# Design of a Three Phase Active Power Filter with Sliding Mode Control and Energy Feedback

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Abstract—Nonlinear and unbalance loads in three phase networks create harmonics and losses. Active and passive filters are used for elimination or reduction of these effects. Passive filters have some limitations. For example, they are designed only for a specific frequency and they may cause to resonance in the network at the point of common coupling. The other drawback of a passive filter is that the sizes of required elements are normally large. The active filter can improve some of limitations of passive filter for example; they can eliminate more than one harmonic and don't cause resonance in the network. In this paper inverter analysis have been done simultaneously in three phase and the RL impedance of the line have been considered. A sliding mode control based on energy feedback of capacitors is employed in the design with this method, the dynamic speed of the filter is improved effectively and harmonics and load unbalance is compensating quickly.

**Abstract**—Shunt active filter, harmonic, inverter, sliding mode control, energy feedback.

## I. INTRODUCTION

CONNECTION of a nonlinear load to the network, cause to appears of current and voltage harmonics and finally increase the power losses. By using a current source in parallel with the load, required harmonic current can be provided for the nonlinear load. Shunt active filters with PWM inverters are normally used for this purpose. A DC bus capacitor is used with inverter that is charged with anti parallel diodes as shown in Fig. 1.

In this paper, various definitions of Electric power are first discussed and a model based on dq-axis is derived for power compensation. In order to take the charging of capacitors by anti- parallel diodes into account, three-phase model of the network is considered. Also a sliding mode inverter with energy feedback is employed to track reference current with fast dynamics.

## II. DEFINITION OF ELECTRIC POWER

There are various definitions for non-sinusoidal transient condition, both in time and frequency domain [1, 2]. The most important definition in time and frequency domain is Fryze and Budeanu respectively [1]. Time domain base control algorithms are faster than frequency domain and in this paper we use time domain.

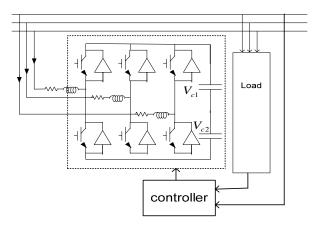


Fig. 1 Shunt active power filter

#### A. Frequency Domain Definition of Electric Power

For the first time the power definition in frequency domain introduced by Budeanu. This definition works in steady state and uses phasor notation for each frequency. RMS voltage:

$$V = \sqrt{\frac{1}{T} \int v^2 dt} = \sqrt{\sum V_n^2}$$
 (1)

Where  $V_n$  is the RMS value of n-th harmonic voltage and the equation for RMS current also is similar to above equation.

The apparent power, active and reactive powers defined as:

$$S' = VI$$

$$P = \sum p_n = \sum V_n I_n \cos \varphi_n$$

$$Q = \sum q_n = \sum V_n I_n \sin \varphi_n$$
(2)

In this method, the reciprocal effects of harmonics are not considered and for balancing of powers, the harmonic power defined as:

$$D^2 = \sqrt{S^2 - p^2 - q^2} \tag{3}$$

It was clear that using of this method for active or passive filters was not appropriate because the effect of voltage and current harmonics with each other is not considered.

#### B. Time Domain Definition of Electric Power

For the first time the power definition in time domain introduced by Fryze. The apparent power, active and reactive powers defined as:

$$S' = VI$$

$$P = \frac{1}{T} \int v(t) i(t) dt = VI_{w}$$

$$Q = \sqrt{S'^{2} - P^{2}}$$
(4)

Reactive power is a part of instantaneous power which does not contribute to active power. Czarnecki improved this definition by dividing the reactive power into four different parts based on their physical origins. Active and reactive power definitions in Frize and budeanu are the same. It must be considered that in Fryze definition, if the power factor is equal one in sinusoidal case, then the voltage and current will be in phase and the power transition will be optimal. However, it is not correct in non-sinusoidal cases.

#### C. Definition of Electric Power in Three-Phase System

In single-phase system, the instantaneous power is equal to production of instantaneous voltage and current. Also the active power is equal to average power and finally the reactive power is equal to the part of the instantaneous power which does not contribute to active power. The instantaneous active three-phase power is defined as:

$$p(t) = v_a(t)i_a(t) + v_b(t)i_b(t) + v_c(t)i_c(t)$$
 (5)

The average of p(t) (P) is equal to active power which transmit from source to load and p(t) minus average power is equal to oscillatory power ( $\widetilde{P}$ ) which oscillate between source and load and its average is equal to zero. The three phase reactive power is part of electric power, which does not contribute to three phase instantaneous active power. In fact, reactive power circulates between three phases of system.

For calculation of active and reactive power component, it is required to divide each phase current into active and reactive components and minimize the Lagrange function of them.

$$i_p = i_{wp} + i_{\mu p}$$
  $p = a,b,c$   

$$L = (i_a - i_{\mu p})^2 + (i_b - i_{\mu b})^2 + (i_c - i_{\mu c})^2$$
(6)

With the constraint of:

$$\Phi(i_{\mu a}, i_{\mu b}, i_{\mu c}) = v_a i_{\mu a} + v_b i_{\mu b} + v_c i_{\mu c} = 0 \tag{7}$$

Finally, we have:

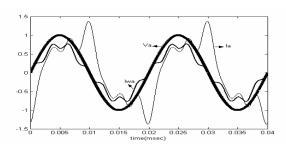
$$\begin{bmatrix} i_{\mu a} \\ i_{\mu b} \\ i_{\mu c} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{p_{3\varphi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$
(8)

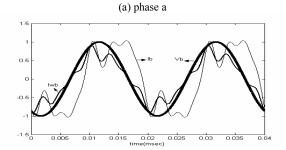
$$\begin{bmatrix} i_{wa} \\ i_{\mu b} \\ i_{\mu c} \end{bmatrix} = \frac{p_{3\varphi}}{v_a^2 + v_b^2 + v_c^2} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}$$
(9)

We can rewrite above equation as:

$$I_{ob} = G_e v_k \quad ; \quad K = (a, b, c) \tag{10}$$

The simulation results for active and reactive three phase currents with a sinusoidal voltage with amplitude equal to 1pu and non-sinusoidal current with  $1^{st}$ ,  $3^{rd}$ ,  $5^{th}$  and  $7^{th}$  harmonic with amplitudes equal to inverse of harmonic order (1, 1/3, 1/5,and 1/7 pu respectively) is shown in Fig. 2. The instantaneous power in two cases (with and without compensation) is shown in Fig. 3 [1-2].





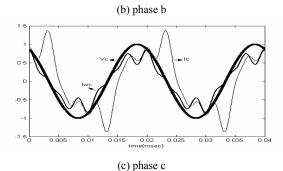


Fig. 2 Phase voltage and current and active component of current (compensated)

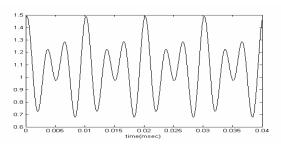


Fig. 3 Instantaneous load power (pu):

$$p(t) = v_a i_a + v_b i_b + v_c i_c = v_a i_w + v_b i_{wb} + v_c i_{wc}$$

## D. Instantaneous Power in dqo-Axis

The voltage and current in dq0 axis can be obtained from abc axis by Clark transformation:

$$\begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-1}{2} \\ 1 & \frac{-\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$
(11)

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix}$$
(12)

Phase voltages in dq0 axis also can be obtained by similar equation. Instantaneous power is equal to

$$P = 1.5(v_d i_d + v_q i_q)$$

$$q = 1.5(v_d i_q - v_q i_d)$$

$$p_0 = 3v_0 i_0$$
(13)

Also we have:

$$p_{3\varphi} = v_{a}i_{a} + v_{b}i_{b} + v_{c}i_{c}$$

$$= 1.5(v_{d}i_{d} + v_{q}i_{q} + 2v_{o}i_{o}) = p + p_{o}$$

$$q_{3\varphi} = 1.5(v_{d}i_{q} - v_{q}i_{d})$$

$$= \frac{-1}{\sqrt{3}} \{ (v_{a} - v_{b})i_{c} + (v_{b} - v_{c})i_{a} + (v_{c} - v_{a})i_{b} \}$$
(14)

We can rewrite the above equation as:

$$\begin{bmatrix} p \\ q \\ p_o \end{bmatrix} = 1.5 \begin{bmatrix} v_q & v_d & 0 \\ v_d & -v_q & 0 \\ 0 & 0 & 2v_o \end{bmatrix} \begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix}$$
 (15)

One of the benefits of (15) is that we can compensate the p, q and p0 individually. If we want to compensate  $\widetilde{p}$ , q,  $p_0$  then:

$$\begin{bmatrix} i_{cq} \\ i_{cd} \end{bmatrix} = \frac{1}{v_d^2 + v_a^2} \begin{bmatrix} v_q & v_d \\ v_d & -v_q \end{bmatrix} \begin{bmatrix} \tilde{p} - p - p_0 \\ -q \end{bmatrix}$$
 (16)

With inverse Clark transformation, we can find reference currents for active filter.

$$\begin{bmatrix} i_{ca} \\ i_{cb} \\ i_{cc} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{cq} \\ i_{cd} \\ 0 \end{bmatrix}$$
 (17)

#### III. DESIGN OF ACTIVE POWER FILTER

The three phase active filter shown in Fig. 1 contains two parts:

- 1. Power circuit (PWM inverter)
- 2. Control circuit

Current source or voltage source inverters can be used as power circuit. In this paper, voltage source inverter with current control is employed. Using split capacitors in DC side of inverter, the zero sequence of current can be compensated. Also if the voltages are not symmetrical, we can find the fundamental positive sequence voltage by using a PLL and then find the reference currents by equations (16) and (17) [4]. The control circuit contains an external sliding mode control loop for inverter control and an internal energy feedback loop for DC voltage regulation [5].

## A. Power Circuit (PWM Inverter)

It is required that the inverter three phase currents  $(i_{fa},i_{fb},i_{fc})$ , track the reference currents  $(i_{ca},i_{cb},i_{cc})$ , with fast dynamic response. One of the inverter legs is shown in Fig. 4. By closing the switch T1, if compensating current is negative  $(i_{fa}<0)$ , C1 will discharge  $(di_{fa}/dt<0)$  and if the current is positive  $(i_{fa}>0)$ , C1 will charge  $(di_{fa}/dt>0)$ . Also By closing the switch T4, if compensating current is positive  $(i_{fa}>0)$ , C2 will discharge  $(di_{fa}/dt<0)$  and if the current is negative  $(i_{fa}<0)$ , C2 will charge  $(di_{fa}/dt>0)$ .

Taking the voltages of capacitors and three phase currents as state variables, the effects of three phases on charge and discharge of capacitors can be taken into account. ( $U_a, U_b, U_c$ ) are the control variables of three phase switches. If a control variable is equal to -1, the upper switch of the respective phase is closed. If it is equal to 1 the lower switch of the respective phase, conducts. If the control variable is equal to zero both switches are open.

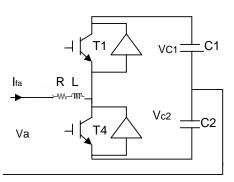


Fig. 4 One inverter leg with DC side capacitors

If only one of the upper switches conduct, for example T1, then we have:

$$u_{a} = -1$$

$$v_{a} - v_{c1} = L_{f} \frac{di_{fa}}{dt}$$

$$i_{fa} - c_{1} \frac{dv_{c1}}{dt} = 0$$

$$(18)$$

If all upper switches conduct:

$$u_{a} = u_{b} = u_{c} = -1$$

$$i_{fa} + i_{fb} + i_{fc} - c_{1} \frac{dv_{c1}}{dt} = 0$$

$$c_{2} \frac{dv_{c2}}{dt} = 0$$
(19)

And if all lower switches conduct:

$$u_{a} = u_{b} = u_{c} = 1$$

$$-c_{1} \frac{dv_{c1}}{dt} = 0$$

$$i_{fa} + i_{fb} + i_{fc} + c_{2} \frac{dv_{c2}}{dt} = 0$$
(20)

Now as an example, if the lower switch of phase a. and upper switch of phase b. and phase c. are on, we have:

$$u_{a} = 1, u_{b} = u_{c} = -1$$

$$i_{fc} - c_{1} \frac{dvc_{1}}{dt} = 0$$

$$i_{fb} + i_{fc} + c_{2} \frac{dvc_{2}}{dt} = 0$$
(21)

State variables are defined as:

$$X_{1} = i_{fa}$$
 $X_{2} = i_{fb}$ 
 $X_{3} = i_{fc}$ 
 $X_{4} = v_{c1}$ 
 $X_{5} = v_{c2}$ 
(22)

If the upper switch of phase, a. is on  $(u_a = -1)$ :

$$\dot{X}_1 = -\frac{R}{L}X_1 - \frac{1}{L}X_4 + \frac{v_a}{L} \tag{23}$$

If the lower switch is on  $(u_a = 1)$ :

$$\dot{X}_1 = \frac{v_a}{L} - \frac{R}{L}X_1 + \frac{1}{L}X_5 \tag{24}$$

Similar expression can be written for other phases. For phase b if:

$$u_b = -1$$

$$\dot{X}_2 = \frac{-R}{I} X_2 - \frac{1}{I} X_4 + \frac{v_b}{I}$$
(25)

And if:

$$u_b = 1$$

$$\dot{X}_2 = \frac{-R}{L} X_2 + \frac{1}{L} X_5 + \frac{v_b}{L}$$
(26)

And for phase c if:

$$u_{c} = -1$$

$$\dot{X}_{3} = \frac{-R}{L} X_{3} - \frac{1}{L} X_{4} + \frac{v_{c}}{L}$$
(27)

And if:

$$u_{c} = 1$$

$$X_{3} = \frac{-R}{L}X_{3} + \frac{1}{L}X_{5} + \frac{v_{c}}{L}$$
(28)

If all upper switches are on:

$$u_{a} = u_{b} = u_{c} = -1$$

$$\dot{X}_{4} = \frac{1}{C_{1}} X_{1} + \frac{1}{C_{1}} X_{2} + \frac{1}{C_{1}} X_{3}$$

$$\dot{X}_{5} = 0$$
(29)

And if all lower switches are on:

$$u_{a} = u_{b} = u_{c} = 1$$

$$X_{4} = 0$$

$$\dot{X}_{5} = -\frac{1}{C_{2}}X_{1} - \frac{1}{C_{2}}X_{2} - \frac{1}{C_{2}}X_{3}$$
(30)

We can rewrite all of the above equations in the form of equation (31) and finally (32).

$$\begin{bmatrix} \dot{X}_{1} \\ \dot{X}_{2} \\ \dot{X}_{3} \\ \dot{X}_{4} \\ \dot{X}_{5} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & 0 & 0 & \frac{-1}{2L} & \frac{1}{2L} \\ 0 & \frac{-R}{L} & 0 & \frac{-1}{2L} & \frac{-1}{2L} \\ 0 & 0 & \frac{-R}{L} & \frac{-1}{2L} & \frac{1}{2L} \\ \frac{1}{2C_{1}} & \frac{1}{2C_{1}} & \frac{1}{2C_{1}} & 0 & 0 \\ \frac{-1}{2C_{2}} & \frac{-1}{2C_{2}} & \frac{-1}{2C_{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{X}_{1} \\ \dot{X}_{2} \\ \dot{X}_{3} \\ \dot{X}_{4} \\ \dot{X}_{5} \end{bmatrix} + \begin{bmatrix} \frac{1}{2C_{1}} & \frac{1}{2C_{1}} & \frac{1}{2C_{1}} & 0 & 0 \\ -\frac{1}{2C_{2}} & \frac{-1}{2C_{2}} & -\frac{1}{2C_{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{a} \\ \dot{X}_{5} \end{bmatrix} + \begin{bmatrix} v_{a} \\ \dot{X}_{5} \\ \dot{X}_{5} \end{bmatrix} + \begin{bmatrix} \frac{1}{2L} (X_{4} + X_{5}) & 0 & 0 \\ 0 & \frac{1}{2L} (X_{4} + X_{5}) & 0 \\ 0 & 0 & \frac{1}{2L} (X_{4} + X_{5}) \end{bmatrix} \begin{bmatrix} u_{a} \\ u_{b} \\ u_{c} \end{bmatrix} + \begin{bmatrix} \frac{v_{a}}{L} \\ \frac{v_{b}}{L} \\ \frac{v_{c}}{L} \end{bmatrix} \\ \frac{1}{2C_{2}} X_{1} & \frac{1}{2C_{2}} X_{2} & \frac{1}{2C_{3}} X_{3} \end{bmatrix} \begin{bmatrix} u_{a} \\ v_{b} \\ \frac{V_{c}}{L} \end{bmatrix}$$

$$\dot{X} = AX + B(X)U + C \tag{32}$$

Equations (31) and (32) are the general form of equations derived in [1] [2] [3]. In references [1-3] single phase current is considered as the state variable, and the resistances of transformer is neglected. If we consider one of the phase currents and neglect the resistance of transformers, then we have (C1=C2=C):

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_4 \\ \dot{X}_5 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2L} & \frac{1}{2L} \\ \frac{1}{4C} & 0 & 0 \\ \frac{-1}{4C} & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_4 \\ X_5 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2L} & \frac{1}{2L} \\ \frac{-1}{4C} & 0 & 0 \\ \frac{-1}{4C} & 0 & 0 \end{bmatrix} U + \begin{bmatrix} \frac{v_a}{L} \\ 0 \\ 0 \end{bmatrix}$$
(33)

## B. Sliding Mode Control

The sliding surfaces  $(\sigma)$  are equal to the error of state variables, which can be express as:

$$\sigma = S(X - X^*) \tag{34}$$

 $X_{5\times 1}$  Is the state variable vector which is defined in (22) and  $X_{5\times 1}^*$  is the state variable reference vector, and  $\sigma = [\sigma_a \ \sigma_b \ \sigma_c]$  are the sliding surfaces and S is a design constant.

$$S = \begin{bmatrix} s_1 & 0 & 0 & \pm s_4 & \pm s_5 \\ 0 & s_2 & 0 & \pm s_6 & \pm s_7 \\ 0 & 0 & s_3 & \pm s_8 & \pm s_9 \end{bmatrix}$$
(35)

The 4<sup>th</sup> and 5<sup>th</sup> column of the above matrix is determined by the switches condition. Thus, positive sign is for positive and negative sign is for negative half cycle of AC input voltage, when connected to inverter capacitors. By replacing (35) in (34) we have:

$$\sigma_{a} = s_{1} \left( X_{1} - X_{1}^{*} \right) \pm s_{4} \left( X_{4} - X_{4}^{*} \right)^{m} \pm s_{5} \left( X_{5} - X_{5}^{*} \right)^{m}$$

$$\sigma_{b} = s_{2} \left( X_{2} - X_{2}^{*} \right) \pm s_{6} \left( X_{4} - X_{4}^{*} \right)^{m} \pm s_{7} \left( X_{5} - X_{5}^{*} \right)^{m}$$

$$\sigma_{c} = s_{3} \left( X_{3} - X_{3}^{*} \right) \pm s_{8} \left( X_{4} - X_{4}^{*} \right)^{m} \pm s_{9} \left( X_{5} - X_{5}^{*} \right)^{m}$$
(36)

m is a coefficient which, if equal to one, the state feedback [5] and if, equal to two, the energy feedback of capacitors is used. This will elaborated in the next section. Control input is

$$u = u_{eq} + u_n \tag{37}$$

 $u_n$  Is sliding surface sign function, which forces the state variable to sliding surface:  $u_n = \begin{bmatrix} -\operatorname{sgn}\sigma_a & -\operatorname{sgn}\sigma_b & -\operatorname{sgn}\sigma_c \end{bmatrix}$ 

$$u_n = \left[ -\operatorname{sgn} \sigma_a - \operatorname{sgn} \sigma_b - \operatorname{sgn} \sigma_c \right] \tag{38}$$

 $u_{eq}$  is the control input, which forces the state variables to the origin (zero error) on the sliding surface. In fact, control uwill force the state variables to track their respective references. If we are out of sliding surface,  $u_n$  will be +1 or -1 with respect to sliding surface sign. This specifies which switch in a leg must be on or off. In this condition  $u_{eq}$  will be zero because we did not reach sliding surface yet. When we reach the sliding surface,  $\sigma = 0$  and  $u_n = 0$  and controller will force the error to zero instantaneously.

Sliding surface stability constraint is:

$$(\dot{\sigma}^t \, \sigma \le \mathbf{0}) \tag{39}$$

On the other hand:

$$\dot{\sigma} = S(\dot{X} - X^*) = S(AX + B(X)U + C) - SX^*$$

$$= (SAX + SB(X)u_{eq} + SC - SX^*) + SB(X)u_n$$
(40)

With respect to  $u_{eq}$  definition:

$$u_{eq} = -(SB(x))^{-1}(SAX + SC - SX^*)$$
 (41)

The simplification of (40) is:

$$\dot{\sigma} = SB(X)u_n \tag{42}$$

From (29), (33), (40) and (37), we will have:

$$\dot{\sigma}^{t} \sigma = \left\{ -S_{1} \frac{1}{2L} (X_{4} + X_{5}) + \frac{1}{2C} X_{1} \right\} \sigma_{a} \operatorname{sgn}(\sigma_{a}) 
+ \left( \frac{-1}{2C} X_{2} \right) \sigma_{a} \operatorname{sgn}(\sigma_{b}) + \left( \frac{-1}{2C} X_{3} \right) \sigma_{a} \operatorname{sgn}(\sigma_{c}) 
+ \left\{ -S_{2} \frac{1}{2L} (X_{4} + X_{5}) + \frac{1}{2C} X_{2} \right\} \sigma_{b} \operatorname{sgn}(\sigma_{a}) 
+ \left( \frac{-1}{2C} X_{2} \right) \sigma_{b} \operatorname{sgn}(\sigma_{b}) + \left( \frac{-1}{2C} X_{3} \right) \sigma_{b} \operatorname{sgn}(\sigma_{c}) 
+ \left\{ -S_{4} \frac{1}{2L} (X_{4} + X_{5}) + \frac{1}{2C} X_{3} \right\} \sigma_{c} \operatorname{sgn}(\sigma_{b}) 
+ \left( \frac{-1}{2C} X_{2} \right) \sigma_{c} \operatorname{sgn}(\sigma_{b}) + \left( \frac{-1}{2C} X_{3} \right) \sigma_{c} \operatorname{sgn}(\sigma_{c}) \le 0$$
(43)

If three individual one-phase analyses are performed, we must finally have [3]:

$$-S_{1}(X_{4} + X_{5}) + X_{1} \le 0$$

$$X_{1} = X_{2} = X_{3}$$

$$S_{1} = S_{2} = S_{3}$$
(44)

It is clear that satisfying equation (44) does not necessarily satisfy, constraint (43), unless  $(s_1, s_2, s_3)$  are selected large enough, which will reduce the speed of dynamic response of closed loop system.

This emphasizes the importance of three phase analysis which makes it possible to find small  $(s_1, s_2, s_3)$  and consequently faster dynamic speed response is achieved. If equation (43) is satisfied, the stability is guaranteed. So if three single phase analysis are used, coefficient of  $(s_1, s_2, s_3)$  must be chosen large to achieve stability.

Considering the similarity of first term with  $2^{nd}$  and  $3^{rd}$  terms in equation (43), if we divide the first term by  $(\sigma_a \operatorname{sgn}(\sigma_a))$ , we have:

$$-S_{1} \frac{1}{2L} (X_{4} + X_{5}) + \frac{1}{2C} X_{1} - \frac{1}{2C} X_{2} \frac{\operatorname{sgn}(\sigma_{b})}{\operatorname{sgn}(\sigma_{a})}$$

$$-\frac{1}{2C} X_{3} \frac{\operatorname{sgn}(\sigma_{c})}{\operatorname{sgn}(\sigma_{a})} \leq 0$$

$$(45)$$

Because the sum of two capacitor voltages is always positive, then if we consider  $(s_1)$  to be high, then the above equation will be satisfied and the stability will be guaranteed.

C. The Control Loop for Voltage Regulation of Capacitors
In equation (36) if m=1 the control loop for voltage regulation is in fact a state feedback [5]. In this paper, we set m equal to 2 and use capacitors energy feedback loop to stabilize the capacitor voltage due to active power oscillation and zero sequence power transition.

The block diagram of control system is shown in Fig. 5. In this control system, the three phase voltages and currents will

be measured and dq0 components will be derived by Clark transformation. Then by calculation of electric power, which must be compensated, the reference currents are calculated. The reference currents and the state variables are then fed to the sliding mode controller. The voltage regulation loop also measures the capacitor voltages and compares this value with reference capacitor voltage. Thus, the required size of energy is computed and fed to the sliding mode controller through a PI controller. A high pass RC filter with cutoff frequency equal to 1000HZ was used in parallel with the active filter, which eliminates high frequency harmonics created by switching.

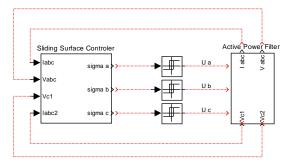


Fig. 5 Block diagram of controller

## IV. SIMULATION RESULTS

Simulation results are shown in Fig. 6 to Fig. 10. The simulation parameters are equal to:

$$\begin{split} L &= 1.5 mH, C_1 = C_2 = 1500 \mu F \\ V_S &= 220 v \quad S_1 = S_2 = S_3 = 1 \\ S_4 &= S_5 = 0.5, S_6 = S_7 = 0.5, S_8 = S_9 = 0.5 \end{split}$$

The source current THD is equal to 38.26% before compensation and after compensation the source current will be sinusoidal with THD equal to 5.35. The source current and voltage is also in phase.

In Fig. 7, the simulation result for capacitor voltage which is obtained by sliding mode control with energy feedback with three phase analysis is compared with the simulation results of conventional controller with single-phase analysis. It is clear that by using energy feedback for voltage control and three-phase analysis, the voltage regulation is better. The load current, compensating current and source current is shown in Fig. 8 in transient condition. For single-phase analysis, the source current THD is equal to 47.9% before compensation and 17.17% after compensation.

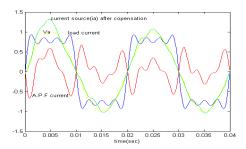


Fig. 6 Load current, compensating current, source current and phase voltage after Compensation

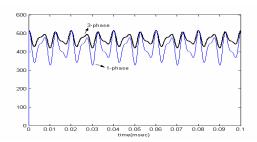


Fig. 7 Capacitors voltages in active power filter

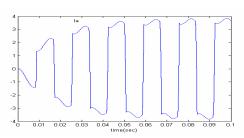


Fig. 8 (a) Single phase rectifier current

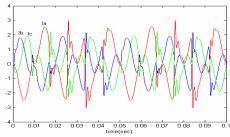


Fig. 8 (b) Three phase current of active power filter

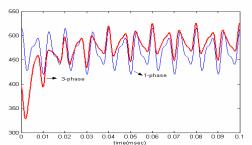


Fig. 9 Voltages capacitors of active power filter for single-phase rectifier load

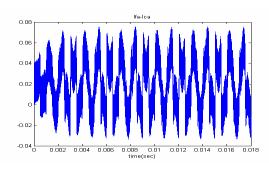


Fig. 10 Error between reference current and active power filter current

Fig. 9 represents the capacitor voltages when the three phase active filter is connected to a single-phase bridge rectifier. Dynamic response and voltage regulation will be better compared with the other methods.

The error between compensating and reference current with sliding mode control and energy feedback is shown in Fig. 10. The error approaches zero in each switching interval, very fast

# V. CONCLUSION

In this paper, it was shown that better performance can be achieved, using three phase analysis instead of three single phase analysis, and considering the resistance of transformers with energy feedback for dc voltage regulation.

Also using four-wire system and dq0 method for control, we can compensate the oscillation of active power and zero sequence power in three-phase system.

When using three-phase analysis in sliding mode control, the stability constraints will be limited. It means that by using three sliding surfaces, the stability constraints will be smaller, which means faster dynamic speed response can be achieved.

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