# An Approximate Engineering Method for Aerodynamic Heating Solution around Blunt Body Nose 

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#### Abstract

This paper is devoted to predict laminar and turbulent heating rates around blunt re-entry spacecraft at hypersonic conditions. Heating calculation of a hypersonic body is normally performed during the critical part of its flight trajectory. The procedure is of an inverse method, where a shock wave is assumed, and the body shape that supports this shock, as well as the flowfield between the shock and body, are calculated. For simplicity the normal momentum equation is replaced with a second order pressure relation; this simplification significantly reduces computation time. The geometries specified in this research, are parabola and ellipsoids which may have conical after bodies. An excellent agreement is observed between the results obtained in this paper and those calculated by others' research. Since this method is much faster than Navier-Stokes solutions, it can be used in preliminary design, parametric study of hypersonic vehicles.


Keywords-Aerodynamic Heating, Blunt Body, Hypersonic Flow, Laminar, Turbulent.

## I. Introduction

T'HE preliminary thermal design of hypersonic vehicles requires precisely and reliably predicting the convective heating over the blunt nose of the vehicle, since aerodynamic heating is a function of $V_{\infty}^{3}$, its consideration in hypersonic flights is more important than aerodynamic forces. Therefore in high speeds the first concern is the large amount of heat production on the surface of vehicle which highly increases the shell temperature.

Methods of solving aerodynamic heating can be the result obtained by numerically solving the Navier- Stokes (NS) equations [1-3], or one of their various subsets such as the parabolized Navier-Stokes (PNS) equations [4-5] and viscous shock layer (VSL) equations [6-9] for the flow field surrounding the vehicle. But these methods require a large computer run times and storage, and are not generally applicable to parametric studies or preliminary design calculations.

Two of the simpler engineering aerodynamic heating methods that are currently used are AROHEAT [10], and

[^0]INCHES [11]. AEROHEAT calculates approximate surface streamlines based on solely on the body geometry, and assumes modified Newtonian theory which is inaccurate for slender bodies. INCHES used of an axisymmetric Maslen technique and an approximate expression for the scale factor in the windward and lee ward planes which describes the spreading of surface streamlines. Another approximate solution for convection heating rate is the LATCH (Langley Approximate Three dimensional Convective Heating) code [12] which provides reasonably accurate heating rates over most of the re-entry vehicles.

In this paper an approximate method has been developed for solving the inverse problem of inviscid perfect gas flow over blunt bodies at zero angle of attack. The procedure is of an inverse nature, that is, a shock wave is assumed and calculations proceeded between the shock and body, until the calculation of all properties after shock. For simplicity, the normal momentum equation is replaced with a Maslen second order pressure equation [13]; this simplification significantly reduces machine computation time and cause resolve simplicity. On the other hand, used to the commercial CFD code for flow field solution. The numerical simulations are able to generate large amount of data that describes the fluid flow, using information from each node and every cell of the grid that models the fluid domain. So, since the grid is sufficiently dense, unstructured mesh, the flow can be reconstructed and the flow features extracted and analyzed. To predict laminar and turbulent surface heating rates, Zoby's convective-heating approximate equation has been used [14]. The method provides a rapid technique such as parabola and ellipsoids.

An excellent agreement is observed between the results obtained in this paper and those calculated by others' research. Since this method is much faster than Navier-Stokes solutions, it can be used in preliminary design, parametric study of hypersonic vehicles and predict axisymmetric viscous hypersonic flows.

## II. ANALYSIS

In this paper, the procedure is of an inverse nature that is assuming a shock-wave shape. In this research, Maslen's method is an inverse method, where a shock wave is assumed and calculations proceeded along rays normal to the shock somewhere between $0<\varphi$. (Fig 1) The solution is iterated until the given body is computed. The inverse method is applied for the calculation of flow field between the shock
wave and the body surface. Calculation body with the analysis body has contrasted and according to the entire errors between them, the shape of shock with the coefficient scales has been approved.


Fig. 1 Shock shape model

The governing equations get use for calculation flow field.

Continuity:

$$
\begin{equation*}
\left(\frac{\partial\left(r^{n} \rho u\right)}{\partial x}\right)_{y}+\left(\frac{\partial\left(r^{n} \rho v\right)}{\partial y}\right)_{x}=0 \tag{1}
\end{equation*}
$$

X-momentum:

$$
\begin{equation*}
u\left(\frac{\partial u}{\partial x}\right)_{y}+v\left(\frac{\partial u}{\partial y}\right)_{x}-\frac{u v}{R_{s}}=-\frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right)_{y} \tag{2}
\end{equation*}
$$

Y-momentum:

$$
\begin{equation*}
u\left(\frac{\partial u}{\partial x}\right)_{y}+v\left(\frac{\partial u}{\partial y}\right)_{x}-\frac{u v}{R_{s}}=-\frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right)_{y} \tag{3}
\end{equation*}
$$

Entropy:

$$
\begin{equation*}
\left(\frac{\partial\left(\mathrm{r}^{\mathrm{n}} \mathrm{pu}\right)}{\partial \mathrm{x}}\right)_{\mathrm{y}}+\left(\frac{\partial\left(\mathrm{r}^{\mathrm{n}} \mathrm{pv}\right)}{\partial \mathrm{y}}\right)_{\mathrm{x}}=0 \tag{4}
\end{equation*}
$$

Assume that the shock layer is thin, and hence the streamlines are essentially parallel to the shock wave, here is the momentum equation normal to a streamline

$$
\begin{equation*}
\frac{\mathrm{pu}^{2}}{\mathrm{R}_{\mathrm{s}}}=\frac{\partial \mathrm{p}}{\partial \mathrm{y}} \tag{5}
\end{equation*}
$$

Define a stream function $\psi$, where $R_{s}$ is the shock radius of curvature and replace stream function in equation (5) [introduce a von Mises transformation such that the independent variables are $((x, \psi)$ rather than $(x, y)]$

$$
\begin{equation*}
\frac{\mathrm{pu}^{2}}{\mathrm{R}_{\mathrm{s}}}=\frac{\partial \mathrm{p}}{\partial \psi}(\rho \mathrm{u}) \tag{6}
\end{equation*}
$$

Or

$$
\begin{equation*}
\frac{\mathrm{u}}{\mathrm{R}_{\mathrm{s}}}=\frac{\partial \mathrm{p}}{\partial \psi} \tag{7}
\end{equation*}
$$

Where all the streamlines are essentially parallel to shock, the velocity just behind the shock, $u \approx u_{s}$. Thus equation (7) becomes

$$
\begin{equation*}
\frac{\mathrm{u}_{\mathrm{s}}}{\mathrm{R}_{\mathrm{s}}}=\frac{\partial \mathrm{p}}{\partial \psi} \tag{8}
\end{equation*}
$$

Integration equation 8 between appoint in the shock layer and just behind shock wave have

$$
\begin{equation*}
\mathrm{p}(\psi)=\mathrm{p}_{\mathrm{s}}+\frac{\mathrm{u}_{\mathrm{s}}}{\mathrm{R}_{\mathrm{s}} \mathrm{r}_{s}^{n}}\left(\psi-\psi_{\mathrm{s}}\right) \tag{9}
\end{equation*}
$$

## III. Aerodynamic Heating Calculations

The zoby's approximate convective heating equations are obtained from the integral form of the axisymmetric boundary layer momentum equation. The equations for laminar flow are as follows [14]
$q_{w l}=0.22\left(R_{e_{\theta l}}\right)^{-1}\left(\frac{\rho^{*}}{\rho_{e}}\right)\left(\frac{\mu^{*}}{\mu_{e}}\right) \rho_{e} V_{e}\left(H_{a w}-H_{w}\right)\left(P r_{w}\right)^{-0.6}$
The variable of $\mathrm{p}^{*}$ and $\mu^{*}$ are introduced to consider the compressibility effects. These variables are evaluated at the Eckert's reference enthalpy [15] defined as:

$$
\begin{equation*}
H^{*}=0.50 H_{e}+0.50 H_{w}+0.22 R \frac{V_{e}^{2}}{2} \tag{11}
\end{equation*}
$$

The laminar boundary layer momentum thickness is:

$$
\begin{equation*}
\theta_{L}=\frac{0.644\left[\int_{0}^{S} \rho^{*} \mu^{*} V_{e} r^{2} d s\right]^{1 / 2}}{\rho_{e} V_{e} r} \tag{12}
\end{equation*}
$$

The adiabatic wall enthalpy is defined as:

$$
\begin{equation*}
H_{a w}=H_{e}+0.5 R V_{e}^{2} \tag{13}
\end{equation*}
$$

Similar equations are developed for the turbulent flow [14]:

$$
\begin{equation*}
q_{w T}=C_{1}\left(R_{e_{\theta T}}\right)^{-m}\left(\frac{\rho^{*}}{\rho_{e}}\right)\left(\frac{\mu^{*}}{\mu_{e}}\right)^{m} \rho_{e} V_{e}\left(H_{a w}-H_{w}\right)\left(P r_{w}\right)^{-04} \tag{14}
\end{equation*}
$$

$$
\begin{equation*}
\theta_{T}=\frac{c_{1}\left[\int_{0}^{s} \rho^{*} \mu^{* m} V_{e} r r^{c 2} d s\right]^{c 4}}{\rho_{e} V_{e} r} \tag{15}
\end{equation*}
$$

The coefficients of $m, C_{1}, C_{2}, C_{3}$ and $C_{4}$ are functions of $N$ and experimental results show that N would be a function of $\mathrm{Re}_{\theta \mathrm{\theta}}$, and are given by:

$$
\begin{aligned}
m & =\frac{2}{N+1}, \quad C_{1}=\left(\frac{1}{C_{5}}\right)^{\frac{2 N}{N+1}}\left[\frac{N}{(N+1)(N+2)^{2}}\right]^{m} \\
c_{2} & =(1+m) c_{1}, \\
C_{4} & =\frac{1}{C_{3}} \quad C_{3}=1+m \\
N & =12.67-6.5 \log \left(\operatorname{Re} \theta_{T}\right)+1.21\left[\log \left(\operatorname{Re} \theta_{T}\right)\right]^{2}
\end{aligned}
$$

The heating rates can be calculated on the surface of body, from either equation 10 or equation 14 . The values of pressure, temperature, density and the velocity magnitude at the edge of the boundary (i.e. $\mathrm{P}_{\mathrm{e}}, \mathrm{T}_{\mathrm{e}}, \rho_{\mathrm{e}}$ and $\mathrm{V}_{\mathrm{e}}$ ) are calculated from the present method and commercial CFD codes.

## IV. Results

The numerical results of this method are compared with other numerical methods results and experimental results. The shock shape over a parabola blunted nose for $\mathrm{M}_{\infty}=10$ and $\gamma=1.4$ are shown in Fig 2. The results obtained from the present approach are compared with those of ref.16. Good agreement between the results of the present method and experimental data is shown in Fig 2. The pressure distribution over spherically blunted nose is shown this Fig 3. These results are compared with pressure distribution ref 16 and ref. 17. Also compares between the aero heating results of the present method, commercial CFD code data and results ref. 18 is shown in Fig 4 (within 10\%) The Mach number and temperature Contours are shown in Fig 5 and Fig 6.


Fig. 2 Shock shape


Fig. 3 Pressure distribution on parabola at $\mathrm{M} \propto=10$ and $\gamma=1.4$


Fig. 4 Convection heating distribution parabola at TW=1256


Fig. 5 Contours of Mach number for parabola at $\mathrm{M} \propto=10$ and $\gamma=1.66$


Fig. 6 Contours of Temperature for parabola at $\mathrm{M} \propto=10$ and $\gamma=1.66$

In Fig. 7, the shock shape over an elliptic blunted nose for $\mathrm{M}_{\infty}=8.08$ and $\gamma=1.4$ is shown. The pressure distribution is shown in Fig. 8.

The convection heating rate over a ellipsoid blunted nose for $\mathrm{M}_{\infty}=8.08$ and $\gamma=1.4$ are shown in Fig 9. These results are compared with results ref 20 and 21 . In this comparison, there is an error of less than $10 \%$ noted. Good agreement between the results of the present method and commercial CFD code data is shown in Fig 9 (less than $8 \%$ disagreement), the maximum qst in Fig 4 is 49343.44(MW/m2).

The commercial CFD code output results are shown in fig 10 and fig 11. These figures are related to contours of Mach and temperature.


Fig. 7 Shock shape


Fig. 8 Pressure distribution on ellipsoid at $\mathrm{M} \propto=8.08$ and $\gamma=1.4$


Fig. 9 Convection heating distribution ellipsoid at TW=1586.89


Fig. 10 Contours of Mach number for ellipsoid at $M \propto=8.08$ and $\gamma=1.4$


Fig. 11 Contours of Temperature for ellipsoid at $\mathrm{M} \propto=8.08$ and $\gamma=1.4$

## V. Conclusion

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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