AHP and Extent Fuzzy AHP Approach for Prioritization of Performance Measurement Attributes

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Abstract—The decision to recruit manpower in an organization requires clear identification of the criteria (attributes) that distinguish successful from unsuccessful performance. The choice of appropriate attributes or criteria in different levels of hierarchy in an organization is a multi-criteria decision problem and therefore multi-criteria decision making (MCDM) techniques can be used for prioritization of such attributes. Analytic Hierarchy Process (AHP) is one such technique that is widely used for deciding among the complex criteria structure in different levels. In real applications, conventional AHP still cannot reflect the human thinking style as precise data concerning human attributes are quite hard to be extracted. Fuzzy logic offers a systematic base in dealing with situations, which are ambiguous or not well defined. This study aims at defining a methodology to improve the quality of prioritization of an employee's performance measurement attributes under fuzziness. To do so, a methodology based on the Extent Fuzzy Analytic Hierarchy Process is proposed. Within the model, four main attributes such as Subject knowledge and achievements, Research aptitude, Personal qualities and strengths and Management skills with their sub-attributes are defined. The two approaches conventional AHP approach and the Extent Fuzzy Analytic Hierarchy Process approach have been compared on the same hierarchy structure and criteria set.

Keywords—AHP, Extent analysis method, Fuzzy AHP, Prioritization.

I. INTRODUCTION

MANPOWER in an organization constitutes an important and essential asset which tremendously contributes to development and growth of that organization by the help of their collective attitude, skills and abilities. The decision to recruit manpower requires clear identification of the criteria or attributes that distinguish successful from unsuccessful performance. Choice of selection criteria should be in synchronization with the organization's strategic directions and culture. The choice of appropriate attributes or criteria in different levels of hierarchy is a multi criteria decision making problem and therefore an appropriate Multi-Criteria Decision Making (MCDM) technique is required for prioritization and assessment of such attributes. The MCDM techniques have an advantage that they can assess a variety of options according to a variety of criteria that have different units. Also they have the capacity to analyze both quantitative and qualitative evaluation criteria together. Analytic Hierarchy Process (AHP)

is one such technique that is widely used for deciding among the complex criteria structure in different levels.

In real applications, conventional AHP [1] still cannot reflect the human thinking style as precise data concerning human attributes are quite hard to be extracted. In addition, decision makers prefer natural language expressions rather than sharp numerical values in assessing various performance attributes. The AHP method is mainly used in nearly crisp (non-fuzzy) decision applications and creates and deals with a very unbalanced scale of judgment. Fuzzy logic offers a systematic base in dealing with situations, which are ambiguous or not well defined [2]. Indeed the uncertainty in expressions such as "low talent", or "high experience" which are frequently encountered in the human capital literature is fuzzy in nature. Fuzzy AHP is a synthetic extension of classical AHP method when the fuzziness of the decision makers is considered. This paper aims at comparing the classical AHP and Fuzzy AHP to a data set which is based on fuzzy linguistic or qualitative evaluations of decision maker, to show the differences of the result and the decisions made after that. To see the distinction between these two approaches, comparison is carried out on the same hierarchy structure and criteria sets.

Paper is organized as follows: Section II introduces the classical AHP & fuzzy AHP methods including the past literatures. Section III presents a comparative case study using both the approaches. Section IV deals with conclusions and future directions for research.

II. AHP AND FUZZY AHP

A. Classical AHP

Analytic Hierarchy Process is a method for ranking decision alternatives, and selecting the best one when the decision maker has multiple criteria [3]. The AHP consists of three main operations, including hierarchy construction, priority analysis, and consistency verification. First of all, the decision makers need to break down complex multiple criteria decision problems into its component parts of which every possible attributes are arranged into multiple hierarchical levels. After that, the decision makers have to compare each cluster in the same level in a pair-wise fashion based on their own experience and knowledge. Since the comparisons are carried out through personal or subjective judgments, some degree of inconsistency may occur. To guarantee that the judgments are consistent, the final operation called consistency verification is incorporated in order to measure the degree of consistency

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among the pair-wise comparisons by computing the consistency ratio. If it is found that the consistency ratio exceeds the limit, the decision makers should review and revise the pair-wise comparisons. Once all pair-wise comparisons are carried out at every level, and are proved to be consistent, the judgments can then be synthesized to find out the priority ranking of each criterion and its attributes.

B. Fuzzy AHP

It has been widely recognized that most decisions made in the real world take place in an environment in which the goals and constraints, because of their complexity, are not known precisely, and thus, the problem cannot be exactly defined or precisely represented in a crisp value [4]. To deal with the kind of qualitative, imprecise information or even ill-structured decision problems, fuzzy set theory is suggested by [4] as a modeling tool for complex systems that can be controlled by humans but are hard to define exactly. In most of the real-world problems, some of the decision data can be precisely assessed while others cannot. The fuzzy AHP technique can be viewed as an advanced analytical method developed from the traditional AHP. Despite the convenience of AHP in handling both quantitative and qualitative criteria of multi-criteria decision making problems based on decision maker's judgments, fuzziness and vagueness existing in many decision-making problems may contribute to the imprecise judgments of decision makers in conventional AHP approaches [5]. Processes compared to the traditional AHP methods. So many researchers [6]-[8] who have studied fuzzy AHP have provided the evidence that fuzzy AHP shows relatively more sufficient description of these kind of decision making There are many fuzzy AHP methods proposed by various authors. These methods are systematic approaches to the alternative selection and justification problem by using the concepts of fuzzy set theory and hierarchical structure analysis. The earliest work in fuzzy AHP appeared in [7], which compared fuzzy ratios described by triangular membership functions. Reference [6] introduces a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons. Reference [9] employed the property of goal programming to solve group decision making fuzzy AHP problem. In this paper Chang's extent analysis method has been used for the prioritization of various performance measurement attributes for the performance measurement of employees in an Indian market research firm.

C. Algorithm of Fuzzy AHP (Chang's Extent Analysis)

Chang's extent analysis [10] on fuzzy AHP depends on the degree of possibilities of each criterion. According to the responses on the question form, the corresponding triangular fuzzy values for the linguistic variables are placed and for a particular level on the hierarchy, the pair-wise comparison matrix is constructed. Sub totals are calculated for each row of the matrix and new (l, m, u) set is obtained, then in order to find the overall triangular fuzzy values for each criterion, $l_i/\Sigma l_i$, $m_i/\Sigma m_i$, $u_i/\Sigma u_i$,($i=1,2,..., n$) values are found and used as the latest $M_i(l_i, m_i, u_i)$ set for criterion M_i in the rest of the process. In the next step, membership functions are constructed for each criterion and intersections are determined by comparing each couple. In fuzzy logic approach, for each comparison the intersection point is found, and then the membership values of the point correspond to the weight of that point. This membership value can also be defined as the degree of possibility of the value. For a particular criterion, the minimum degree of possibility of the situations, where the value is greater than the others, is also the weight of this criterion before normalization. After obtaining the weights for each criterion, they are normalized and called the final importance degrees or weights for the hierarchy level. To apply the process depending on this hierarchy, according to the method of Extent analysis, each criterion is taken and extent analysis for each criterion, g_i ; is performed on, respectively. Therefore, *m extent* analysis values for each criterion can be obtained by using following notation [2], [11]:

$$M^1_{gi}, M^2_{gi}, M^3_{gi}, M^4_{gi}, M^5_{gi}, ..., M^m_{gi}$$; where g_i is the goal set ($i = 1, 2, 3, 4, 5, ...n$) and all the M^j_{gi} ($j = 1, 2, 3, 4, 5,........, m$) are Triangular Fuzzy Numbers (TFNs). The steps of Chang's analysis can be given as in the following:

Step 1: The fuzzy synthetic extent value (S_i) with respect to the ith criterion as follows :

$$S_i = \sum_{j=1}^{m} M^j_{gi} \otimes [\sum_{i=1}^{n} \sum_{j=1}^{m} M^j_{gi}]^{-1} \qquad (1)$$

This involves

1. Computation of
$$\sum_{j=1}^{m} M^j_{gi} \qquad (2)$$

Perform the "fuzzy addition operation" of m extent analysis values for a particular matrix given in (3) below, at the end step of calculation, new (l, m, u) set is obtained and used for the next:

$$\sum_{j=1}^{m} M^j_{gi} = (\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j) \qquad (3)$$

Where l is the lower limit value, m is the most promising value and u is the upper limit value and to obtain (4);

2. Computation of
$$[\sum_{i=1}^{n} \sum_{j=1}^{m} M^j_{gi}]^{-1} \qquad (4)$$

Perform the "fuzzy addition operation" of M^j_{gi} ($j = 1, 2, 3, 4, 5, ..., m$) values given $\sum_{i=1}^{n} \sum_{j=1}^{m} M^j_{gi} = (\sum_{j=1}^{m} l_i, \sum_{j=1}^{m} m_i, \sum_{j=1}^{m} u_i)$

$$(5)$$

and then compute the inverse of the vector in (5) . (6) is then obtained such that:

$$[\sum_{i=1}^{n} \sum_{j=1}^{m} M^{j}_{gi}]^{-1} = \left[\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right] \quad (6)$$

Step 2: The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as equation (7):

$$V(M_2 \geq M_1) = \sup_{y \geq x}[\min(\mu_{M1}(x), \mu_{M2}(y))] \quad (7)$$

and *x and y* are the values on the axis of membership function of each criterion. This expression can be equivalently written as given in equation (8) below:

$$V(M_2 \geq M_1) = \begin{cases} 1, if m_2 \geq m_1, \\ 0, if m_2 \geq m_1, \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} \, otherwise \end{cases} \quad (8)$$

where d is the highest intersection point $(\mu_{M1} and \mu_{M2})$ [13] (see Fig. 1).

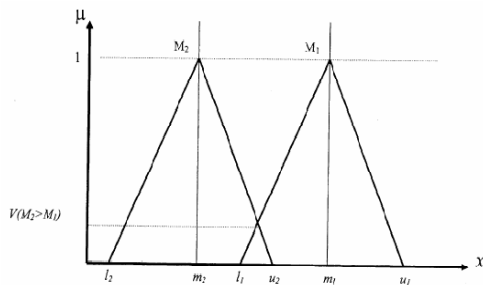To compare M_1 and M_2; we need both the values of $V(M_2 \geq M_1)$ and $V(M_1 \geq M_2)$

Fig. 1 Intersection between M_1 and M_2

Step 3. The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i *(i = 1, 2, 3, 4, 5, ..., k)* can be defined by
$V(M \geq M_1, M_2, M_3, M_4, M_5, M_6, ..., M_k)$
$= V[(M \geq M_1) and (M \geq M_2) and (M \geq M_3) and (M \geq M_4),$
$and.....and(M \geq M_k)] = min \, V(M \geq M_i)$, *i = 1, 2, 3, 4, 5, ., k.*
Assume that equation (9) is:
$d(A_i) = min \, V(S_i \geq S_k)$ (9)

For k = 1, 2, 3, 4, 5, ..., n; k≠ i. Then the weight vector is given by equation (10):

$Wι = (d(A_1), d(A_2), d(A_3), d(A_4), d(A_5), ..., d(A_n))^T$ (10)

Where A_i *(i = 1, 2, 3, 4, 5, 6, ..., n)* are n elements.
Step 4. Via normalization, the normalized weight vectors are given in (11):

$W = (d(A_1), d(A_2), d(A_3), d(A_4), d(A_5), d(A_6), ..., d(A_n))^T$ (11)

Where W is non-fuzzy numbers.

After the criteria have been determined as given in Fig. 1, a question form has been prepared to determine the importance levels of these criteria. To evaluate the questions, people only select the related linguistic variable, then for calculations they are converted into the following scale including triangular fuzzy numbers & generalized for such analysis as given in Table I.

TABLE I
TRIANGULAR FUZZY SCALES BY TOLGA & CHANG

TFN Values [12]	Triangular fuzzy scale & fuzzy reciprocal scale [6]
Absolute (7/2, 4, 9/2)	Just equal (1, 1, 1) (1, 1, 1)
Very strong (5/2, 3, 7/2)	Equally important (1/2, 1, 3/2) (2/3, 1, 2)
Fairly strong (3/2, 2, 5/2)	Weakly important (1, 3/2, 2) (1/2, 2/3, 1)
Weak (2/3, 1, 3/2)	Strongly more important
Equal (1, 1, 1)	(3/2, 2, 5/2) (2/5, 1/2, 2/3)
	Very strongly more important (2, 5/2, 3) (1/3, 2/5, 1/2)
	Absolutely more important (5/2, 3, 7/2) (2/7, 1/3, 2/5)

III. EMPIRICAL CASE STUDY

In this paper, a case of an Indian market research company has been considered which assess its employees on the basis of four main attributes for the entry level of recruitment. Firstly, the outlines of the employee selection and the attributes taken into consideration have been discussed. Various attributes as well as their sub attributes are compared with the help of pair wise comparison matrices.
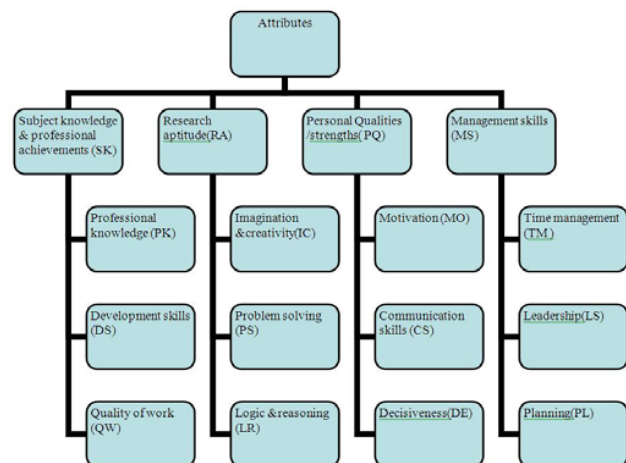

Fig. 2 Hierarchical model for the prioritization of performance measurement attributes

According to the management board of the company the following attributes set is constructed as given in Fig. 2. A group of experts consisting of academics and professionals were asked to make pair-wise comparisons for main and sub-attributes. The overall results could be obtained by taking the geometric mean of individual evaluations.

Subject Knowledge and Professional Achievements: It includes the theoretical as well as practical knowledge of the subject as well as the experience gained in the relevant subject area.

Research Aptitude: It is the ability to publish research papers, technical reports, patents etc. It also includes the imagination and creativity of the individual.

Personal Qualities and Strengths: It includes the candidate's ability to motivate others, his loyalty, dependability, communication skills etc.

Management Skills: This is particularly important for the senior level management people. It includes capabilities such as leadership abilities, time management etc.

After structuring the hierarchy of short listed attributes that are vital for performance measurement, opinion of the experts has been obtained on the following issues:

a) Comparative effects of various attributes on employee performance measurement viz. Subject Knowledge and professional achievements, Aptitude for research, Management skills and Personal qualities

b) Comparative contribution of sub attributes to each attributes as identified in the figure given below.

The question form developed for this study includes all questions for each level of hierarchy, i.e. the sample questions with respect to the overall goal "selecting the best performer for the company" are given as follows:

Q-1: How important is "SK" when it is compared with "RA" at entry level of managerial position?

Q-2: How important is "SK" when it is compared with "PQ" at entry level of managerial position?

The qualitative opinion is obtained from the experts in a specified format.

Attribute A (SK)   Attribute B (RA)

Managerial positions	ES	VS	S	MS	EQ	MS	S	VS	ES
Entry level			√						
Middle level				√					
Senior level				√					

Fig. 3 Pair wise comparison of attributes

From the fuzzy numbers in Table I, following calculations are performed to reach the importance values of the first level.

The values of the fuzzy synthetic extents with respect to the main attributes are calculated as below. Here SS defines the fuzzy synthetic extent values for Subject Knowledge.

SS = ((1+3/2+3/2+2);(1+2+2+5/2);(1+5/2+5/2+3)) ⊗ (1/(3+3+2+1+5/2+3+1+1+5/2+1+1/2+1/2+1+2/3+2/3+1/2);1/(5/2+5/2+3/2+1+2+5/2+1+2/3+2+1+2/5+2/5+1+1/2+1/2+2/5);(1/(2+2+1+1+3/2+2+1+1/2+3/2+1+1/3+1/3+1+2/5+2/5+1/3))

i.e. SS= (6.0; 7.5; 9.0) ⊗ (1/24;1/19.9/;1/16.3)

Similarly

SR = (5.4; 6.5; 7.7) ⊗ (1/24;1/19.9/;1/16.3)

SP = (2.74; 3.4; 4.2) ⊗ (1/24;1/19.9/;1/16.3)

SM = (2.2; 2.5; 3.0) ⊗ (1/24;1/19.9/;1/16.3)

i.e. SS= (0.25,0.377,0.552)

SR=(0.225,0.327,0.4724)

SP=(0.1142,0.17,0.26)

SM=(0.0917,0.126,0.184)

Using these vectors, the degree of possibility is calculated as:

$V(SS \geq SR) = 1$

$V(SS \geq SP) = 1$

$V(SS \geq SM) = 1$

$V(SR \geq SS) = 0.835$

$V(SR \geq SP) = 1$

$V(SR \geq SM) = 1$

$V(SP \geq SM) = 1$

$V(SP \geq SR) = 0.1823$

$V(SP \geq SS) = 0.183$

$V(SM \geq SS) = 0.112$

$V(SM \geq SR) = 0.133$

$V(SM \geq SP) = 0.622$

From these values, the minimum degree of possibility is calculated as:

Min {V (SS≥SR); V (SS≥SP) ; V (SS≥SM) }=1;

Similarly,

Min {V (SR≥SS); V (SR≥SP) ; V (SR≥SM) }=0.835;

Min {V (SP≥SM); V (SP≥SR) ; V (SP≥SS) }=0.1823;

Min {V (SM≥SS); V (SM≥SR) ; V (SM≥SP) }=0.112.

Thus, the weight vector can be generated as :

$Wgoal$ = (1; 0.835; 0.1823 ; *0.112*)^T.

Via normalization, the importance weights of the main attributes are calculated as follows:

$Wgoal$ =(0.47; 0.392; 0.0856; *0.053*)^T.

This means according to this person, the main criteria for the recruitment in the company at the entry level are 'subject knowledge' and 'research aptitude' with importance values as 1 and 0.835 respectively. The next step consists of

operations to calculate the local importance values or weight vector of the second level in hierarchy. For each branch, each criteria group in the second level is subject to a pair wise comparison in itself. The criteria sets are calculated with the same approach and procedure is ended when global and local importance levels are obtained.

Chang's fuzzy AHP uses intersection operation while evaluating comparison results. The result of the fuzzy intersection can be obtained as zero which means that the corresponding criterion has no importance. Fuzzy pair wise comparisons provide that if a criterion is less important than all of the others, then relatively this criterion has no importance and weight is zero. Even if it is declared that a criterion is handled for the decision making process, it has no importance when compared with the others. In the classic AHP method, deterministic values and operations do not permits such a situation "having zero weight", but if a criterion is evaluated as "less than all of the others", then the numerical result of this situation, the weight of this criterion would be near to zero, furthermore the weight can descend to 0.01 which means that this criterion is not so important on the final decision. Fuzzy AHP totally neglects the criterion which is less important than the others whereas classical AHP uses this criterion with so small weight.

These were the weights for the main attributes, for level 2 analysis that is weightages for sub attributes from fuzzy AHP approach is as follows:

$$W_{subject\ knowledge} = (\ 1;\ 0.8;\ 0.795;\ 0.762\ )^T.$$

Via normalization it becomes

$$W_{subject\ knowledge} = (0.3;\ 0.24;\ 0.236;\ 0.23)^T.$$
$$W_{research\ aptitude} = (\ 0.61;\ 0.18;\ 0.19\ )^T.$$
$$W_{management\ skills} = (0.25;\ 0.6;\ 0.15\ )^T.$$
$$W_{personal\ qualities} = (0.25;\ 0.24;\ 0.25;\ 0.26\ )^T.$$

Incorporation of Attributes Weights in Candidate's Performance Evaluation:

The performance P of an employee is computed as follows:
$$P = w_1* SK + w_2 * RA + w_3* PQ + w_4*MS$$

The weights of the sub attributes are computed using second level of comparisons and are given below.

$$SK = 0.3*PK + 0.24 * DS + 0.236 *QW + 0.23 *TA$$

$$RA = 0.61 *IC + 0.18 *PS + 0.19 *LR$$

$$MS = 0.25 *TM + 0.6 *CS + 0.15 *PN$$

$$PQ = 0.25 *MO + 0.24 *CS + 0.25 *DE + 0.26 *TW$$

In order to quantify the judgments made by the interviewers for the candidates, a six level grading system viz: A+= 90, A=80, B+=70, B=60, C+=50, C=40 has been employed. The interviewers assign the above grades as per the abilities of the candidates. For example, for each attribute as per the ability

assessed for a candidate, the performance evaluation is carried out for a sample case for recruitment at all levels is as follows:

$$\begin{bmatrix} P\,K \\ D\,S \\ Q\,W \\ T\,A \end{bmatrix} = \begin{bmatrix} A+ \\ A \\ A+ \\ B+ \end{bmatrix} \quad \begin{bmatrix} I\,C \\ P\,S \\ L\,R \end{bmatrix} = \begin{bmatrix} B+ \\ B+ \\ A \end{bmatrix} \quad \begin{bmatrix} M\,O \\ C\,S \\ D\,E \\ T\,W \end{bmatrix} = \begin{bmatrix} A+ \\ A \\ B+ \\ A \end{bmatrix}$$

$$\begin{bmatrix} T\,M \\ C\,S \\ P\,N \end{bmatrix} = \begin{bmatrix} B+ \\ C+ \\ C \end{bmatrix}$$

By substituting the values corresponding the grades, we get
SK = 0.3 *90 + 0.24 * 80+ 0 .236 *90+0 .23 *70=83
RA = 0.61*70+ 0.18 *70+ 0.19 *80=70.5
PQ= 0.25 *90+0.24 *80+ 0.25 *70+ 0.26 *80=80
MS = 0.25 *70+0 .6 *50+ 0 .15 *40=53.5

So Performance (P) of the employee at the entry level using the arithmetic aggregation rule is: 0.47*SK+0.392*RA+ 0.0856*MS+0.053*PQ = 91.866.

IV. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

As the nature of the human being, linguistic values can change from person to person. In these circumstances, taking the fuzziness into account will provide less risky decisions. This study proposes a fuzzy AHP framework using Chang's extent analysis method to prioritize the performance measurement attributes and then assess the performance based on the grades obtained by the employee. Other than AHP, other fuzzy multi-attribute approaches such as fuzzy TOPSIS and fuzzy outranking methods can be used for the prioritization of performance measurement attributes.

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