# Force on a High Voltage Capacitor with Asymmetrical Electrodes 

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#### Abstract

When a high DC voltage is applied to a capacitor with strongly asymmetrical electrodes, it generates a mechanical force that affects the whole capacitor. This phenomenon is most likely to be caused by the motion of ions generated around the smaller of the two electrodes and their subsequent interaction with the surrounding medium. A method to measure this force has been devised and used. A formula describing the force has also been derived. After comparing the data gained through experiments with those acquired using the theoretical formula, a difference was found above a certain value of current. This paper also gives reasons for this difference.


Keywords-Capacitor with asymmetrical electrodes, Electrical field, Mechanical force, Motion of ions.

## I. INTRODUCTION

CAPACITOR is a basic electrical device widely known for a long time. Yet after we apply high voltage to it, even this basic component, if its electrodes are very asymmetrical, shows an unexpected behaviour. It generates a mechanical force. This is not just a simple Coulomb attraction force generated by oppositely charged electrodes, because the force tends to move with the whole capacitor in one direction. Although the force is relatively small, a potential application may arise in the form of a new type of propulsion. Even a force ranging in milinewtons may gain in effectiveness particularly on a micro- or nanoscale. Beside reliable studies from NASA and British Ministry of Defence there are many amateur and somewhat unreliable researches, so the quest for serious and reliable literature on this matter is not an easy matter.

The majority of sources agree that the effect is caused by the motion of charged particles (ions), which are accelerated by an electric field and their interaction with the surrounding medium, in most cases air. The most reliable set of experiments was carried out by NASA [1]. However we did
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not find an agreement between the results predicted by NASA's formula and our own experimental data. That is why we have tried to derive a new theoretical formula for this force and to verify it against the experimental data.

## II. ELECTROSTATIC HOVERCRAFT - LIFTER

A "Lifter" is a device, which is usually used to demonstrate the effects of the force generated by a capacitor with asymmetrical electrodes (see Fig. 1).


Fig. 1: Photo of a lifter
The frame is made from rods of lightweight insulating material (balsa or Styrofoam). A thin aluminium foil forms the lower electrode and the upper electrode is made from a thin wire (in our case a bare copper wire 0.1 mm in diameter). The frame holds both electrodes at a constant distance. After a positive high voltage of approximately 20 kV DC is applied to the upper smaller electrode (while the lower larger electrode is grounded), the whole construction is lifted into the air. As long as the voltage is applied, the force is present. This experiment clearly demonstrates that a mechanical force appears on capacitors with asymmetrical electrodes, yet it is not suitable for measuring the actual force. For this purpose we have created a so called element.

## III. ELEMENT AND A METHOD OF MEASUREMENT

In this text we define an element as one third of the device known as a lifter (see Fig. 1) i.e. a single capacitor with asymmetrical electrodes (see Fig. 2).


Fig. 2: Photo of an element
The upper electrode is again formed by a thin copper wire with a diameter of 0.1 mm , which is suspended at a distance of 30 mm above the larger lower electrode. This electrode is built from a Styrofoam block ( $10 \mathrm{~mm} \times 50 \mathrm{~mm} \times 100 \mathrm{~mm}$ ), which is covered in a thin aluminium foil.

Finding an effective and simple method for measuring the force originating on the capacitor was a relatively difficult task, because to measure precisely very small forces (ranging in milinewtons) is generally very complicated. That is why we had to find a way to transform the force into some other more easily measurable quantity. Finally we decided to put the element on a balance and to measure the force as a change in weight (the difference being before and after applying the high voltage). As the force is equivalent to a weight ranging in hundreds of milligrams, it was measured as a weight difference of the whole capacitor with asymmetrical electrodes (i.e. element) on a KERN balance, which is able to measure with the accuracy of 0.001 g . But to proceed with the experiment, it was necessary to meet two requirements. First we had to make sure that the damping of the balance is adequate to annul all possible oscillations. The second condition was to ensure the insulation of the parts of the setup from the digital balance, where the high voltage is present. Through experimentation we were able to ascertain that, at the values of voltage used, the oscillation of the measured values of weight is negligible and so is the effect of toughness of the connecting wires. For insulation of the balance we have used a support stand, built from Styrofoam, which due to low density is quite light. The support stand was made high enough to separate the capacitor from the sensitive electronics of the digital balance and, keeping in mind the fragility of Styrofoam, even quite durable. To its lower side a thin aluminium foil is fixed. The foil is kept grounded during experiments and thus the balance is further protected.

The whole measuring setup (digital balance KERN, insulating support stand and the element), which was used during all the experiments carried out so far, can be seen in Fig. 3.


Fig. 3: Photo of the measuring setup - digital balance, support stand and the element

As the method of measurement is based on measuring the differences of close values the result would be affected by an unomittable relative error. For this reason we repeated the measurement for each value of voltage ten times and used their mean value.

## IV. THEORY

Let us consider, in accordance with the other researchers, that the force originating on capacitors with asymmetrical electrodes has its cause in the motion of ions. In that case we see the mechanism of the origin of the force thus (see Fig. 4):

1) At the wire radius 0.1 mm and voltage 10 kV the electric field strength at a distance of 1 mm from the wire is ranging in $\mathrm{MV} / \mathrm{m}$. If the surrounding medium is air, it is ionized and a large amount of charged particles of both polarities will be created around the wire. Corona discharge clearly visible around the wire can be considered as proof.
2) Charged particles with the opposite charge to the polarity of the wire electrode are almost immediately drawn to it and discharged. Particles with the same polarity as the wire electrode are accelerated in the direction of the field strength vector to the drift velocity $v_{d}$ and thus move away from the smaller electrode with the same polarity towards the grounded larger electrode.


Fig. 4: Mechanism of the origin of the force
3) As they move through the surrounding medium (in our case air), the charged particles hit the neutral air molecules and because of the large number of these neutral molecules, the collisions are quite numerous (approximately $10^{7}$ times per second).
4) Because the kinetic energy of the charged particles accelerated by the electric field does not reach the value necessary to cause ionization of the neutral molecules, each hit results only in a transfer of momentum.
5) We also consider the neutral molecules to be stationary, as the thermal motion velocity is negligible with regard to the velocity of the charged particles in the electric field.
6) The law of conservation of momentum implies that after the collision the neutral molecule and the charged particle would travel in opposite directions. Before the collision the charged particle was moving away from the electrode, because of the electric field affecting it. So after the collision it should start moving back towards the electrode. Yet as the charged particle is still affected by the electric field, it is prevented from moving back and is again accelerated away from the electrode with the same polarity. But the momentum which the charged particle received from the collision is transferred to the electrode and thus to the whole construction of the capacitor.
7) After the collision the neutral molecule carrying the equal and opposite momentum moves away and loses the momentum through further collisions with other air molecules. As these are not drawn to or repelled from either of the electrodes, their momentum is not transferred to the capacitor.
8) Thus the force originates from this very difference - i.e. only the charged particles give their momentum to the capacitor and the neutral molecules exert no influence.
9) On the very end of its journey the charged particle hits the larger electrode, towards which it was travelling all along, but momentum transferred in this way is insignificant with regard to the sum of momentum in the opposite direction which the capacitor gained from the collisions.
10) This mechanism works, only if the ionization area around the thin wire is small enough. If the electric field strength between the electrodes rises to values where it is sufficient to cause the avalanche ionization effect, the force originating on the capacitor is drastically weakened, since it relies on the neutral molecules remaining neutral after
the collisions.
11) We also consider the current flow through the capacitor to be caused solely by the accelerated charged particles.
These are the necessary premises and simplifications, which could cause the following calculations to be inaccurate, however each of the approximations will be commented on later in the text.
To calculate the force we will use a formula for a force affecting a charged particle in an electric field. As we want to calculate only the value of the force, we will not use the vector form of the formula:

$$
\begin{equation*}
F=Q \cdot E, \tag{1}
\end{equation*}
$$

where $Q$ is the value of electric charge and $E$ is the electric field strength. Yet in our case there is a very large number of charged particles of the same polarity moving (in the same direction) through the electric field. That is why we will use a sum of all charges:

$$
\begin{equation*}
F=E \cdot \sum Q_{i} \tag{2}
\end{equation*}
$$

We can carry out this operation because we intend to find only the numerical value of the velocity of the charged particles and because of the fact that the principle of superposition can be applied on equations of electrostatics and motion.
Our next goal is to find a formula, which will describe the electric field surrounding our capacitor with asymmetrical electrodes, i.e. the element (see Fig. 5).


Fig. 5: Scheme of the element and its dimensions
Although the design seems rather simple, the formula cannot be found analytically. Using FEM (finite element method) a description of the field could be procured, but according to [2] we can use the following formula with sufficient accuracy:

$$
\begin{equation*}
E=\frac{U}{\ln \left(\frac{d}{R}\right)} \cdot \frac{1}{x} \tag{3}
\end{equation*}
$$

# International Journal of Mechanical, Industrial and Aerospace Sciences <br> ISSN: 2517-9950 <br> Vol:4, No:9, 2010 

where $U$ is the voltage applied to the electrodes, $d$ is the distance between the electrodes, $R$ is the radius of the wire electrode and $x$ is the distance from its axis (see Fig. 5).

Furthermore the current flow through the circuit can be defined as follows:

$$
\begin{equation*}
I=\frac{\sum Q_{i}}{t} \tag{4}
\end{equation*}
$$

where $t$ is the amount of time it takes for a charged particle to travel from one electrode to the other - the distance $d$.

If $v_{d}$ is the drift velocity of the charged particles, then according to [3] we can state the following formula:

$$
\begin{equation*}
v_{d}=k \cdot E \tag{5}
\end{equation*}
$$

where $k$ represents the ion mobility. This constant depends on many factors such as pressure, temperature etc. For our purpose let us take the standard value for air under standard conditions (atmospheric pressure, room temperature etc.). The last thing which remains is to establish whether the charged particles have positive or negative charge, as the value of the ion mobility constant depends on the polarity. We will use the value for positive ions, as the smaller electrode is positive during all our measurements - i.e. $k=2.1 \cdot 10^{-4} \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$. If we use the electric field $E$ from (3) on (5), we will get the following:

$$
\begin{equation*}
v_{d}=k \cdot \frac{U}{\ln \left(\frac{d}{R}\right)} \cdot \frac{1}{x} \tag{6}
\end{equation*}
$$

We can clearly see that the drift velocity at a given voltage and geometry (which was not changed during our experiments) depends only on the distance of the charged particle from the wire electrode axis. To advance in the calculations we will use (6) to get a formula describing the time it takes the charged particle to travel the whole distance $d$. According to the basic motion equations the time the charged particle needs to move the distance $d$ moving at the velocity $v_{d}$ can be defined as:

$$
\begin{equation*}
t=\int_{0}^{d} \frac{1}{v_{d}} d x \tag{7}
\end{equation*}
$$

Using $v_{d}$ from (6) on (7) and after integration and applying the limits we obtain the following formula:

$$
\begin{equation*}
t=\frac{\ln \left(\frac{d}{R}\right)}{2 k \cdot U} \cdot d^{2} \tag{8}
\end{equation*}
$$

To get the formula for the total force, we apply the sum of
all charges $\sum Q_{i}$ from (4) on (2):

$$
\begin{equation*}
F=I \cdot t \cdot E . \tag{9}
\end{equation*}
$$

Again we use the time $t$ from (8) and the electric field strength $E$ from (3) and apply them both on (9). Thus we derive the final form of the formula describing the originating force:

$$
\begin{equation*}
F=I \cdot \frac{d}{2 k} \tag{10}
\end{equation*}
$$

where $I$ is the current flow through the circuit, $d$ is the distance between electrodes and $k$ is the ion mobility constant. From this formula we can also see that the direction of the resulting force does not depend on the polarity of the applied voltage, i.e. the direction of the current. If we were to switch the poles, the only change this would cause would be that the charged particles moving between the electrodes would be negative instead of positive. As the surrounding medium is air, the ion mobility of negative charged particles is under normal circumstances greater than that of the positive charged particles. Thus our original setup, where the smaller electrode is charged positively and the larger electrode is grounded, yields greater force. This fact, which was concluded out of (10), was also experimentally proven.

## V. COMPARISON OF THE THEORETICAL DATA WITH EXPERIMENTS

To compare the theoretically calculated data we obtained using the final formula (10) and those that came from measuring the element, we put both sets into a graph in order to see any differences clearly (see Fig. 6).


Fig. 6: Theoretical and experimental values of the force in relation to current

From this graph it is obvious that at currents up to $30 \mu \mathrm{~A}$ the theoretically calculated and experimentally measured force is practically identical. The reason for the sets of values disagreeing at currents higher then $30 \mu \mathrm{~A}$ is as follows. A very important condition, on which (10) may be considered valid, is that the momentum, which the neutral molecules of the surrounding medium received from the collisions with the charged particles, never returns to the capacitor (particularly
the grounded larger electrode). This condition is fulfilled only if the energy the electric field gives to the charged particles is not sufficient for avalanche ionization of the surrounding medium. If the electric field strength between the electrodes reached values sufficient for the avalanche ionization, the neutral molecules would no longer remain neutral after the collisions with the charged particles and thus would become charged particles themselves. As such they would be drawn towards one of the electrodes and in the end their momentum would be transferred back to the construction of the capacitor. Then their contribution to the force originating on the capacitor would be zero. They would of course hit other neutral molecules and if they were far enough from the smaller electrode the momentum transfer would occur, but the area where only momentum is transferred between the charged and neutral particles would become smaller as the electric field increases. The more the neutral molecules become ionized, the more the momentum would return to the capacitor and thus the increase of the force would be smaller and the whole device less efficient.

As the Townsend discharge theory [4] implies, the point where the avalanche ionization begins to take effect, is the same as the point where the theoretical values began to differ from our experimental data. To prove this further, we used the data to make a volt-ampere characteristic of the element (see Fig. 7).


Fig. 7: Volt-ampere characteristic of the element
From Fig. 6 we can also determine a certain point of optimal efficiency of the capacitor, i.e. the point, above which the force grows only minimally and at the price of a massive increase of current. From this point on the current above the optimal $30 \mu \mathrm{~A}$ is caused mainly by the avalanche ionization and it contributes less and less to the force originating on the capacitor.

## VI. CONCLUSION

We proposed a theoretical explanation of the origin of the force generated on a capacitor with asymmetrical electrodes. This lead to the derivation of the formula (10), which describes the situation on a real element very well up to the current of $30 \mu \mathrm{~A}$. Above this value the conditions, for which
the formula was derived, cease to be fulfilled and the formula is no longer valid. Yet the formula is crucial for practical applications of said phenomenon, as it describes the behaviour of the element up to and including the point of the highest efficiency, i.e. the working area of the element.
We also designed and carried out a method of measuring the force which allowed us to experimentally verify the theoretically calculated data we acquired using the derived formula. We believe that this method will allow further and hopefully more detailed research in the field of forces originating on any capacitor with asymmetrical electrodes even from angles this paper has not yet touched upon. Simultaneous with this research concerning the capacitor in a gas medium we conducted another study of the effect of liquid and solid dielectric media surrounding the capacitor on this phenomenon. But that will be the subject of another paper in due course.

## Acknowledgment

The authors would like to thank Prof. Aleš Richter and Prof. Miloslav Košek for their support and creative ideas.

This work was supported by the grant of Czech Science Foundation No. GA 102/08/H081 - Non-standard application of physical fields.

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