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The countabilities of soft topological spaces

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Abstract—Soft topological spaces are considered as mathematical tools for dealing with uncertainties, and a fuzzy topological space is a special case of the soft topological space. The purpose of this paper is to study soft topological spaces. We introduce some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft separable spaces, and some basic properties of these concepts are explored.

Keywords—soft sets, soft first-countable spaces, soft second-countable spaces, soft separable spaces, soft Lindelöf.

I. INTRODUCTION

THE real world is too complex for our direct and immediate understanding, for example, many disciplines, including engineering, medicine, economics, and sociology, are highly dependent on the task of modeling uncertain data. When the uncertainty is highly complicated and difficult to characterize, classical mathematical approaches are often insufficient to derive effective or useful models. There are some theories: the theory of fuzzy sets [14], the theory of vague sets [2], and the theory of rough sets [11], which can be considered as mathematical tools for dealing with uncertainties. However, all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov in [7]. Molodtsov [7] introduced the concept of a soft set in order to solve complicated problems. In [7], Molodtsov presented the fundamental results of the new theory and successfully applied it to several directions such as game theory, operations research, theory of probability, Riemann-integration, Perron integration, smoothness of functions etc. A soft set is a collection of approximate descriptions of an object. Molodtsov [7] also proved how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, game theory, rough set theory and probability theory. Soft systems provide a very general framework with the involvement of parameters. Therefore, research works on soft set theory and its applications in various fields are progressing rapidly.

Recently, Shabir and Naz [12] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Then some authors studied some of basic concepts and properties of soft topological spaces, see [1], [3], [5], [9], [12], [13]. In particular, Zorlutuna, Akdag, Min and Atmaca [13] showed that a fuzzy topological space is a special case of the soft topological space, and that an ordinary topological space can be considered a soft topological space. However, every soft topological space is not an ordinary topological space in general.

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In the present study, we introduce some new concepts in soft topological spaces such as soft first-countable spaces, soft second-countable spaces and soft separable spaces, and some properties and relations of there concepts are discussed.

II. PRELIMINARIES

Definition 2.1: [7] Let U be an initial universe and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A).

Definition 2.2: [8] For two soft sets (F, A) and (G, B) over a common universe U, $(F; A)$ is a *soft subset* of $(G; B)$, denoted by $(F, A)\widetilde{\subseteq}(G, B)$, if $A \subset B$ and $e \in A$, $F(e) \subseteq G(e)$.

(F, A) is said to be a *soft superset* of (G, B), if (G, B) is a soft subset of (F, A), $(F, A)\widetilde{\supseteq}(G, B)$.

Definition 2.3: [8] Two soft sets (F, A) and (G, B) over a common universe U are said to be *soft equal* if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4: [4] The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c : A \to \mathcal{P}(U)$ is a mapping given by $F^c(e) = U - F(e)$ for each $e \in A$. F^c is called the *soft complement function* of F. Clearly, $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 2.5: [8] A soft set (F, A) over U is said to be a *NULL soft set* denoted by $\emptyset$ if for all $e \in A$, $F(e) = \emptyset$(null set).

Definition 2.6: [8] A soft set (F, A) over U is said to be an *absolute soft set*, denoted by U_A, if $e \in A$, $F(e) = U$.

Clearly, we have $U_A^c = \emptyset_A$ and $\emptyset_A^c = U_A$.

Definition 2.7: [8] The union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B, \\ G(e), & \text{if } e \in B \setminus A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.8: [10] The intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

In order to efficiently discuss, we consider only soft sets (F, E) over a universe U in which all the parameter set E are same. We denote the family of these soft sets by $\mathcal{SS}(U)_E$.

Definition 2.9: [13] Let I be an arbitrary index set and $\{(F_i, E)\}i \in I$ be a subfamily of $\mathcal{SS}(U)_E$.

1) The union of these soft sets is the soft set (H, E), where $H(e) = \cup_{i \in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\cup}_{i \in I}(F_i, E) = (H, E)$.

2) The intersection of these soft sets is the soft set (M, E), where $M(e) = \cap_{i\in I} F_i(e)$ for each $e \in E$. We write $\widetilde{\cap}_{i\in I}(F_i, E) = (M, E)$.

Definition 2.10: [12] Let τ be a collection of soft sets over a universe U with a fixed set E of parameters, then $\tau \subseteq \mathcal{SS}(V)_E$ is called a *soft topology* on U with a fixed set E if

1) $\emptyset_E, U_E$ belong to τ;
2) the union of any number of soft sets in τ belongs to τ;
3) the intersection of any two soft sets in τ belongs to τ.

Definition 2.11: [13] A soft set (G, E) in a soft topological space (U, τ, E) is called a *soft neighborhood* of the soft set (F, E) if there exists a soft open set (H, E) such that $(F, E)\widetilde{\subseteq}(H, E)\widetilde{\subseteq}(G, E)$.

Definition 2.12: Let (U, τ, E) be a soft topological space and let (G, E) be a soft set over U.

1) The *soft closure* [12] of (G, E) is the soft set $\overline{(G, E)} = \widetilde{\cap}\{(S, E) : (S, E)$ is soft closed and $(G, E)\widetilde{\subseteq}(S, E)\}$;
2) The *soft interior* [13] of (G, E) is the soft set $(G, E)^\circ = \widetilde{\cup}\{(S, E) : (S, E)$ is soft open and $(S, E)\widetilde{\subseteq}(G, E)\}$.

Definition 2.13: [6] The soft set $(F, E) \in \mathcal{SS}(U)_E$ is called a *soft point* in U_E if there exist $x \in U$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by e_x.

Theorem 2.14: [6] Let (U, τ, E) be a soft topological space. A soft point $e_x\widetilde{\in}\overline{(A, E)}$ if and only if each soft neighborhood of e_x intersects (A, E).

Definition 2.15: [5] Let $\mathcal{SS}(U)_A$ and $\mathcal{SS}(V)_B$ be families of soft sets. Let $u : U \to V$ and $p : A \to B$ be mappings. Then a mapping $f_{pu} : \mathcal{SS}(U)_A \to \mathcal{SS}(V)_B$ is defined as:

(1) Let (F, A) be a soft set in $\mathcal{SS}(U)_A$. The image of (F, A) under f_{pu}, written as $f_{pu}(F, A) = (f_{pu}(F), p(A))$, is a soft set in $\mathcal{SS}(V)_B$ such that

$$f_{pu}(F)(y) \in \begin{cases} \cup_{x\in p^{-1}(y)\cap A} u(F(x)), & p^{-1}(y) \cap A \neq \emptyset, \\ \emptyset, & \text{otherwise} . \end{cases}$$

for all $y \in B$.

(2) Let (G, B) be a soft set in $\mathcal{SS}(V)_B$. Then the inverse image of (G, B) under f_{pu}, written as $f^{-1}_{pu}(G, B) = (f^{-1}_{pu}(G), p^{-1}(B))$, is a soft set in $\mathcal{SS}(U)_A$ such that

$$f^{-1}_{pu}(G)(x) \in \begin{cases} u^{-1}(G(p(x))), & p(x) \in B, \\ \emptyset, & \text{otherwise} . \end{cases}$$

for all $x \in A$.

Definition 2.16: [6] Let (U, τ, A) and (V, τ^*, B) soft topological spaces. Let $u : U \to V$ and $p : A \to B$ be mappings. Let $f_{pu} : \mathcal{SS}(U)_A \to \mathcal{SS}(V)_B$ be a function.

(1) The function f_{pu} is *soft continuous* [13] if $f^{-1}_{pu}(H, B) \in \tau$ for each $(H, B) \in \tau^*$.

(2) The function f_{pu} is *soft open* if $f_{pu}(G, A) \in \tau^*$ for each $(G, A) \in \tau$.

Definition 2.17: [6] Let $\{(U^\alpha, \tau_\alpha, E_\alpha)\}_{\alpha\in I}$ be a family a soft spaces. Let us take as a basis for a soft topology on the product space $(\prod_{\alpha\in I} U^\alpha, \prod_{\alpha\in I} \tau_\alpha, \prod_{\alpha\in I} E_\alpha)$ the collection of all soft sets
$\{(\prod_{\alpha\in I} F_\alpha, \prod_{\alpha\in I} E_\alpha)$: There is a finit set $J \subset I$ such that $(F_\alpha, E_\alpha) = U^\alpha_{E_\alpha}$ for each $\alpha \in I \setminus J\}$.

The notations $\mathbf{R}, \mathbf{Q}, \mathbf{N}$ are real numbers, rational numbers and positive natural numbers respectively. Readers may refer to [8], [10], [12], [13] for notations and terminology not explicitly given here.

III. SOFT FIRST-COUNTABLE AND SOFT SECOND SPACES

Definition 3.1: [1] Let (U, τ, A) be a soft topological space. A subcollection γ of τ is said to be a *base* for τ if every member of τ can be expressed as a union of members of γ.

Definition 3.2: Let (U, τ, A) be a soft topological space, and let $\mathcal{U}$ be a family a soft neighborhood of some soft point e_x. If, for each soft neighborhood (F, A) of e_x, there exists a $(H, E) \in \mathcal{U}$ such that $e_x\widetilde{\in}(H, E)\widetilde{\subseteq}(F, A)$, then we say that $\mathcal{U}$ is a *soft neighborhoods base* at e_x.

Definition 3.3: Let (U, τ, A) be a soft topological space, and let e_x be a soft point in U_A. If e_x has a countable soft neighborhoods base, then we say that U_A is *soft first-countable* at e_x. If each soft point in U_A is oft first-countable, then we say that U_A is *soft first-countable*.

Proposition 3.4: Let (U, τ, A) be a soft topological space, and let e_x be a soft point in U_A. Then U_A is soft first-countable at e_x if and only if there exists a countable soft open neighborhoods base $\{(F_nA)\}_{n\in\mathbf{N}}$ at e_x such that $(F_{n+1}A)\widetilde{\subseteq}(F_nA)$ for each $n \in \mathbf{N}$.

Proof: Sufficiency. It is obvious.

Necessity. Let $\{(U_n, E)\}_{n\in\mathbf{N}}$ be a soft base at e_x. For each $n \in \mathbf{N}$, put $(F_n, A) = \widetilde{\cap}^n_{i=1}(U_{ni}, E)$. Then it is easy to see that $\{(F_n, A)\}_{n\in\mathbf{N}}$ is a soft base at e_x and $(F_{n+1}, A)\widetilde{\subseteq}(F_n, A)$ for each $n \in \mathbf{N}$. ∎

Definition 3.5: Let (U, τ, A) be a soft topological space. If U_A has a countable soft base, then we say that U_A is *soft second-countable*.

Proposition 3.6: (1) Each soft second space is soft first-countable;

(2) Soft first-countability and soft second-countability are hereditarily.

Remark 3.7: (1) There exists a soft space which is not soft first-countable.

(2) There exists a soft first-countable space which is not soft second-countable.

Example 3.8: Let $\mathbf{R}$ be real line, and let $A = \{e\}$. Then $\mathbf{R}$ be an uncountable set. Let $\tau = \{(H, A) : H(e) = \mathbf{R} \setminus F, F$ is a finite set in $\mathbf{R}\}, \sigma = \{(H, A) : H(e) = F, F \subseteq \mathbf{R}\}$. Then $(\mathbf{R}, \tau, A)$ and $(\mathbf{R}, \sigma, A)$ are soft spaces. However, $(\mathbf{R}, \tau, A)$ is not soft first-countable, and $(\mathbf{R}, \sigma, A)$ is a soft first-countable space which is not soft second-countable.

Theorem 3.9: (1) The product of countably many soft first-countable spaces is soft first-countable;

(2) The product of countably many soft second-countable spaces is soft second-countable.

Proof: We only prove (2).

Let (U^n, τ_n, A_n) be a family of countable many soft second-countable spaces. For each $n \in \mathbf{N}$, let $\mathcal{B}_n$ be a countable soft base of $U^n_{A_n}$. Put $\mathcal{B} = \{\prod_{n\in\mathbf{N}} U_n$: where $U_n \in \mathcal{B}_n$ and U_n equals $U^n_{A_n}$ except for finitely many valuse $n\}$. Then $\mathcal{B}$ is countable and a soft base for $(\prod_{n\in\mathbf{N}} U^n, \prod_{n\in\mathbf{N}} \tau_n, \prod_{n\in\mathbf{N}} A_n)$. ∎

Theorem 3.10: The image of soft first-countable spaces under a soft open continuous map are soft first-countable.

Proof: Let (U,τ,A) and $(V,\mathcal{G},B)$ be two soft spaces, where (U,τ,A) is soft first-countable, and let f be an onto soft pu-continuous open mapping from U to V. For any soft point e_y in V_B, there exists a soft point e_x in U_A such that $f(e_x)=e_y$. Since (U,τ,A) is soft first-countable, there exists a countable soft neighborhoods base $\{(F_n,A)\}_{n\in\mathbf{N}}$ at e_x. Then it is easy to see that $\{f(F_n,A)\}_{n\in\mathbf{N}}$ is soft neighborhoods base at e_y. ■

Definition 3.11: Let (U,τ,A) be a soft topological space. If there exists a countable many soft points $\{e_{x_n}:n\in\mathbf{N}\}$ such that $\widetilde{\cup}_{n\in\mathbf{N}}e_{x_n}=U_A$, then we say that (U,τ,A) is *soft separable*.

Proposition 3.12: Each soft second-countable space is soft separable.

Proof: Let (U,τ,A) be a soft second-countable space, and let $\mathcal{B}=\{(F_n,A)\}_{n\in\mathbf{N}}$ be a countable soft open base of U_A. For each $n\in\mathbf{N}$, take a soft point $e_{x_n}\widetilde{\in}(F_n,A)$, and put $B=\{e_{x_n}:n\in\mathbf{N}\}$. Then we have $\widetilde{\cup}B=U_A$. ■

Definition 3.13: A soft space (U,τ,A) is *soft Lindelöf* if each soft open covering $\mathcal{A}$ of U_A has a countable subcover.

Proposition 3.14: Each soft second-countable space is soft Lindelöf.

Proof: Let (U,τ,A) be a soft second-countable space, and let $\mathcal{B}$ be a countable soft open base of U_A. Take an arbitrary soft open covering $\mathcal{C}$ of U_A. Put $\mathcal{B}'=\{B\in\mathcal{B}:$ There is $C\in\mathcal{C}$ such that $B\widetilde{\subseteq}C\}$. Then $\mathcal{B}'$ is countable. Denote $\mathcal{B}'$ by $\{(F_n,A):n\in\mathbf{N}\}$. For each $n\in\mathbf{N}$, there exists a $C_n\in\mathcal{C}$ such that $(F_n,A)\widetilde{\subseteq}C_n$. Then $\{C_n:n\in\mathbf{N}\}$ is countable soft subfamily of $\mathcal{C}$. Next we shall prove that $\{C_n:n\in\mathbf{N}\}$ is a soft cover of (U,τ,A).

Take an arbitrary soft point $e_x\widetilde{\in}U_A$. Since $\mathcal{C}$ is a soft covering of U_A, there exists a $C\in\mathcal{C}$ such that $e_x\widetilde{\in}C$. Then there is a $B\in\mathcal{B}$ such that $e_x\widetilde{\in}B\widetilde{\subseteq}C$ since $\mathcal{B}$ is a soft base of U_A. Hence $B\in\mathcal{B}'$, and therefore, there is a $n\in\mathbf{N}$ such that $B=(F_n,A)$. Thus $e_x\widetilde{\in}(F_n,A)$. Hence $\{C_n:n\in\mathbf{N}\}$ is a soft cover of (U,τ,A). ■

Definition 3.15: Let (U,τ,A) be a soft topological space over U, and let (G,A) be a soft closed set in U and soft point e_x such that $e_x\widetilde{\notin}(G,A)$. If there exist soft open sets (F_1,A) and (F_2,A) such that $e_x\widetilde{\in}(F_1,A)$, $(G,A)\widetilde{\subseteq}(F_2,A)$ and $(F_1,A)\widetilde{\cap}(F_2,A)=\emptyset$, then (U,τ,A) is called a *soft regular space*.

Definition 3.16: Let (U,τ,A) be a soft topological space over U, and let (G_1,A) and (G_2,A) be two disjoint soft closed set in U. If there exist two soft open sets (F_1,A) and (F_2,A) such that $(G_2,A)\widetilde{\subseteq}(F_1,A)$, $(G_2,A)\widetilde{\subseteq}(F_2,A)$ and $(F_1,A)\widetilde{\cap}(F_2,A)=\emptyset$, then (U,τ,A) is called a *soft regular*.

Theorem 3.17: Each soft regular and soft Lindelöf space is soft normal.

Proof: Let (U,τ,A) be a soft regular and soft Lindelöf space. Let (F_1,A) and (F_2,A) be two disjoint soft closed sets in (U,τ,A). For each soft point $e_x\widetilde{\in}(F_1,A)\widetilde{\subseteq}(F_2,A)^c$, and since (U,τ,A) is soft regular, there exists a soft open neighborhood (G_x,A) of e_x such that $e_x\widetilde{\in}\overline{(G_x,A)}\widetilde{\subseteq}(F_2,A)^c$, that is, $\overline{(G_x,A)}\widetilde{\cap}(F_2,A)=\emptyset$. Let $\mathcal{G}=\{(G_x,A):e_x\widetilde{\in}(F_1,A)\}$; then $\mathcal{G}\cup\{(F_1,A)^c\}$ is a soft open cover of U_A. Since (U,τ,A) is soft Lindelöf, there exists a countable subcover $\{(G_{x_n},A):n\in\mathbf{N}\}\cup\{(F_1,A)^c\}$. Put $U_n=(G_{x_n},A)$ for each $n\in\mathbf{N}$; then $(F_1,A)\widetilde{\subseteq}\widetilde{\cup}_{n\in\mathbf{N}}U_n$ and each $U_n\widetilde{\cap}(F_2,A)=\emptyset$. Similarly, there exists a countably many soft open sets $\{V_n:n\in\mathbf{N}\}$ such that $(F_2,A)\widetilde{\subseteq}\widetilde{\cup}_{n\in\mathbf{N}}V_n$ and each $V_n\widetilde{\cap}(F_1,A)=\emptyset$.

For each $n\in\mathbf{N}$, put

$$U_n'=U_n\widetilde{\cap}(\widetilde{\cup}_{i=1}^n\overline{V}_i)^c, V_n'=V_n\widetilde{\cap}(\widetilde{\cup}_{i=1}^n\overline{U}_i)^c.$$

Then for $n,m\in\mathbf{N}$, we have $U_n'\widetilde{\cap}V_m'=\emptyset$. Put

$$G=\widetilde{\cup}_{n\in\mathbf{N}}U_n', H=\widetilde{\cup}_{n\in\mathbf{N}}V_n'.$$

Then we have $(F_1,A)\widetilde{\subseteq}G$, $(F_2,A)\widetilde{\subseteq}H$ and $G\widetilde{\cap}H=\emptyset$. Therefore, (U,τ,A) is soft normal. ■

Corollary 3.18: Each soft regular and soft second-countable space is soft normal.

Proposition 3.19: [12] Let (U,τ,E) be a soft space. Then the collection $\tau_\alpha=\{F(\alpha):(F,E)\in\tau\}$ for each $\alpha\in E$, defines a topology on U.

Proposition 3.20: Let (U,τ,E) be a soft space, where $E=\{e\}$. Then (U,τ,E) is soft Lindelöf if and only if the topological space (U,τ_e) is Lindelöf.

Remark 3.21: There exists a soft first-countable, soft separable and soft Lindelöf space which is not soft second-countable.

Example 3.22: Let $\mathbf{R}$ be real line, and let $A=\{e\}$. Then $\mathbf{R}$ be an uncountable set. Let $\psi=\{(F,A):F(e)=[a,b),a<b\}$, and let τ be the soft topology generated by ψ as a base. Then $(\mathbf{R},\tau,A)$ is soft first-countable, soft separable and soft Lindelöf space. Indeed, for each soft point e_x, it is easy to see that $\{(F_n^x,A):F_n^x(e)=[x,x+\frac{1}{n}),n\in\mathbf{N}\}$ is a soft neighborhoods base at e_x. Let $A=\{e_x:x\in\mathbf{Q}\}$. Then it is easy to see that $\overline{A}=\mathbf{R}_A$. Since it is well known that $(\mathbf{R},\tau_e)$ is Lindelöf, it follows from Proposition 3.20 that $(\mathbf{R},\tau,A)$ is soft Lindelöf. Next, we shall prove that $(\mathbf{R},\tau,A)$ is not soft second-countable.

Let $\mathcal{B}$ be an arbitrary soft base for $(\mathbf{R},\tau,A)$. For arbitrary soft point $e_x\widetilde{\in}\mathbf{R}_A$ and soft open neighborhood (F_1^x,A), there exists a soft open set $(B_x,A)\in\mathcal{B}$ such that $e_x\widetilde{\in}(B_x,A)\widetilde{\subseteq}(F_1^x,A)$, and hence $x\in\inf B_x(e)$. If $e_x\neq e_y$, then $(B_x,A)\neq(B_y,A)$. Therefore, the set $\{(B_x,A):e_x\widetilde{\in}\mathbf{R}_A\}$ is an uncountable subfamily of $\mathcal{B}$, and thus $\mathcal{B}$ is an uncountable family.

IV. Conclusion

In the present work, we have continued to study the properties of soft topological spaces. We mainly introduce soft first-countable spaces, soft second-countable spaces, soft separable spaces and soft Lindelöf spaces. Moreover, we have also established several interesting results and presented its fundamental properties with the help of some examples. We hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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