

# A Lifetime-Guaranteed Routing Scheme in Wireless Sensor Networks

Jae Keun Park, Sung Je Hong, Kyong Hoon Kim, Tae Heum Kang, and Wan Yeon Lee

**Abstract**—In this paper, we propose a routing scheme that guarantees the residual lifetime of wireless sensor networks where each sensor node operates with a limited budget of battery energy. The scheme maximizes the communications QoS while sustaining the residual battery lifetime of the network for a specified duration. Communication paths of wireless nodes are translated into a directed acyclic graph(DAG) and the maximum-flow algorithm is applied to the graph. The found maximum flow are assigned to sender nodes, so as to maximize their communication QoS. Based on assigned flows, the scheme determines the routing path and the transmission rate of data packet so that any sensor node on the path would not exhaust its battery energy before a specified duration.

**Keywords**—Sensor network, battery, residual lifetime, routing scheme, QoS

## I. INTRODUCTION

Wireless sensor networks consist of sensor nodes and one or more sink nodes. Sensor nodes monitor physical or environmental conditions (e.g., image, sound, temperature, pressure, vibration) and communicate with each other and sink nodes over wireless channels. Sensor nodes transmit their monitored data to the sink nodes. In wireless networks, the operation of each sensor node resorts to the battery energy and thus its operation lifetime is limited by available budget of residual battery energy. Moreover, although a sensor node can operate with available residual energy, it cannot transmit its monitored data to the sink node if any intermediate node located in the communication path is powered off due to depletion of battery energy. Hence, in order to sustain the communication path for a specified duration, the routing scheme must manage the battery energy of intermediate nodes as well as the sensor node.

In this paper, we propose a routing scheme for many-to-one communications where many sender nodes collect physical or environmental conditions (e.g., image, sound, temperature, pressure, vibration) and send the collected data to a single sink node via wireless channels. The proposed scheme maximizes the total transmission rate of many-to-one communications, while sustaining the residual lifetime of the wireless networks for a specified duration. Based on the information about the residual battery energies of all nodes, the proposed scheme determines the routing path and the transmission rate so that any wireless node on the path would not exhaust its battery energy before a given time.

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The proposed scheme first generates a directed acyclic graph(DAG) from a wireless network, where each node has the maximum flow. Next, in order to directly apply the maximum-flow algorithm [1], the generated graph is modified into a semantically equivalent graph where each edge has the maximum flow. The found flow capacity using the maximum-flow algorithm is assigned to sensor nodes, so as to maximize their total quality reward with distributed flow capacity. Finally, the scheme finds the routing path from sensor nodes to the sink node with their own assigned flow capacity. The transmission rate of each sensor node is determined to be the number of assigned flow capacity divided by the specified lifetime. The sensor node continues to operate during the specified lifetime without depletion of available battery energy when each sensor node sends the data packets at the determined transmission rate.

In order to conserve the battery energy, some studies [2], [3], [4], [5] have considered the control mechanisms that degrade the performance of a single wireless system with a single battery, as a means to maintain its residual battery energy for a specified critical duration. These studies are not applicable to wireless networks with multiple batteries because sensor nodes consume the battery energy of intermediate nodes on the communication path as well as their own battery energy. Many conventional routing schemes [6], [7] are proposed to maximize the lifetime of wireless networks, which is the first failure time of available communication paths due to battery depletion of intermediate nodes. Maximum-lifetime routing schemes do not guarantee the residual lifetime of communication paths for the specified duration because they adopt a greedy mechanism to select the path whose minimum residual energy is the largest among those of viable paths. In other words, these maximum-lifetime schemes cannot be applied to the problem of sustaining the residual lifetime of networks for a specified duration, because they just select the best routing path minimizing the lifetime reduction but cannot measure the residual communication capacity in advance.

The rest of this paper is organized as follows; Section 2 describes the assumptions and notations used in this paper. Section 3 describes the proposed routing scheme in detail. Finally, Section 4 concludes this paper.

## II. PRELIMINARIES

In the considered wireless sensor network,  $n$  sensor nodes are randomly deployed. The wireless network has the following properties:

- This network is a static densely deployed network. Every node is stationary. It means a large number of sensor

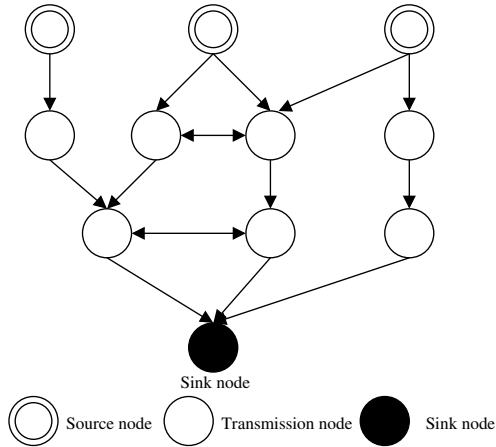


Fig. 1. An example of generated DAG

nodes are densely deployed in a two-dimensional geographic space, forming a network and these nodes do not move any more after deployment.

- There exists only one base station, called *sink node*, which is deployed at a fixed place. The sink node collects all the data generated from the sensor nodes and has sufficient hardware, software and constant power supply.
- All nodes are location-aware, i.e., their location information can be obtained in advance through other mechanisms such as GPS or position algorithm.
- The radio power of wireless nodes cannot be controlled, i.e., the transmission power of each node (the transmission range of each node) is fixed.
- There is no cycle among communication paths, i.e., the routing direction of wireless nodes is biased to the sink node. The routing direction going farther from the sink node is not allowed.
- The battery energy of wireless nodes cannot be recharged.

The sensor nodes generate the data packets periodically at a fixed rate. The size of the generated data packets is fixed. The transmission node only transmits data packets to the next neighbor node along their paths going closer to the sink node. The sink node collects all the data generated from the sensor nodes and has sufficient hardware, software and constant power supply. The information about the residual battery energy and the transmission energy of all wireless nodes can be obtained in advance using the existing aggregation protocol [8].

The wireless network is modeled as a directed acyclic graph (DAG)  $G = (V, E)$  where  $V$  is the set of nodes in the network and  $E$  is the set of edges.  $V = V_s \cup V_t \cup b$  where  $V_s$  represents the set of sensor nodes,  $V_t$  represents the set of intermediate transmission nodes, and  $b$  represents a sink node. There is a directed edge  $(u, v) \in E$  from node  $u$  to node  $v$  if node  $v$  is within the transmission range of node  $u$ . The edges between sensor nodes do not exist. Fig. 1 shows an example of DAG generated from a wireless sensor network.

The maximum flow of a given DAG is denoted as  $MM$ . The residual energy and the transmission energy of node  $u$

are denoted as  $re(u)$  and  $te(u)$ , respectively. The maximum number of data packets transmittable from node  $u$  to neighbor nodes with its own residual energy  $re(u)$  is denoted  $MP(u)$ , and calculated as follows:

$$MP(u) = \lfloor \frac{re(u)}{te(u)} \rfloor.$$

The data generation rate of node  $u$  is denoted as  $DG(u)$ , which is the number of data packets generated per a second. Each sensor node has its own quality function which represents the quality value, e.g., PSNR (peak signal to noise ratio), obtained from the input transmission rate. The quality reward value of sensor node  $u$  with its input transmission rate  $tr$  is denoted as  $Q(u, tr)$ . The specified network lifetime is denoted as  $T_{sp}$ , which is the required communication time to run without depletion of batteries. A routing path from sensor node  $s_i$  to sink node  $b$  is denoted as  $(s_i, t_1, t_2, \dots, b)$  which is a sequence of nodes on the path. There may exist multiple routing paths from sensor node  $s_i$  to sink node  $b$ . The  $j$ -th routing path from sensor node  $s_i$  to the sink node with  $k_j$  transmission capacity is denoted as  $r_j(s_i, k_j) = (s_i, t_1, t_2, \dots, b : k_j)$ . A set of all feasible routing paths from  $s_i$  to  $b$  is denoted as  $SR(s_i) = \{r_1(s_i, k_1), \dots, r_q(s_i, k_q)\}$  where  $q$  is the number of all feasible routing paths.

### III. PROPOSED SCHEME

To guarantee a specified lifetime of sensor nodes, we need to determine the transmission rate of each sensor node and to assign data packets to some paths from the sensor nodes to the sink node. The transmission rate of the sensor node is calculated from dividing the maximum number of data packets by a specified lifetime. Given a wireless sensor network, we propose a lifetime-guaranteed routing scheme which maximizes the number of data packets from the sensor nodes to the sink node, where the sink node knows the graph  $G$ , the residual energy and transmission energy of all the nodes, the rate generating the data sensed in all the sensor nodes, and the quality functions of the sensor nodes.

The proposed scheme consists of two phases. In Phase 1, the sink node calculates the maximum flow (the number of transmitted data packets) to be sent from all the sender nodes. The maximum-flow algorithm [1] is used to find the maximum number of transmittable packets in sensor networks with energy constraints. In Phase 2, the found maximum flow are distributed to some paths from the sender nodes to the sink node.

In Phase 1, the sink node generates a graph where the flow capacity of each node represents its residual battery energy available and the transmission energy to/from neighbor nodes. In our generated graph, each node has the flow capacity whereas each edge has the flow capacity in the conventional graph applied to the maximum-flow algorithm. Hence, in order to utilize the maximum-flow algorithm, the generated graph needs to be modified into another graph where each edge has the flow capacity but each node does not have the capacity. The generated graph  $G$  is transformed into a graph  $G'=(V', E')$  as follows:

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**DAG Generation Procedure**

- 1) Split each node  $u$  into two sub-nodes except the sink node,  $u'$  and  $u''$ . Add  $u'$  and  $u''$  to  $V'$ .
  - 2) For each  $u \in V$ , add edge  $(u', u'')$  to  $E'$ . For each  $(u, v) \in E$ , add edge  $(u'', v')$  to  $E'$ .
  - 3) Add a start node  $st$  to  $V'$ , which is the node connected to all the sensor nodes. For each sensor node  $u$ , add edge  $(st, u')$  to  $E'$ .
  - 4) Set  $MP'(u', u'')$  to  $MP(u)$ , which is the maximum number of data packets that node  $u'$  transmits to neighbor node  $u''$ .
  - 5) For each  $u \in Vs$  and each  $v1, v2 \in Vt$ , each  $MP'(st, u')$  and each  $MP'(v1'', v2')$  are set to  $\infty$ . For each neighbor node  $v''$  of the sink node  $d$ ,  $MP'(v'', d)$  is set to  $\infty$ .
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Fig. 2 shows a working example of the above DAG Generation Procedure. There are two sensor nodes ( $s_1$  and  $s_2$ ) and three transmission nodes ( $r_1$ ,  $r_2$  and  $r_3$ ).  $re(u)$  and  $te(u)$  are the residual energy (energy unit) and transmission energy (data packets/energy unit) of node  $u$  as shown in Fig. 2(a). The specified lifetime  $T_{sp}$  is set to 10 seconds. The data generation rate  $DG(s_1)$  and  $DG(s_2)$  are set to 0.5 data/second and 1.0 data/second. In order to directly apply the maximum-flow algorithm, the graph  $G$  in Fig. 2(a) is transformed to the graph  $G'$  in Fig. 2(b) using the above DAG Generation Procedure.

The graph  $G$  in Fig. 2(a) is semantically equivalent to the transformed graph  $G'$  in Fig. 2(b), because  $r$  (or  $s$ ) is virtually divided into  $r'$  and  $r''$  (or  $s'$  and  $s''$ ). In the transformed graph  $G'$ , there are one source node and one destination node, each edge has its own flow capacity, and each node does not have the flow capacity. In this case, we can directly apply the maximum-flow algorithm [1] in order to find the maximum flow  $MM$  of the graph. In Fig. 2(b),  $MM$  is 14 because the minimum cut of the maximum flow is a set of edge  $(r'_1, r''_1)$ , edge  $(r'_2, r''_2)$ , and edge  $(r'_3, r''_3)$  in graph  $G'$ .

In Phase 2, the sink node assigns the maximum flow (the maximum number of data packets) found in Phase 1 to the sensor nodes, and finds routing paths with assigned flows in order to transmit assigned data packets to the sink node. Let  $TM_{s_i}$  denote the number of data packets assigned to sensor node  $s_i$ . When assigning the number of data packets to be transmitted, three conditions must be satisfied as follows.

$$C1. TM(s_i) \leq DG(s_i) \times T_{sp}$$

$$C2. TM(s_i) \leq MP(s_i)$$

$$C3. \sum_{s_i \in Vs} TM(s_i) \leq MM$$

The first condition C1 is that the total number  $TM(s_i)$  of data packets assigned to the sensor node  $s_i$  is no larger than the number of data generated in sensor node during the specified lifetime  $T_{sp}$ ,  $DG(s_i) \times T_{sp}$ . If the number of transmittable data packets assigned to the sensor node is larger than that of data packets generated in sensor node  $s_i$  for the residual lifetime,  $(TM(s_i) - DG(s_i) \times T_{sp})$  data packets are useless in sensor node because there is no more data sensed. The second condition C2 is that the number of data packets assigned to the sensor nodes is no larger than the maximum number of data packets  $MP(s_i)$  which the sensor node can transmit

using its own residual energy. Sensor node  $s_i$  can transmit the data packets no larger than  $(MP(s_i))$  because of its battery constraint. The third condition C3 is that the sum of the data packets assigned to all sensor nodes is no larger than the maximum number of the data packets  $MM$  found in Phase 1. When more data packet than  $MM$  are assigned to the sensor nodes, there are no available path due depletion of residual energy in some transmission node after the sensor nodes transmit  $MM$  data packets.

When assigning the maximum flow (the maximum number of transmittable data packets) found in Phase 1, we consider two model: Even Distribution and QoS-Maximum Distribution. In the Even Distribution model, it is assumed that the sensor nodes are homogeneous and have the same quality function with each other. In the QoS-Maximum Distribution model, it is assumed that the sensor nodes are heterogeneous and their quality functions are different. The found maximum flow are assigned to the sensor nodes, so as to maximize the quality sum of all the sensor nodes.

In the Even Distribution model, the found maximum flow are assigned evenly to each sensor node as follows,

$$TM(s_i) = \frac{MM}{n} \quad (1)$$

where  $n$  is the number of the sensor nodes. If the first condition C1 is not satisfied in sensor node  $s_i$ ,  $TM(s_i) - DG(s_i) \times T_{sp}$  data packets are assigned to other sensor nodes whose assigned data packets are less than the number of generated (from sensing) data packets. If the second condition C2 is not satisfied in sensor node  $s_i$ ,  $TM(s_i) - M(s_i)$  packets are assigned to other sensor nodes whose assigned data packets are less than the maximum number of data packets transmittable with the residual battery energy. The Even Distribution model is formally described as follows.

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**Even Distribution Model**

- 1) Assign data packets  $TM(s_i) = \lfloor \frac{MM}{n} \rfloor$  for each sensor node  $s_i$ . Assign  $\frac{MM}{n} - \lfloor \frac{MM}{n} \rfloor$  data packets to sensor nodes in the decreasing order of their maximum data packets.
  - 2) If  $DG(s_i) \times T_{sp} \leq TM(s_i)$  for each sensor node  $s_i$ , assign  $TM(s_i) - DG(s_i) \times T_{sp}$  data packets evenly to each sensor node  $s_j \in A$  where  $A$  is the set of sensor nodes whose assigned data packets are less than data packets generated from their sensing.
  - 3) If  $M(s_i) \leq TM(s_i)$  for each sensor node  $s_i$ , assign  $TM(s_i) - M(s_i)$  data packets evenly to each sensor node  $s_j \in (A \cap B)$  where  $R$  is the set of sensor nodes whose assigned data packets are less than maximally transmittable data packets with their own residual battery energy.
  - 4) Find paths to transmit  $TM(s_i)$  data packets from sensor node to the sink node for each sensor node  $s_i$ .
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In the example of Fig. 2(b), the maximum flow is distributed evenly to sensor node  $s_1$  and  $s_2$  when the sensor nodes are homogeneous and have the same quality function. However,

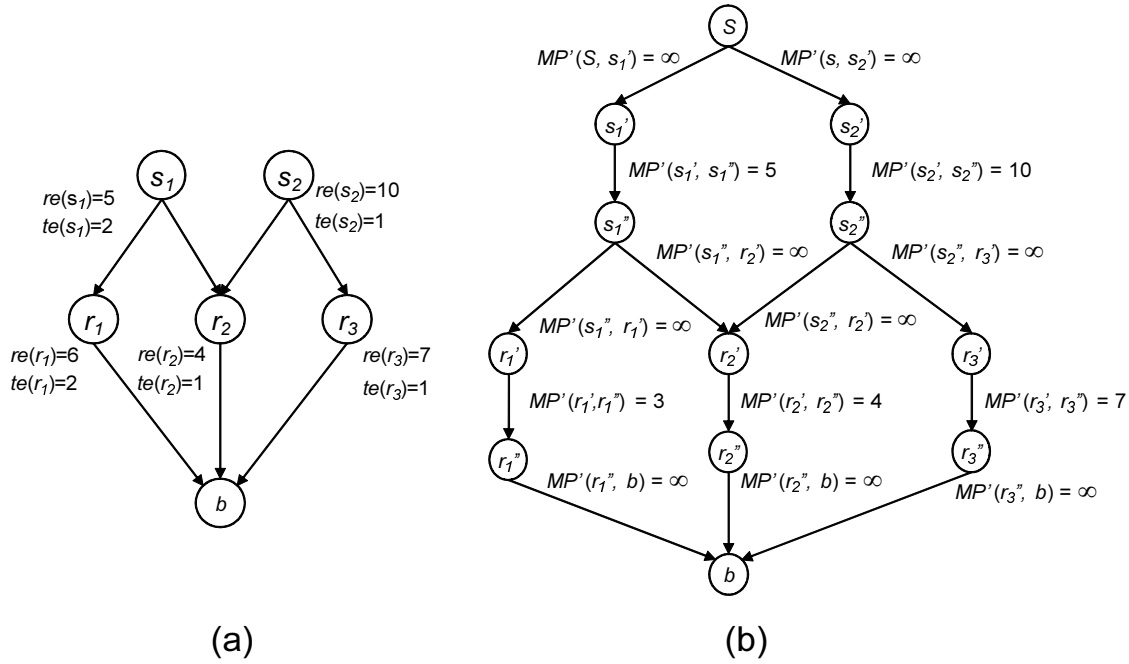


Fig. 2. An example of finding the maximum flow in sensor networks

sensor node  $s_1$  cannot not transmit more than 6 data packets because the number of data generated in sensor node  $s_1$  for 10 second are 5. As a result, 2 data packets assigned to sensor node  $s_1$  are assigned to sensor node  $s_2$ . Finally,  $TM(s_1)$  and  $TM(s_2)$  are 5 and 9, respectively.  $SR(s_1)$  and  $SR(s_2)$  are  $\{(s_1, r_1, b : 3), (s_1, r_2, b : 2)\}$  and  $\{(s_2, r_2, b : 2), (s_2, r_3, b : 7)\}$ , respectively.

In the QoS-Maximum Distribution model, the sensor nodes have different quality functions with an input of the number of transmitted data packets per unit time. When  $\Psi^i$  denotes the quality value of sensor node  $s_i$  derived from the number  $TM(s_i)$  of its assigned data packet and there are  $n$  sensor nodes, the problem of maximizing total quality value with the found maximum flow can be formulated as follows:

$$\text{Maximize } \sum_{i=1}^n \Psi^i \quad \text{subject to } \sum_{i=1}^n TM(s_i) \leq MM. \quad (2)$$

As the number of transmitted packets per unit time (transmission rate)  $TR(s_i) = \frac{TM(s_i)}{T_{sp}}$  increases, the quality reward  $\Psi^i$  increases, but the utility increment per unit transmission rate  $\frac{\Psi^i}{TR(s_i)}$  decreases in most multimedia communications [5]. In this case, we can apply the Lagrange Multiplier method [9] to obtain the solution to Eq. (2). Then

$$L(TM(s_1), \dots, TM(s_n), \lambda) =$$

$$\sum_{i=1}^n \Psi^i + \lambda \cdot (MM - \sum_{i=1}^n TM(s_i)),$$

$$\frac{\partial L}{\partial \lambda} = MM - \sum_{i=1}^n TM(s_i) = 0,$$

$$\text{and } \frac{\partial L}{\partial TM(s_k)} = \frac{\Psi^k}{TM(s_k)} - \lambda = 0 \quad \text{for } 1 \leq k \leq n.$$

The above equations verify that the maximum of Eq. (2) occurs when  $\sum_{i=1}^n TM(s_i) = MM$  and the values of  $\frac{\Psi^i}{TM(s_k)}$  for

$1 \leq i \leq N$  are equal to the largest. In other words, the solution to Eq. (2) is derived by incrementally assigning the capacity  $MM$  to the sensor node with the largest quality increment from a number of transmitted packet, where the quality reward increment decreases as the number of assigned packets increases for each sensor node. The QoS-Maximum Distribution model is formally described as follows, where  $Q(s_i, TM(s_i))$  is the quality reward value of sensor node  $s_i$  with its assigned transmission capacity  $TM(s_i)$ .

#### QoS-Maximum Distribution Model

- 1) Find  $(TM(s_1), TM(s_2), \dots, TM(s_n))$  to maximize  $\sum_{i=1}^n Q(s_i, \frac{TM(s_i)}{T_{sp}})$ .
- 2) If  $DG(s_i) \times T_{sp} \leq TM(s_i)$  for each sensor node  $s_i$ , assign  $(TM(s_i) - DG(s_i) \times T_{sp})$  transmission capacity to each sensor node  $s_j \in A$ , so as to maximize  $\sum_{s_j \in A} Q(s_j, \frac{TM(s_j)}{T_{sp}})$ .
- 3) If  $DG(s_i) \times T_{sp} \leq TM(s_i)$  for each sensor node  $s_i$ , assign  $TM(s_i) - DG(s_i) \times T_{sp}$  transmission capacity to each sensor node  $s_j \in (A \cap B)$ , so as to maximize  $\sum_{s_j \in (A \cap B)} Q(s_j, \frac{TM(s_j)}{T_{sp}})$ .
- 4) Find paths to transmit  $TM(s_i)$  data packets from sensor node  $s_i$  to the sink node in order for each sensor node  $s_i$ .

Table I shows an example of quality reward versus transmission rates, where the quality reward increment decreases as the transmission rate increases. If  $TM(s_1) = 4$  and  $TM(s_2) = 10$  when  $MM$  is 14, the quality reward sum  $Q(s_1, 0.4) + Q(s_2, 1.0)$  of two sensor nodes is 39. In contrast,

TABLE I  
EXAMPLE OF QoS VERSUS TRANSMISSION RATES

| Transmission Rate | $s_1$ | $s_2$ |
|-------------------|-------|-------|
| 0.4               | 14    | 11    |
| 0.5               | 15    | 14    |
| 0.6               | 16    | 17    |
| 0.7               | 17    | 19    |
| 0.8               | 17.5  | 21    |
| 0.9               | 18    | 23    |
| 1.0               | 18.3  | 25    |

if  $TM(s_1) = 5$  and  $TM(s_2) = 9$ , then the quality reward sum  $Q(s_1, 0.5) + Q(s_2, 0.9)$  of two sensor nodes becomes 38.

Let us apply the quality reward functions of Table I to the example of Fig. 2(b), where  $MM$  is 14,  $T_{sp}$  is 10 seconds, and  $DG(s_1)$  and  $DG(s_2)$  are 0.5 and 1.0, respectively. The QoS-Maximum Distribution model first allocates 4 transmission capacity among the maximum transmission capacity 14 to sensor node  $s_1$ , because the quality reward increment per unit transmission rate of  $s_1$ ,  $\frac{14}{0.4}$ , is larger than that of  $s_2$ ,  $\frac{11}{0.4}$ . Next, the QoS-Maximum Distribution model allocates 4 transmission capacity among the remaining transmission capacity 10 to  $s_2$ , because the quality reward increment per unit transmission rate of  $s_2$ ,  $\frac{11}{0.4}$ , is larger than that of  $s_1$ ,  $\frac{(15-14)}{0.1} = \frac{1}{0.1}$ . Finally, the QoS-Maximum Distribution model allocates all the remaining transmission capacity 6 to  $s_2$ , because the quality reward increment per unit transmission rate of  $s_2$ ,  $\frac{3}{0.1}$  or  $\frac{2}{0.1}$ , is larger than that of  $s_1$ ,  $\frac{1}{0.1}$ . In this case,  $SR(s_1)$  and  $SR(s_2)$  are  $\{(s_1, r_1, b : 3), (s_1, r_2, b : 1)\}$  and  $\{(s_2, r_2, b : 3), (s_2, r_3, b : 7)\}$ , respectively.

#### IV. CONCLUSIONS

In this paper, we propose a lifetime-guaranteed routing scheme for sensor nodes in wireless sensor networks. The proposed scheme generates a directed acyclic graph(DAG) from a wireless sensor network, where each node has the maximum flow. In order to directly apply the maximum-flow algorithm, the generated graph is modified into a semantically equivalent graph where each edge has the maximum flow. The found flow capacity using the maximum-flow algorithm is distributed to sensor nodes, so as to maximize their total quality reward with distributed flow capacity. The transmission rate of the sensor node is determined to be the number of assigned flow capacity divided by the specified lifetime. The sensor node continues to operate during the specified lifetime without depletion of available battery energy when each sensor node sends the data packets at the determined transmission rate. In future work, we will study a problem of finding the maximum flow of sensor networks with more practical assumptions: for example, unfixed communication paths between wireless nodes (neighbor nodes are dynamically changed due variable transmission range of sensor node with different energy consumptions) and flexible routing direction to allow cycles among communication paths.

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