Fuzzy Separation Bearing Control for Mobile Robots Formation

A. Bazoula, and H. Maaref

Abstract—In this article we address the problem of mobile robot formation control. Indeed, the most work, in this domain, have studied extensively classical control for keeping a formation of mobile robots. In this work, we design an FLC (Fuzzy logic Controller) controller for separation and bearing control (SBC). Indeed, the leader mobile robot is controlled to follow an arbitrary reference path, and the follower mobile robot use the FSBC (Fuzzy Separation and Bearing Control) to keep constant relative distance and constant angle to the leader robot. The efficiency and simplicity of this control law has been proven by simulation on different situation.

Keywords—Autonomous mobile robot, Formation control, Fuzzy logic control, Multiple robots, Leader-Follower.

I. Introduction

FORMATION control of multiple autonomous mobile robots and vehicles has been studied extensively over the last decade for both theoretic research and practical applications.

Various approaches and strategies have been proposed for the formation control of multiple robots.

Multi-robot coordination methods can be partitioned into three classes approaches: virtual structure approach, behavioral approach and leader follower approach. Each of them has several advantages and weaknesses.

The virtual structure approach treats the entire formation as a single virtual rigid structure [1]-[3]. Desired motion is assigned to the virtual structure as a whole, as a result which will trace out trajectories for each robot in the formation to follow. It is easy to prescribe the behavior of the whole group, and maintain the formation very well during the maneuvers. The main disadvantage of the current virtual structure implementation is the centralization, which leads a single point of failure for the whole system.

By behavior based approach, several desired behaviors are prescribed for each robot, and the final action of each robot is derived by weighting the relative importance of each behavior. Possible behaviors include obstacle avoidance, collision avoidance, goal seeking and formation keeping [3]-[5]. The limitation of behavior-based approach is that it is difficult to analyze mathematically, therefore it is hard to guarantee a precise formation control.

In the leader following approach [7], [8], one of the robots is designated as the leader, with the rest being followers. The follower robots need to position themselves relative to the leader and to maintain a desired relative position with respect to the leader. In order, to prescribe a formation maneuver, we need only to specify the leader's motion and the desired relative positions between the leader and the followers.

When the motion of the leader is known, the desired positions (desired distance and orientation) of the followers relative to the leader can be achieved by local control law on each follower. Therefore, in a certain sense, the formation control problem can be seen as a natural extension of the traditional trajectory-tracking problem.

Among all the approaches to formation control reported in the literature, the leader-following method has been adopted by many researchers [2], [3], [6], [8], [9]. In this method, each robot takes another neighboring robot as a reference point to determine its motion. The referenced robot is called a leader, and the robot following it called a follower. Thus, there are many pairs of leaders and followers and complex formations can be achieved by controlling relative positions of these pairs of robots respectively. This approach is characterized by simplicity, reliability and no need for global knowledge and computation.

In this paper, we will develop a method based on the leader-following approach to investigate formation control problem in a group of nonholonomic mobile robots. For this purpose, we design a new controller based on fuzzy logic to drive a fleet of mobile robots in a leader-follower configuration.

The rest of this paper is organized as follows. First motion modeling is revealed. Then, the method of path following used by the leader robot is exposed. After that, the architecture of the fuzzy controller is described. We conclude the paper by some simulation and results.

II. Mobile Robot Modeling

The mobile robots considered in this study are the popular wheeled mobile robots of unicycle type, shown in Fig. 1. The configuration of the robot denoted by $q = (x, y, \theta)^T \in \Re^3$ in the Earth fixed inertial coordinate system $X - Y$, where $(x(t), y(t))$ represents the position of the mobile robot by the fixed cartesian coordinates where t is time, and the angle $\theta(t)$, $(-\pi \le \theta(t) \prec \pi)$ its orientation relatively to the x-axis. The linear and angular velocities are respectively

$v(t)$ and $\omega(t)$. The equation of motion of the mobile robot is given by:

$$\begin{aligned}\dot{x}(t) &= v(t)\cos\theta(t)\\ \dot{y}(t) &= v(t)\sin\theta(t)\\ \dot{\theta}(t) &= \omega(t)\end{aligned} \qquad (1)$$

$v(t)$ and $\omega(t)$ are considered as the inputs to the mobile robot. Their magnitudes are constrained as follows:

$$|v(t)| \le v_{max}, \quad |\dot{v}(t)| \le a_{max}, \quad |\omega(t)| \le \omega_{max}$$

where v_{max}, a_{max} and ω_{max} are the maximum admissible values for $v(t)$, $a(t)$ and $\omega(t)$, respectively. Furthermore, from equation (1), the mobile robot behavior is subject to an additional nonholonomic constraint:

$$\dot{x}(t)\sin\theta(t) - \dot{y}(t)\cos\theta(t) = 0 \qquad (2)$$

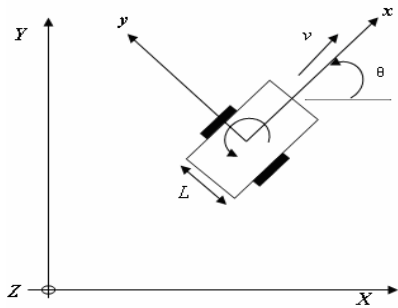This constraint means that the robot can not move in the direction of the wheel axis (i.e. y).

Fig. 1 Coordinates system

III. PATH FOLLOWING

For a given reference trajectory $(x_r(t), y_r(t))$ defined in a time interval $t \in [0, T]$, we can derive a control law. From the inverse kinematics the robot inputs are calculated which drive the robot on a desired path. The needed robot inputs, tangential velocity v_r and angular velocity ω_r are calculated from the reference path. The tangential velocity reads:

$$v_r(t) = \pm\sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \qquad (3)$$

Where "+" for the forward direction and "–" for inverse direction. The tangent angle of each point on the path can be determined as:

$$\theta_r(t) = \arctan2(\dot{y}_r(t), \dot{x}_r(t)) + k\pi \qquad (4)$$

Where $k = 0, 1$ defines the desired drive direction (0 for

forward and 1 for reverse). The reference angular velocity can be obtained by deriving previous equation [9], [10]:

$$\omega_r(t) = \frac{\dot{x}_r(t)\ddot{y}_r(t) - \dot{y}_r(t)\ddot{x}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)} \qquad (5)$$

We can write:

$$\omega_r(t) = v_r(t)\kappa(t)$$

where $\kappa(t)$ represents the path curvature. By following relations from (1) to (4) and the defined reference robot path $q_r(t) = [x_r(t), y_r(t), \theta_r(t)]^T$ robot inputs $v_r(t)$ and $\omega_r(t)$ are calculated. The necessary condition in the path design procedure is twice differentiable path and nonzero tangential velocity $v_r(t) \ne 0$. If for some time t tangential velocity is $v_r(t) = 0$, the robot rotates at a fixed point with the angular velocity $\omega_r(t)$.

IV. CONTROL DESIGN FOR PATH FOLLOWING

For solving the tracking control problem, the following global change of co-ordinates was proposed by Kanayama et al. [12] (Fig. 2). In the frame of real robot, the error between controlled robot and reference path can be expressed as:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \qquad (6)$$

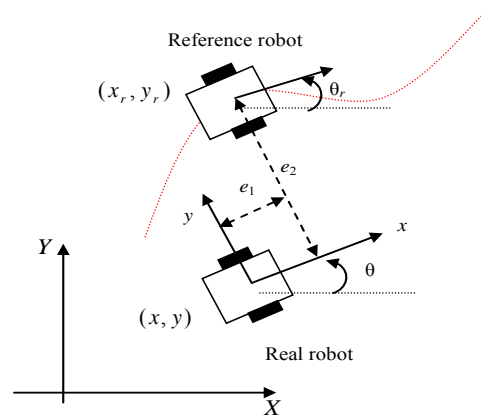The control algorithm should be designed to enforce the robot to follow the reference path precisely.

Fig. 2 The new error coordinates

Considering the robot kinematics (1), the error dynamics of the mobile robot can be derived from the time derivative of the above equation as

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} \cos e_3 & 0 \\ \sin e_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{r1} \\ u_{r2} \end{bmatrix} + \begin{bmatrix} -1 & e_2 \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (7)$$

where u_{r1} is the feedforward tangential velocity and u_{r2} is feedforward angular velocity. The robot inputs, regarding relation (6), can be expressed in the following form:

$$u_1 = u_{r1}\cos e_3 - v_1$$
$$u_2 = u_{r2} - v_2 \tag{8}$$

u_{r1}, u_{r2} represent feedforward inputs and v_1, v_2 are the inputs from the closed loop.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & u_2 & 0 \\ -u_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \sin e_3 \\ 0 \end{bmatrix}u_{r1} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{9}$$

By linearizing (9) around the operating point $\left(e_1 = e_2 = e_3 = 0, \ v_1 = v_2 \right)$ we get the following linear system:

$$\Delta\dot{e} = \begin{bmatrix} 0 & u_{r2} & 0 \\ -u_{r2} & 0 & u_{r1} \\ 0 & 0 & 0 \end{bmatrix}.\Delta e + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\Delta v \tag{10}$$

This system can be written under space state form as:
$$\Delta\dot{q} = A.\Delta q + B\Delta u \tag{11}$$
and can be verified as controllable because Kalman controllability matrix is full rank $rank(B, AB, A^2 B) = 3$. The resulting control can be schematized as indicated in Fig. 3.
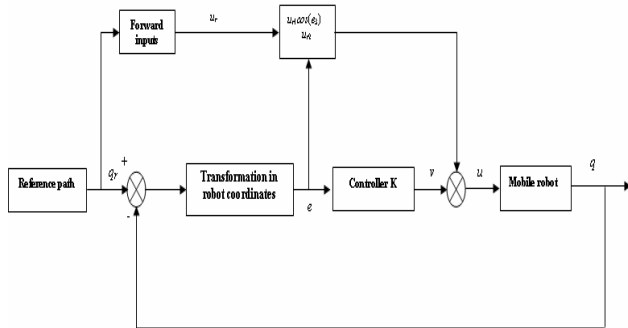

Fig. 3 Architecture of schematic control

The control law, according to [11], can be written as:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -sign(u_{r1})k_2 & -k_3 \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{12}$$

where

$$v_1 = -k_1 e_1$$
$$v_2 = -\bar{k}_2 u_{r1} e_2 - k_3 e_3 \tag{13}$$

This control can be verified as stable, and can make convergence of errors [11].

V. MODELING OF LEADER-FOLLOWER FORMATION

Let us consider the situation shown in Fig. 4. A leader and a follower robot are denoted as R_l and R_f, respectively. According to the notation defined in Section II, the states and the inputs of R_l and R_f are newly denoted as (x_l, y_l, θ_l), (x_f, y_f, θ_f) (v_l, ω_l) and (v_f, ω_f), where the subscript "l" and "f" mean *leader* and *follower*, respectively. The equations of motion of both robots are given by (1). The relative distance between the leader and the follower robot is denoted as $d_{lf} = d_{fl}$ and the relative angles from their heading are denoted as θ_{lf}, They are given by:

$$d_{lf} = \sqrt{(x_l - x_f)^2 + (y_l - y_f)^2}$$
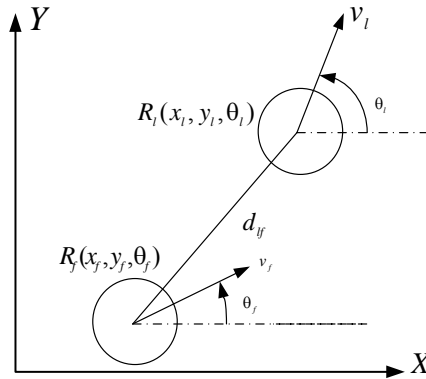$$\theta_{lf} = \theta_f - \theta_l \tag{14}$$

Fig. 4 Leader-follower configuration

VI. FUZZY SBC CONTROL

The main goal of the FSBC is to make the follower mobile robot pursue the leader mobile robot. The parameters used to construct the FSBC are shown in Fig. 4, where (x_l, y_l) are the position of center of the leader mobile robot, θ_l is its orientation angle corresponding to the *x*-axis. (x_f, y_f) are the position of center of the follower mobile robot, θ_f is its orientation angle corresponding to the *x*-axis.

For translation velocity and angular velocity controls, two two-input–single-output FSBC are adopted, respectively. Let the input variables of the FSBC be defined as:

$$e(k) = d_{des}(k) - d_{act}(k) \quad (15)$$

$$\Delta e(k) = e(k) - e(k-1) \quad (16)$$

$$\theta(k) = \theta_f(k) - \theta_i(k) \quad (17)$$

$$\Delta\theta(k) = \theta(k) - \theta(k-1) \quad (18)$$

where d_{des} is the desired distance between leader and the follower robot. We want to design the FSBC for the follower mobile robot such that it can pursue the target mobile robot within a constant distance. The fuzzy control rules can be represented as a mapping from input linguistic variables e, Δe, θ and $\Delta\theta$ to output linguistic variables v and ω as follows:

$$u(k+1) = \text{FSBC}(e(k), \Delta e(k), \theta(k), \Delta\theta(k)) \quad (19)$$

where u includes the translation and angular velocity. Because translation and angular velocities of the mobile robot can be driven individually, thus we can decompose (19) as follows:

$$v(k+1) = \text{FSBC}(e(k), \Delta e(k)) \quad (20)$$

$$\omega(k+1) = \text{FSBC}(\theta(k), \Delta\theta(k)) \quad (21)$$

The membership functions of input linguistic variables e, Δe, θ and $\Delta\theta$ and the membership functions of output linguistic variables v and ω are shown in Fig. 5 respectively.
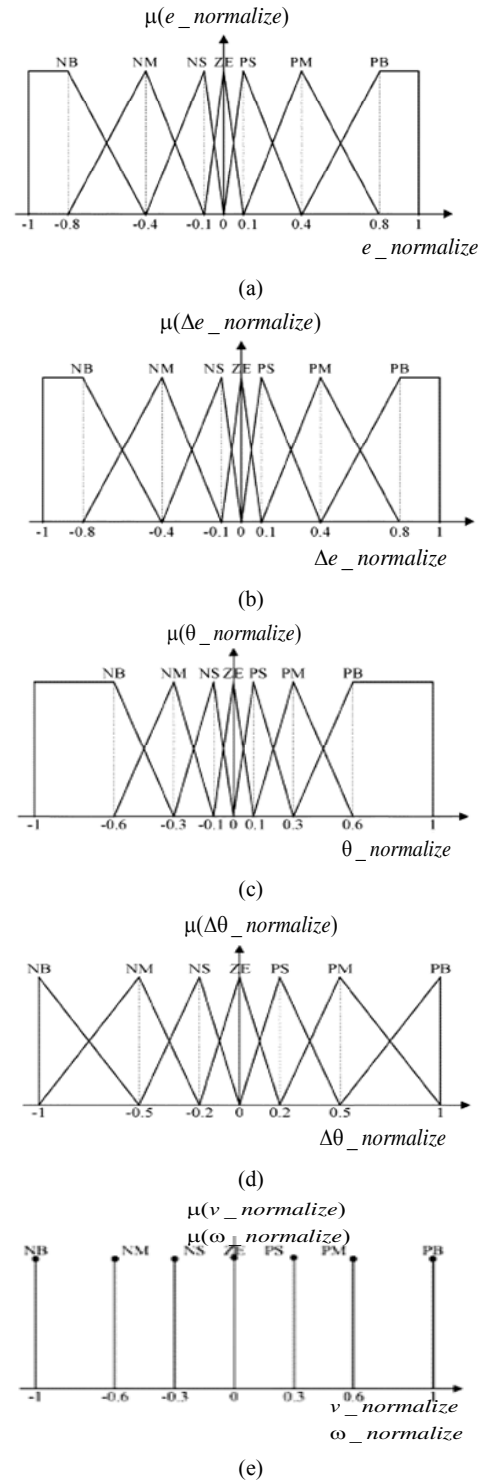
TABLE I
RULE TABLE OF FSBC

dE / E	NB	NM	NS	ZE	PS	PM	PB
PB	ZE	NS	NM	NB	NB	NB	NB
PM	PS	ZE	NS	NM	NB	NB	NB
PS	PM	PS	ZE	NS	NM	NB	NB
ZE	PB	PM	PS	ZE	NS	NM	NB
NS	PB	PB	PM	PS	ZE	NS	NM
NM	PB	PB	PB	PM	PS	ZE	NS
NB	PB	PB	PB	PB	PM	PS	ZE

(case $u = v$, $E = e$, $dE = \Delta e$,
case $u = \omega$, $E = \theta$, $dE = \Delta\theta$)

Fig. 5 (a) Membership function of e (b) Membership function of Δe (c) Membership function of θ (d) Membership function of $\Delta\theta$ (e) Membership functions of v and ω

They all are decomposed into seven fuzzy partitions, such as: negative big (**NB**), negative medium (**NM**), negative small (**NS**), zero (**ZE**), positive small (**PS**), positive medium (**PM**), and positive big (**PB**).

Since each input is divided into seven fuzzy sets, 49 fuzzy rules for speed control and steering angle control must be determined, respectively. Following the FSMC concept [13]-[15], a diagonal type rule table is adopted (see Table 1). The FSMC provides control action to drive state trajectories toward a sliding surface (in fact, the sliding line in this application) in the state–space, and to maintain the state trajectories sliding on the sliding surface until stable equilibrium state is reached. In this paper, we first have to choose the sliding surface that represents the control purpose. The sliding surfaces for angular velocity control and angular velocity control are defined as $s = e + \Delta e$ and $s = \theta + \Delta \theta$. The defuzzification strategy is implemented by the weighted average method.

$$u = \frac{\sum_{j=1}^{49} \mu_j(u_j) \cdot u_j}{\sum_{j=1}^{49} \mu_j(u_j)} \qquad (22)$$

where u may be the linear velocity or angular velocity command of the follower mobile robot, μ_j is the support of each fuzzy set j, and $\mu_j(u_j)$ is the membership function value of each rule.

VII. SIMULATION

To illustrate the efficiency of the proposed controller, we simulate a team of 3 nonholonomic mobile robots as shown in Fig. 6. Initially, the position of the leader (red color) is Rob1(100, 0, 0). The position of the two followers are Rob2(90, 0,0) and Rob3(70, 0,0) (green and blue respectively) the distance inter-robots are initially at $d_{12} = 20$, $d_{23} = 30$, and we want to keep the relative distance and angle between two consecutive robots as a constant value ($d = 20$ and $\theta = 0$). The simulation results for several different cases are listed as below in Fig. 6 to Fig. 7. In each figure, the three plots are the simulation results of the robot team evolution, translation velocity of the leader, the angular velocity of the leader, inter-distance error, and angular error form the top to bottom respectively.

Fig. 6 shows the case when the reference path is a straight line with a slope of 0.2. The leader goes along a straight line with a constant linear speed (2m/s) and constant angular speed (0rad/s). Fig. 6 shows the result when the followers robots keep a constant relative angle ($\theta_d = 0$) with respective to the leader.

Fig. 7 shows the case of a reference path, with an initial curvature equal to -0.05 ($\omega = -0.01 rad / s$) and changes later to 0.05 ($\omega = 0.01 rad / s$) at $t = 50s$.

VIII. CONCLUSION

In this paper, we present a new controller based on fuzzy logic for the leader following formation of multiple nonholonomic mobile robots. For keeping the distance and bearing between two consecutive mobile robots in desired values, we design two controllers the first for translation velocity its inputs are distance and its derivative, the second one is for angular velocity its inputs are the difference between the mobile robots orientations and its derivative. Successful simulation have been conducted for various situations and showed the efficiency of the proposed approach.
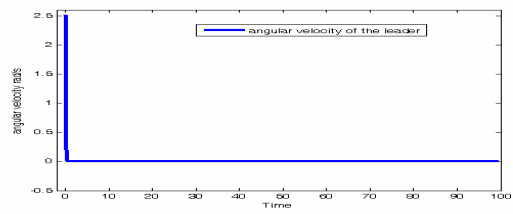
Authors' future work will be focused on comparison with classical techniques, experimental verification of presented approach and the optimization of the fuzzy logic in real time.
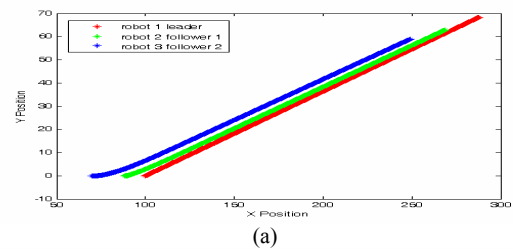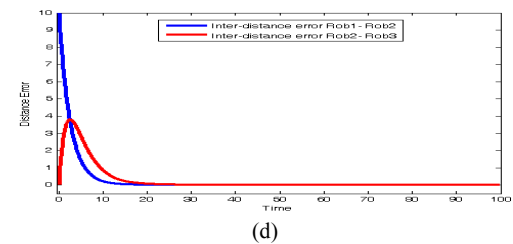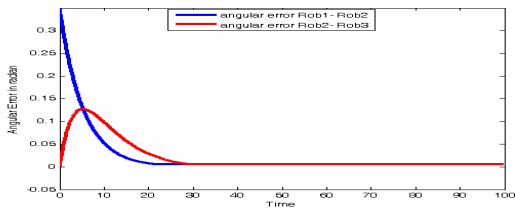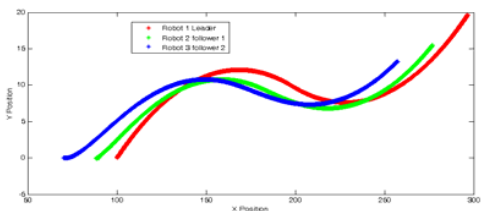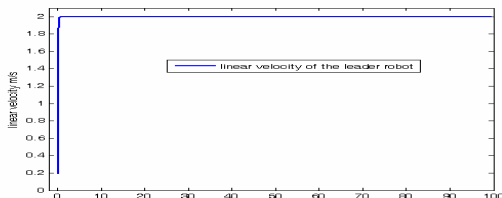

(a)
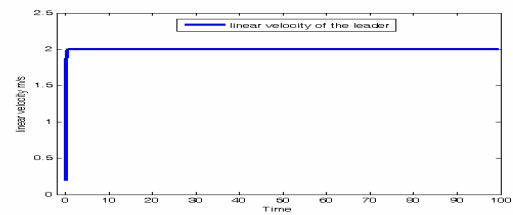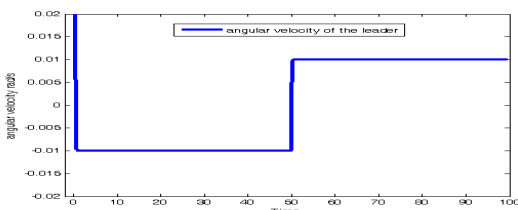

(b)

(c)

(d)

(e)

Fig. 6 Leader Robot goes on straight line and the follower keeps a constant relative distance and angel with respect to leader
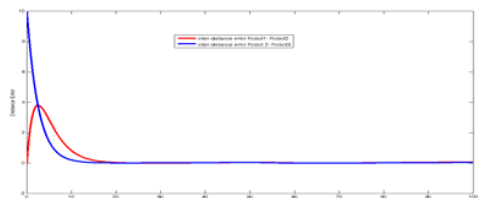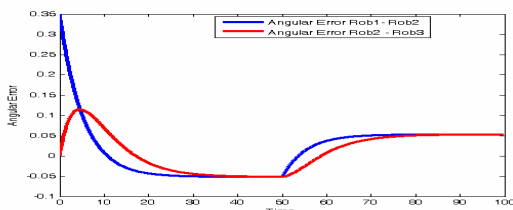

(a)

(b)

(c)

(d)

(e)

Fig. 7 Leader Robot goes on reference path that has initially its curvature equal to -0.05 and change the curvature to 0.05 at time equal to 50s

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