamma}{\bar{\gamma}}\right)^K \prod_{k=1}^{K}\sum_{m=1}^{M}\frac{1}{\bar{\beta}_{km}} \qquad (10)$$

Next, we will use the derived cdf of γ_s^{AF} to study the performance metrics namely, the outage probability and SEP of MUD in TCMRNs.

A. Outage Probability

To derive a closed form expression of the outage probability of FDF-based multiuser TCMRN with MUD, defined as the probability that the channel capacity falls below a specified target transmission rate R is calculated as:

$$P_{out}^{FDF} = Pr[I_s^{FDF} < R] = Pr[\gamma_s^{FDF} < 2^{2R} - 1] = F_{\gamma_s^{FDF}}(2^{2R} - 1) = \left(\frac{2^{2R} - 1}{\bar{\gamma}}\right)^K \prod_{k=1}^{K}\sum_{m=1}^{M}\frac{1}{\bar{\beta}_{km}} \qquad (11)$$

B. Symbol Error Probability (SEP)

In general, the conditional SEP for a given system SNR of the most commonly used signal modulation techniques, including *M*-ary quadratic-amplitude modulation (*M*-QAM), *M*-ary phase-shift keying (*M*-PSK), and *M*-ary pulse amplitude modulation (*M*-PAM), can be expressed as the uniform expression [10]

$$P_s(\gamma_s) = AQ\left(\sqrt{B\gamma_s}\right) \qquad (12)$$

where the constants A and B specify the constellations and modulation type [11] and {Q} is the Gaussian Q function, for certain SNR the average SEP for Rayleigh channel can be expressed as:

$$P_s^{FDF} = E_{\gamma_s^{FDF}}\left\{AQ\left(\sqrt{B\gamma_s^{FDF}}\right)\right\} = AE_{\gamma_s^{FDF}}\left\{\left(\sqrt{B\gamma_s^{FDF}}\right) < X\right\} = AE_{\gamma_s^{FDF}}\left\{\gamma_s^{FDF} < \frac{X^2}{B}\right\} = AE_X\left\{F_{\gamma_s^{FDF}}\left(\frac{X^2}{B}\right)\right\} \qquad (13)$$

where:

X is a normally distributed random variable.

$E_{\gamma_s^{FDF}}\{\cdot\}$denotes the expectation operator over the distribution of γ_s^{FDF}.

Using (7), (13) can be asymptotically approximated as:

$$P_s^{FDF} \approx A\int_0^\infty \left(\frac{x^2}{B\bar{\gamma}}\right)^K \prod_{k=1}^{K}\sum_{m=1}^{M}\frac{1}{\bar{\beta}_{km}}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx = \frac{A}{\sqrt{2\pi}(B\bar{\gamma})^K}\prod_{k=1}^{K}\sum_{m=1}^{M}\frac{1}{\bar{\beta}_{km}}\int_0^\infty x^{2K}e^{-\frac{x^2}{2}}dx = \frac{A}{(B\bar{\gamma})^K}\frac{[2K-1]!}{2^K[K-1]!}\times\prod_{k=1}^{K}\sum_{m=1}^{M}\frac{1}{\bar{\beta}_{km}} \qquad (14)$$

in which we use [12]

$$\int_0^\infty x^{2n}e^{-px^2}dx = \frac{\sqrt{(\pi/p)}(2n-1)!}{2^{2n}(n-1)!\,p^n}$$

IV. RESULTS AND DISCUSSION

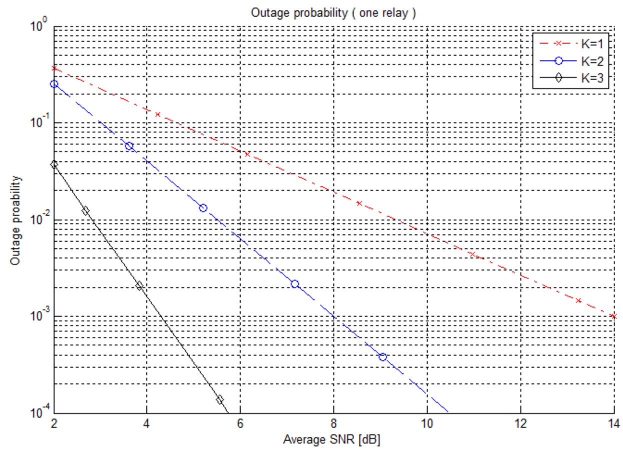

Fig. 2 Outage probability vs. SNR (1 relay)

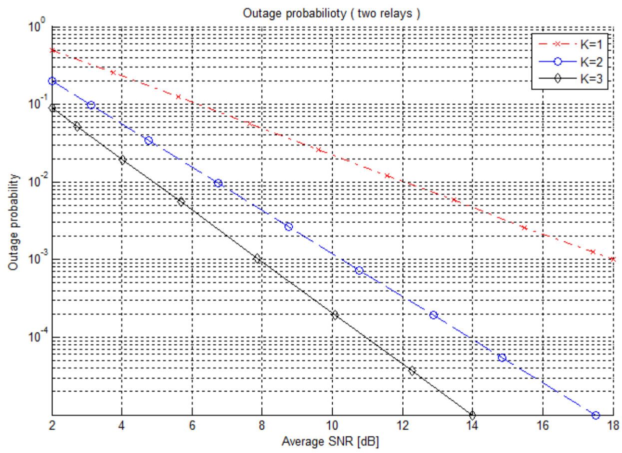

Fig. 3 Outage probability vs. SNR (2 relays)

We present some results based on the analysis provided in the previous section. Fig. 2 shows the outage probability for one relay and different number of users K = 1, 2 and 3 respectively, as shown in the figure, P_{out} decreases rapidly as the number of users increases. For example, Given that the outage probability is 10^{-3}, when we upgrade k from 2 to 3, we would approximately have more than 2.5 dB in the performance enhancement which confirms the principle of multiuser diversity in cooperative relays.

Fig. 3 shows P_{out} in case of two relays and different number of users. From the figure we can conclude that the system of two relays provide better performance than the one with one relay.

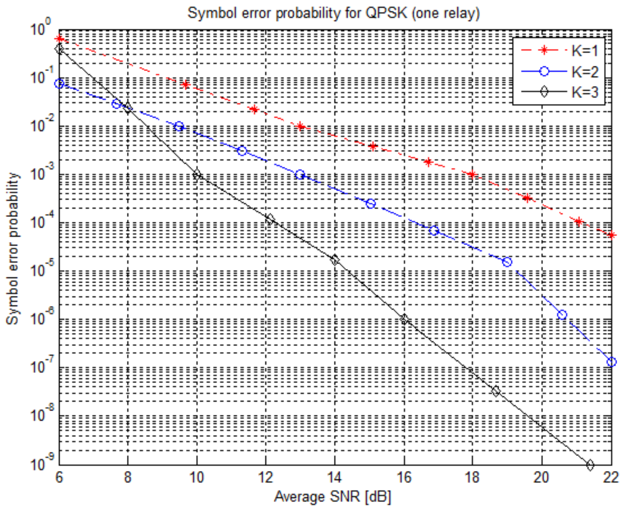

Fig. 4 SEP vs. SNR for one relay

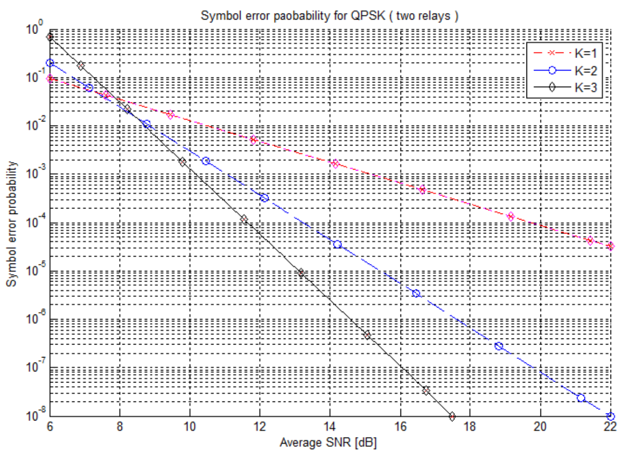

Fig. 5 SEP vs. SNR for two relays

Fig. 4 and Fig. 5 show the SEP performance of (QPSK) for different users using one relay and two relays respectively. From the two figures and in multiuser TCMRNs, when we upgrade the number of users, the system will have a much better performance in terms of the SEP, for example, for one relay (Fig. 4), and given that the SEP is 10^{-3}, we would gain a performance enhancement of 3.5 dB for K = 2 over that of K = 1 and 1.5 dB for K = 3 over that of K = 2. Meanwhile, the relative performance gain decreases with the increase of the number of users, that is, increasing the number of users cannot infinitely decrease the SEP.

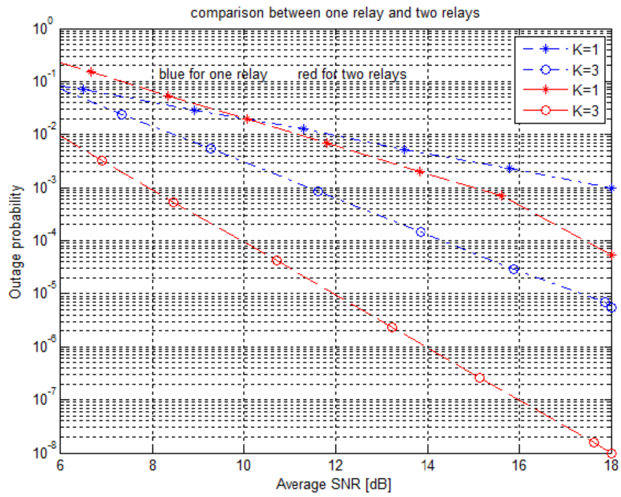

Fig. 6 Outage probability vs. SNR for different number of relays

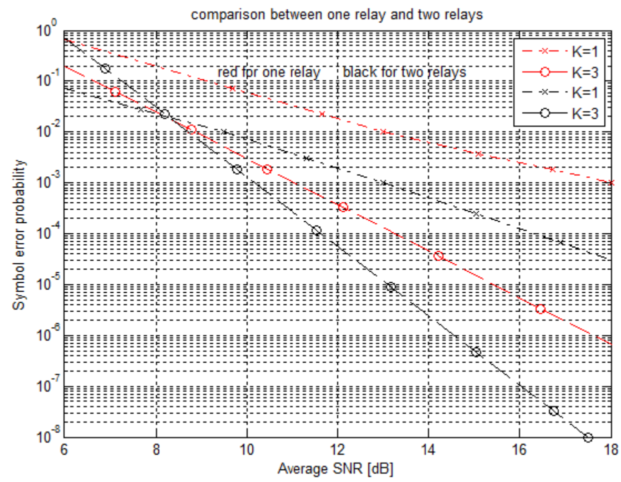

Fig. 7 SEP vs. SNR for different number of relays

To explain more the effect of the number of relays, Fig. 6 and Fig. 7 show the outage probability and SEP respectively, for one relay and two relays plotted on the same figures for k = 1 and 3. From the two figures we can see that the system with two relays provide better performance than the one with one relay. For example, for three users and for outage probability = 10^{-3} when we use two relays we will have a performance gain of 3.5 dB gain over that with only one relay. Also, we conclude that in TCMRNs, the number of the intermediate relaying nodes has a great impact on the system performance in terms of SEP, for example, with one user (Fig. 7), given that the SEP is 10^{-3}, there would be an enhancement in the system performance of more than 2 dB for two relays (M = 2) over that of one relay (M = 1), whereas, for three users (K = 3), we see that there is a performance enhancement of 1 dB for two relays (M = 2) over that of one relay (M = 1).

Meanwhile, the relative performance gain decreases with the increase of the number of relays, that is, increasing the number of relays cannot infinitely decrease the SEP.

V. CONCLUSION

We have derived tight close form expressions of the outage probability and the symbol error probability for MUD in multiuser cooperative multi-relay system. We have k diversity order (k number of users) and (M+1) cooperative diversity (M available relays and one direct path), these assumptions and derivations are valid for Rayleigh channels and for high SNR. From the results one can see that the system with multi-relays provide better performance than the one with only one relay.

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