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A New Quantile Based Fuzzy Time Series Forecasting Model

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Abstract**—**Time series models have been used to make predictions of academic enrollments, weather, road accident, casualties and stock prices, etc. Based on the concepts of quartile regression models, we have developed a simple time variant quantile based fuzzy time series forecasting method. The proposed method bases the forecast using prediction of future trend of the data. In place of actual quantiles of the data at each point, we have converted the statistical concept into fuzzy concept by using fuzzy quantiles using fuzzy membership function ensemble. We have given a fuzzy metric to use the trend forecast and calculate the future value. The proposed model is applied for TAIFEX forecasting. It is shown that proposed method work best as compared to other models when compared with respect to model complexity and forecasting accuracy.

Keywords**—**Quantile Regression, Fuzzy time series, fuzzy logical relationship groups, heuristic trend prediction.

I. Introduction

IT is obvious that forecasting activities play an important role in our daily life. The traditional statistical approaches for time series can predict problems arising from new trends, but fail to forecast the data with linguistic facts. Furthermore, the traditional time series requires more historical data along with some assumptions like normality postulates.

In recent years, many researchers used fuzzy time series to handle forecasting problems. Song and Chissom [1-2] presented the concept of fuzzy time series based on the historical enrollments of the University of Alabama. Song and Chissom [3] presented the time-invariant fuzzy time series model and the time-variant fuzzy time series model based on the fuzzy set theory for forecasting the enrollments of the University of Alabama. Chen [4] presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series. It has the advantage of reducing the calculation, time and simplifying the calculation process using simple fuzzy number arithmetic operations. Song et al. presented some forecasting methods [3], [5], [6] to forecast the enrollments of the University of Alabama. Hwang, Chen and Lee [7] used the differences of the enrollments to present a method to forecast the enrollments of the University of Alabama based on fuzzy time series. Huarng [8-9] used simplified calculations with the addition of heuristic rules to forecast the enrollments using [4]. Chen [10] presented a forecasting method based on high-order fuzzy time series for forecasting the enrollments problem. Chen and Hwang [11] presented a method based on fuzzy time series to forecast the daily temperature. Tsaur, Yang and Wang [12] proposed a fuzzy relation matrix to represent a time-invariant relation. Based on the concept of fuzziness in information theory, the concept of entropy is applied to measure the degrees of fuzziness when a time-invariant relation matrix is derived. Li and Kozma [13] presented a dynamic neural network method for time series prediction using the KIII model. Su and Li [14] presented a method for fusing global and local information in predicting time series based on neural networks. Sullivan and Woodall [15] reviewed the first-order time-variant fuzzy time series model and the first-order time-invariant fuzzy time series model presented by [1], where their models are compared with each other and with a time-variant Markov model using linguistic labels with probability distributions. Lee et al. [16] presented two factor high order fuzzy time series for forecasting TAIFEX. Jilani, Burney and Ardil [17] and Jilani and Burney [18] presented new fuzzy metrics for high order multivariate fuzzy time series forecasting for car road accident casualties in Belgium. Li and Cheng [19] proposed a novel deterministic forecasting model to manage the issue of interval lengths in the sense of accuracy, robustness and reliability. He applied there model to forecasting enrollments in the University of Alabama.

In this paper a comprehensive concept is proposed for promoting performance and facing future changing trends. The new proposed approach is based on prediction of the trend using third order fuzzy relationships. We have applied this new method for TAIFEX forecasting. The results reveal that the proposed method is comparably better than other fuzzy time series methods with respect to model complexity and model forecasting accuracy.

II. Some Basic Concepts

The concept of fuzzy logic and fuzzy set theory [20] was introduced to cope with the ambiguity and uncertainly of most of the real world problems. Thus a time series introduced with

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fuzziness is termed as fuzzy time series. In this section, the basic concepts of fuzzy set theory as well as quantile regression are viewed and some of the essentials are being reproduced to make the study self contained.

Let $U = \{x_1, x_2, ..., x_n\}$ be the universe of discourse and A_i be the fuzzy set of U defined as

$$A_i = f_{A_i}(x_1) + f_{A_i}(x_2) + ... + f_{A_i}(x_n)$$

where $f_{A_i}(x_1)$ is the membership function of the fuzzy set A_i, and $f_{A_i}(x_1)$ represents degree of membership of x_j in A_i.

Let $Y(t), (t = ..., 0,1,2,...)$ be the universe of discourse and $Y(t) \subseteq R$. Assume that $f_i(t),\ i = 1,2,...$ is defined in the universe of discourse $Y(t)$ and $F(t)$ is a collection of $f(t_i), (i = ..., 0,1,2,...)$, then $F(t)$ is called a fuzzy time series of $Y(t),\ i = 1,2,...$. Using fuzzy relation, we define $F(t) = F(t-1) \circ R(t, t-1)$ where $R(t, t-1)$ is a fuzzy relation and "$\circ$" is the max–min composition operator, then $F(t)$ is caused by $F(t-1)$ where $F(t)$ and $F(t-1)$ are fuzzy sets.

Let $F(t)$ be a fuzzy time series and let $R(t, t-1)$ be a first-order model of $F(t)$. If $R(t, t-1) = R(t-1, t-2)$ for any time t, then $F(t)$ is called a time-invariant fuzzy time series. If $R(t, t-1)$ is dependent on time t, that is, $R(t, t-1)$ may be different from $R(t-1, t-2)$ for any t, then $F(t)$ is called a time-variant fuzzy time series. Song and Chissom [1-2] proposed the time-variant fuzzy time-series model and forecasted the enrollments of the University of Alabama based on the model.

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1)$, $F(t-2)$,…, $F(t-n)$, then the nth-order fuzzy logical relationship is represented by

$$F(t-n), ..., F(t-2), F(t-1) \rightarrow F(t)$$

where $F(t-1)$, $F(t-2)$,…, $F(t-n)$ and $F(t)$ are all fuzzy sets, where $F(t-1)$, $F(t-2)$,…, $F(t-n)$ is called the antecedent and $F(t)$ is called the consequent of the nth order fuzzy logical relationship. A set of nth-order fuzzy logical relationships have same antecedents that form an nth-order fuzzy logical relationship group.

Quantile regression, developed by Koenker [21], is an extension of the classical least squares estimation of the conditional mean to a collection of models for different conditional quantile functions. As the median (quantile) regression estimator minimizes the symmetrically weighted sum of absolute errors to estimate the conditional median (quantile) function, other conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors, where the weights are functions of the quantile of interest. We have used triangular weight functions with fixed parameters 0.5, 1, 0.5 for symmetric case, although asymmetric weighted sum is also possible Thus, quantile regression is robust to the presence of outliers. This technique has been used widely in the past decade in many areas of applied econometrics; applications include investigations of wage structure.

TABLE I
THE HISTORICAL DATA OF TAIFEX

Date	Actual TAIFEX index	Date	Actual TAIFEX index
8/3/1998	7552	9/1/1998	6409
8/4/1998	7560	9/2/1998	6430
8/5/1998	7487	9/3/1998	6200
8/6/1998	7462	9/4/1998	6430.2
8/7/1998	7515	9/5/1998	6697.5
8/10/1998	7365	9/7/1998	6722.3
8/11/1998	7360	9/8/1998	6859.4
8/12/1998	7330	9/9/1998	6769.6
8/13/1998	7291	9/10/1998	6709.75
8/14/1998	7320	9/11/1998	6726.5
8/15/1998	7300	9/14/1998	6774.55
8/17/1998	7219	9/15/1998	6762
8/18/1998	7220	9/16/1998	6952.75
8/19/1998	7285	9/17/1998	6906
8/20/1998	7274	9/18/1998	6842
8/21/1998	7225	9/19/1998	7039
8/24/1998	6955	9/21/1998	6861
8/25/1998	6949	9/22/1998	6926
8/26/1998	6790	9/23/1998	6852
8/27/1998	6835	9/24/1998	6890
8/28/1998	6695	9/25/1998	6871
8/29/1998	6728	9/28/1998	6840
8/31/1998	6566	9/29/1998	6806
		9/30/1998	6787

III. Proposed Method Based on Trend Prediction

In this section we present a new method for TAIFEX forecasting, based on high order fuzzy logical relationships and our proposed approach. Table I shows the TAIFEX [16]. First based on Table I, we define the universe of discourse $U = [U_{min} - U_1, U_{max} - U_2]$, where U_{min} and U_{max} are the minimum and maximum values in the universe of discourse U and U_1, U_2 are two real positive numbers in the universe of discourse to divide the universe of discourse into n equal length intervals $u_1, u_2, ..., u_n$. The proposed method is now presented as follows:

Step 1: Define the linguistic term $A_1, A_2, ..., A_n$ for each interval $u_1, u_2, ..., u_n$ of the universe of discourse U, as follows:

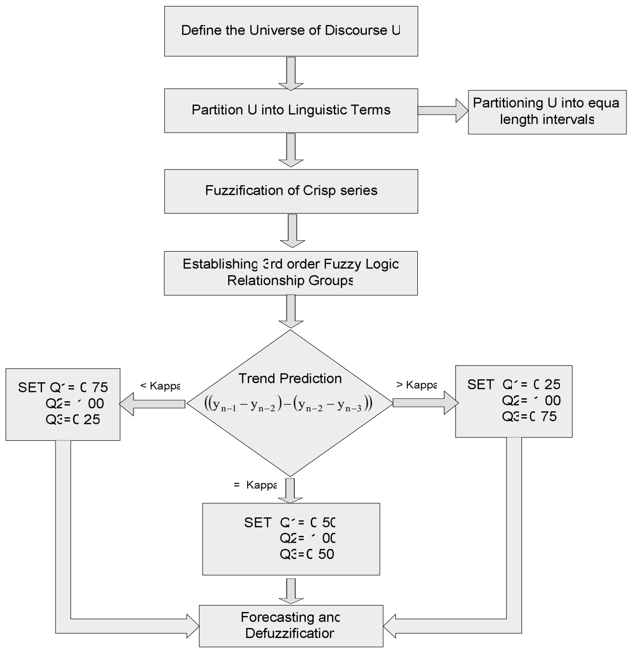

Fig. 1 Process Flow Diagram of the Proposed Method

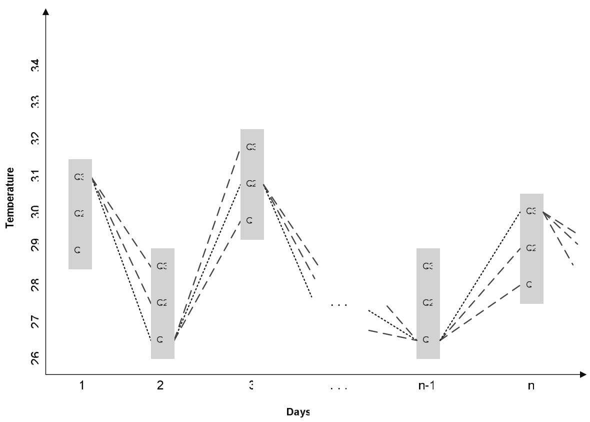

Fig. 2 FTS Forecasting model based on trend prediction

$$A_1 = {1}/{u_1} + {0.5}/{u_2} + {0}/{u_3} + {0}/{u_4} + {0}/{u_5} + ... + {0}/{u_{n-2}} + {0}/{u_{n-1}} + {0}/{u_n}$$

$$A_2 = {0.5}/{u_1} + {1}/{u_2} + {0.5}/{u_3} + {0}/{u_4} + {0}/{u_5} + ... + {0}/{u_{n-2}} + {0}/{u_{n-1}} + {0}/{u_n}$$

$$A_3 = {0}/{u_1} + {0.5}/{u_2} + {1}/{u_3} + {05}/{u_4} + {0}/{u_5} + ... + {0}/{u_{n-2}} + {0}/{u_{n-1}} + {0}/{u_n}$$

$$\vdots$$

$$A_n = {0}/{u_1} + {0}/{u_2} + {0}/{u_3} + {0}/{u_4} + {0}/{u_5} + ... + {0}/{u_{n-2}} + {0.5}/{u_{n-1}} + {1}/{u_n}$$

where $u_1, u_2, ..., u_n$ are equal length partitions of the universe of discourse U and $A_1, A_2, ..., A_n$ are the corresponding fuzzy terms of $u_1, u_2, ..., u_n$. If the average TAIFEX of Day i lie in the interval u_j, then that value is fuzzified into A_j with membership grade $\mu_{A_j}(x) = 1$ where $1 \le j \le n$.

Step 2: Construct higher order fuzzy logical relationship groups (FLRGs) as follows:

$$A_{i1}, A_{j1}, A_{k1} \rightarrow A_{q1}$$
$$A_{i2}, A_{j2}, A_{k2} \rightarrow A_{q2}$$
$$\vdots$$
$$A_{im}, A_{jm}, A_{km} \rightarrow A_{qm}$$

where the FLRG $A_{i1}, A_{j1}, A_{k1} \rightarrow A_{q1}$ denotes that, If the fuzzified value of year p is A_{i1}, year q is A_{j1} and year r is A_{k1}, then the fuzzified value of year s is A_{q1}.

Step 3: For the 3rd order fuzzy logical relationship group, the forecasted value of Day j is calculated using any one of the three following conditions. In our proposed quantile based fuzzy time series forecasting method, we have introduced a "Trend" parameter that determines the direction of the series for Day i. The trend value is determined using last three forecasted values y_{n-2}, y_{n-1} and y_n.

If $((y_{n-1} - y_{n-2}) - (y_{n-2} - y_{n-3})) > \kappa$, then the trend will go upward and the forecasted value will be

$$t_j = \frac{2}{\frac{0.25}{m_{j-1}} + \frac{1.0}{m_j} + \frac{0.75}{m_{j+1}}} \quad (1)$$

If $((y_{n-1} - y_{n-2}) - (y_{n-2} - y_{n-3})) < \kappa$, then the trend will go downward and the forecasted value will be

$$t_j = \frac{2}{\frac{0.75}{m_{j-1}} + \frac{1.0}{m_j} + \frac{0.25}{m_{j+1}}} \quad (2)$$

If $((y_{n-1} - y_{n-2}) - (y_{n-2} - y_{n-3})) = \kappa$, then the trend will remain unchanged and the forecasted value will be

$$t_j = \frac{2}{\frac{0.5}{m_{j-1}} + \frac{1.0}{m_j} + \frac{0.5}{m_{j+1}}} \quad (3)$$

where m_{j-1}, m_j and m_{j+1} are the mid points of the intervals u_{j-1}, u_j and u_{j+1} with corresponding linguistic terms A_{j-1}, A_j and A_{j+1}.

The parameter kappa(κ) is a threshold value and determines the expected trend of the series for day i. Selection of the parameter κ is a trial-and-error approach and has prominent affects on the overall fitness of the proposed forecasting method. In order to compare the proposed method with existing methods, we use the average forecasting error rate (AFER) Eq. (4) and mean square error (MSE) Eq. (5) as the fitness values for TAIFEX forecasting.

$$AFER = \frac{\sum_{j=1}^{n} \left| F_j - A_j \right| / A_j}{n} \quad (4)$$

$$MSE = \frac{\sum_{j=1}^{n} \left(F_j - A_j \right)^2}{n} \quad (5)$$

where A_j is the actual value of Day j and F_j is the forecasted value of day j.

IV. Experiments

Based on the steps defined in Section III, initially, we partition n the universe of discourse U into partitions of equal length and associate linguistic measures for each of them. For the simplicity of calculations, we have used triangular membership function in defining linguistic terms $A_1, A_2, ..., A_n$. Finally, using Eqs. (1-3), forecasted values are calculated and the accuracy is measured using Eq. (4-5). Fig. 2 is an illustrative presentation of the proposed method. The dotted lines show the actual trend movement of the series and dashed lines represents expected (possible) trend movement. The trend based forecasting for n observations of the series is performed based on the trend predictions. The method outlined in Section III is now implemented here for TAIFEX forecasting problem.

Step1: First based on Table II, we define $U = [6200, 7600]$ and partition it into seven intervals $u_1 = [6200, 6400]$, $u_2 = [6400, 6600]$, $u_3 = [6600, 6800]$, $u_4 = [6800, 7000]$ $u_5 =$, $[7000, 7200]$, $u_6 = [7200, 7400]$ and $u_7 = [7400, 7600]$. Now define fuzzy membership function for each interval to convert non-fuzzy data into fuzzy sets as shown in Step 1. Associate each observation to a linguistic term A_i with $\mu_{A_i}(x) = 1$

Step2: Construct the 3rd order FLRGs as shown in Table III.

Step3: For the third order FLRGs, the forecast value is alculated using the third order conditions defined in Eqs. (1-3). We have applied many values of the "Trend" parameter and corresponding fitness values of AFER and MSE are reported in Table V. In Table VI, a comparison is shown among the proposed method and [4], [8], [9] and [16]. It is clear that proposed method gives near to the best results with respect to model complexity and forecasting accuracy. The parameter κ is used to determine the trend of the series and we can control the possible move of the trend using this parameter.

From Table VI, it is clear that the proposed method has slightly lower accuracy rate as compared to [16]. But Lee's model [16] is based on two factors and involve1s fuzzification, fuzzy logical groups' formation and defuzzification of two factors. Thus Lee's model [16] is computationally expensive as compared to proposed method. Therefore, the proposed method has better performance as compared to [16] with respect to model complexity and better forecasting accuracy than [4], [8] and [9].

TABLE II
THIRD-ORDER FUZZY LOGICAL RELATIONSHIP GROUPS FOR TAIFEX PREDICTION DATA

Group 1:	$A_1, A_3, A_8 \rightarrow A_{12}$	Group 15:	$A_{10}, A_8, A_8 \rightarrow A_{12}$
Group 2:	$A_3, A_8, A_{12} \rightarrow A_{11}$	Group 16:	$A_8, A_8, A_{12} \rightarrow A_{12}$
Group 3:	$A_8, A_{12}, A_{11} \rightarrow A_{10}$	Group 17:	$A_8, A_{12}, A_{12} \rightarrow A_{13}$
Group 4:	$A_{12}, A_{11}, A_{10} \rightarrow A_{11}$	Group 18:	$A_{12}, A_{12}, A_{13} \rightarrow A_{13}$
Group 5:	$A_{11}, A_{10}, A_{11} \rightarrow A_{10}$	Group 19:	$A_{12}, A_{13}, A_{13} \rightarrow A_7$
Group 6:	$A_{10}, A_{11}, A_{10} \rightarrow A_8$	Group 20:	$A_{13}, A_{13}, A_7 \rightarrow A_4$
Group 7:	$A_{11}, A_{10}, A_8 \rightarrow A_{10}$	Group 21:	$A_{13}, A_7, A_4 \rightarrow A_3$
Group 8:	$A_{10}, A_8, A_{10} \rightarrow A_9$	Group 22:	$A_7, A_4, A_3 \rightarrow A_4$
Group 9:	$A_8, A_{10}, A_9 \rightarrow A_6$	Group 23:	$A_4, A_3, A_4 \rightarrow A_2$
Group 10:	$A_{10}, A_9, A_6 \rightarrow A_7$	Group 24:	$A_3, A_4, A_2 \rightarrow A_6$
Group 11:	$A_9, A_6, A_7 \rightarrow A_3$	Group 25:	$A_4, A_2, A_6 \rightarrow A_4$
Group 12:	$A_6, A_7, A_3 \rightarrow A_{10}$	Group 26:	$A_2, A_6, A_4 \rightarrow A_8$
Group 13:	$A_7, A_3, A_{10} \rightarrow A_8$	Group 27:	$A_6, A_4, A_8 \rightarrow A_{12}$
Group 14:	$A_3, A_{10}, A_8 \rightarrow A_8$		

V. Conclusion And Future Research

In this paper we have proposed a new method for time series forecasting having simple computational algorithm of complexity of linear order. The proposed method first predicts the trend of the future value and then use the proposed quantile based fuzzy forecasting approach. The method is found to be robust and can handle the problem of inaccuracy in the data set. As compared to other methods, the complexity of the proposed model is lower than other methods. The suitability of the method is examined for TAIFEX forecasting and in both the applications it's found near to be the superior in nature in terms of accuracy in forecast, robustness and complexity. As future plans, we will extend this paper to obtain further improved results using other soft computing approaches.

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TABLE V
A COMPARISON OF FORECASTING VALUES OF THE TAIFEX FOR DIFFERENT FORECASTING METHODS

Date	Actual TAIFEX	Chen's Method (1996)	Huarng's method (2001a)	Huarng's method (2001b)	Lee et al.'s ,method (2006)	Proposed Method
8/3/1998	7552					--
8/4/1998	7560	7450	7450	7450		--
8/5/1998	7487	7450	7450	7450		--
8/6/1998	7462	7500	7450	7500	7450	7413
8/7/1998	7515	7500	7500	7500	7550	7459
8/10/1998	7365	7450	7450	7450	7350	7348
8/11/1998	7360	7300	7350	7300	7350	7248
8/12/1998	7330	7300	7300	7300	7350	7348
8/13/1998	7291	7300	7350	7300	7250	7248
8/14/1998	7320	7183.33	7100	7188.33	7350	7348
8/15/1998	7300	7300	7350	7300	7350	7348
8/17/1998	7219	7300	7300	7300	7250	7248
8/18/1998	7220	7183.33	7100	7100	7250	7248
8/19/1998	7285	7183.33	7300	7300	7250	7348
8/20/1998	7274	7183.33	7100	7188.33	7250	7348
8/21/1998	7225	7183.33	7100	7100	7250	7248
8/24/1998	6955	7183.33	7100	7100	6950	6847
8/25/1998	6949	6850	6850	6850	6950	6847
8/26/1998	6790	6850	6850	6850	6750	6747
8/27/1998	6835	6775	6650	6775	6850	6847
8/28/1998	6695	6850	6750	6750	6650	6747
8/29/1998	6728	6750	6750	6750	6750	6647
8/31/1998	6566	6775	6650	6650	6550	6547
9/1/1998	6409	6450	6450	6450	6450	6447
9/2/1998	6430	6450	6550	6550	6450	6497
9/3/1998	6200	6450	6350	6350	6250	6384
9/4/1998	6403.2	6450	6450	6450	6450	6447
9/5/1998	6697.5	6450	6550	6550	6650	6747
9/7/1998	6722.3	6750	6750	6750	6750	6747
9/8/1998	6859.4	6775	6850	6850	6850	6847
9/9/1998	6769.6	6850	6750	6750	6750	6747
9/10/1998	6709.75	6775	6650	6650	6750	6647
9/11/1998	6726.5	6775	6850	6775	6750	6747
9/14/1998	6774.55	6775	6850	6775	6817	6747
9/15/1998	6762	6775	6650	6775	6817	6747
9/16/1998	6952.75	6775	6850	6850	6817	6847
9/17/1998	6906	6850	6950	6850	6950	6947
9/18/1998	6842	6850	6850	6850	6850	6847
9/19/1998	7039	6850	6950	6850	7050	7048
9/21/1998	6861	6850	6850	6850	6850	6947
9/22/1998	6926	6850	6950	6850	6950	6847
9/23/1998	6852	6850	6850	6850	6850	6947
9/24/1998	6890	6850	6950	6850	6850	6847
9/25/1998	6871	6850	6850	6850	6850	6947
9/28/1998	6840	6850	6750	6750	6850	6847
9/29/1998	6806	6850	6750	6850	6850	6847
9/30/1998	6787	6850	6750	6750	6750	6747
MSE		9668.94	7856.5	5437.58	1364.56	3736.64
RMSE		98.3308	88.6369	73.7399	36.9400	61.13
AFER		1.05%	1.03%	0.89%	0.42%	0.72%

TABLE III
DAILY TAIFEX FORECASTING USING TREND PREDICTOR KAPPA (κ)=-10

Date	TAIFEX	Fuzzy Rule	FLRG	κ	TREND	Forecasting
8/3/1998	7552	A_7	A_6, A_7	--	--	--
8/4/1998	7560	A_7	A_6, A_7	--	--	--
8/5/1998	7487	A_7	A_6, A_7	--	--	--
8/6/1998	7462	A_7	A_6, A_7	-81	Downward	7413
8/7/1998	7515	A_7	A_6, A_7, A_8	48	Upward	7459
8/10/1998	7365	A_6	A_5, A_6, A_7	78	Upward	7348
8/11/1998	7360	A_6	A_5, A_6, A_7	-203	Downward	7248
8/12/1998	7330	A_6	A_5, A_6, A_7	145	Upward	7348
8/13/1998	7291	A_6	A_5, A_6, A_7	-25	Downward	7248
8/14/1998	7320	A_6	A_5, A_6, A_7	-9	Upward	7348
8/15/1998	7300	A_6	A_5, A_6, A_7	68	Upward	7348
8/17/1998	7219	A_6	A_5, A_6, A_7	-49	Downward	7248
8/18/1998	7220	A_6	A_6, A_7, A_8	-61	Downward	7248
8/19/1998	7285	A_6	A_5, A_6, A_7	82	Upward	7348
8/20/1998	7274	A_6	A_5, A_6, A_7	64	Upward	7348
8/21/1998	7225	A_6	A_5, A_6, A_7	-76	Downward	7248
8/24/1998	6955	A_4	A_3, A_4, A_5	-38	Downward	6847
8/25/1998	6949	A_4	A_3, A_4, A_5	-221	Downward	6847
8/26/1998	6790	A_3	A_2, A_3, A_4	264	Upward	6747
8/27/1998	6835	A_4	A_3, A_4, A_5	-153	Downward	6847
8/28/1998	6695	A_3	A_2, A_3, A_4	204	Upward	6747
8/29/1998	6728	A_3	A_2, A_3, A_4	-185	Downward	6647
8/31/1998	6566	A_2	A_1, A_2, A_3	173	Upward	6547
9/1/1998	6409	A_2	A_1, A_2, A_3	-195	Downward	6447
9/2/1998	6430	A_2	A_1, A_2, A_3	5	Upward	6497
9/3/1998	6200	A_1	A_1, A_2	178	Upward	6384
9/4/1998	6430.2	A_2	A_1, A_2, A_3	-251	Downward	6447
9/5/1998	6697.5	A_3	A_2, A_3, A_4	460.2	Upward	6747
9/7/1998	6722.3	A_3	A_2, A_3, A_4	37.1	Upward	6747
9/8/1998	6859.4	A_4	A_3, A_4, A_5	-242.5	Downward	6847
9/9/1998	6769.6	A_3	A_2, A_3, A_4	112.3	Upward	6747
9/10/1998	6709.75	A_3	A_2, A_3, A_4	-226.9	Downward	6647
9/11/1998	6726.5	A_3	A_2, A_3, A_4	29.95	Upward	6747
9/14/1998	6774.55	A_3	A_2, A_3, A_4	76.6	Upward	6747
9/15/1998	6762	A_3	A_2, A_3, A_4	31.3	Upward	6747
9/16/1998	6952.75	A_4	A_3, A_4, A_5	-60.6	Downward	6847
9/17/1998	6906	A_4	A_3, A_4, A_5	203.3	Upward	6947
9/18/1998	6842	A_4	A_3, A_4, A_5	-237.5	Downward	6847
9/19/1998	7039	A_5	A_4, A_5, A_6	-17.25	Downward	7048
9/21/1998	6861	A_4	A_3, A_4, A_5	261	Upward	6947
9/22/1998	6926	A_4	A_3, A_4, A_5	-375	Downward	6847
9/23/1998	6852	A_4	A_3, A_4, A_5	243	Upward	6947
9/24/1998	6890	A_4	A_3, A_4, A_5	-139	Downward	6847
9/25/1998	6871	A_4	A_3, A_4, A_5	112	Upward	6947
9/28/1998	6840	A_4	A_3, A_4, A_5	-57	Downward	6847
9/29/1998	6806	A_4	A_3, A_4, A_5	-12	Downward	6847
9/30/1998	6787	A_3	A_2, A_3, A_4	-3	Upward	6747
MSE						3736.64
RMSE						61.13
AFER						0.7173%

TABLE IV
FITNESS OF PROPOSED MODEL WITH VARYING VALUES OF TREND PREDICTION FOR TAFEX PREDICTION

	Kappa (κ)						
	-50	-25	-10	0	10	25	50
MSE	4755.21	4659.46	3736.64	4456.78	3879.45	4150.83	4562.11
RMSE	68.96	68.26	61.13	66.76	62.29	64.43	67.54
AFER	0.8037%	0.8049%	0.7173%	0.7825%	0.7327%	0.7472%	0.7977%

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