Finite Element Analysis for Damped Vibration Properties of Panels Laminated Porous Media

Y. Kurosawa and T. Yamaguchi

Abstract—A numerical method is proposed to calculate damping properties for sound-proof structures involving elastic body, viscoelastic body, and porous media. For elastic and viscoelastic body displacement is modeled using conventional finite elements including complex modulus of elasticity. Both effective density and bulk modulus have complex quantities to represent damped sound fields in the porous media. Particle displacement in the porous media is discretised using finite element method. Displacement vectors as common unknown variables are solved under coupled condition between elastic body, viscoelastic body and porous media. Further, explicit expressions of modal loss factor for the mixed structures are derived using asymptotic method. Eigenvalue analysis and frequency responded were calculated for automotive test panel laminated viscoelastic and porous structures using this technique, the results almost agreed with the experimental results.

Keywords—Damping, Porous Media, Finite Element Method, Computer Aided Engineering.

I. INTRODUCTION

DAMPING and sound-insulation measures are strictly applied to automotive body panels to prevent noise in the vehicle cabin. Automotive body panels, which are made of steel sheet press-molded into a required form, are laminated with damping materials to reduce the vibration level. Furthermore, porous media, resin sheets (surface) a carpet are laminated on the damping materials. Sandwiching the porous media between the panel and the rubber sheet realizes a double-walled sound insulation structure (Fig. 1). In this way, solid materials (elastic and viscoelastic), porous media and gas (air) coexist in the sound isolation structure for the automotive body panels. Fig. 2 shows the results of vibration level measurement of the front floor (acceleration response). In this measurement the front suspension mounting part was selected as vibration excitation point of estimated road noise input. The vibration was measured under the panel ("panel" in the figure) and on the rubber sheet ("pane + felt + resin sheet"). The difference of the vibration levels of the two areas is small until about 180 Hz, while it becomes greater at larger noises.

From the above, for predicting the high-frequency road noise (180 to 500 Hz), it is essential to predict the vibration noise characteristics of the sound-proof structure, especially the surface sheet which emits in-vehicle noise, and numerical value calculation is a possible technique for this [1], [2]. This study

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proposes a numerical analysis method for a sound-proof structure where elastic materials, viscoelastic materials, porous media and air, designed with complex sound-proof structures of automotive body panels. The finite element method is used to handle any forms and boundaries. Besides the single damping of the viscoelastic materials and the porous media, the method is designed to solve coupled problems of solid materials, porous media and air. In addition, an approximate calculation method is proposed for the modal loss factor of the complex sound-proof structure. With this new technique, a vibration analysis of a simple panel that mimics the automotive panels was performed and the results were compared with experimental results for accuracy verification.
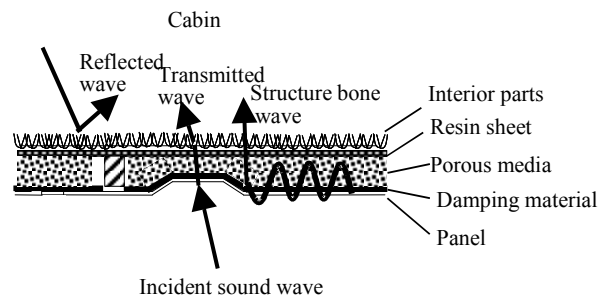

Fig. 1 Automotive panel laminated with viscoelastic body and porous media
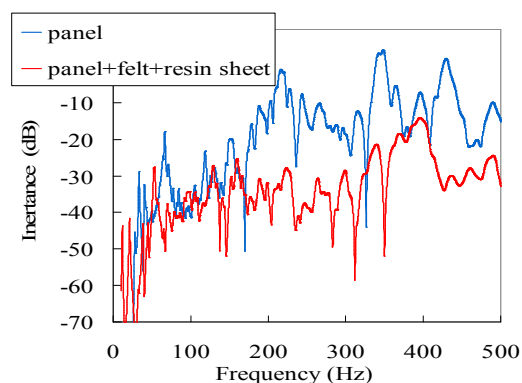

Fig. 2 Effect of porous media to reduce vibration of front floor (Excitation point: front cross member)

II. ANALYSIS METHOD

This chapter introduces a numerical analysis method for vibration damping characteristics of coupled problems of vibration and acoustics in a field where an elastic, a viscoelastic, porous media and gas. These components are

expressed as a finite element and stacked in consideration of coupling in order to handle any structures regardless their forms. First, the Section II A will suggest a numerical analysis method by discretizing particle displacement in a damped sound field. Then the Section II B will explain finite elements of the displacing field for solid bodies (elastic and viscoelastic). The Section II C will explain the discrete equation for the global coordinate where solid bodies, porous media and gas coexist. In the Section II D, an equation will be derived that approximately calculates the modal loss factor of the global coordinate by applying the asymptotic method. Finally, the Section II E will introduce an equation for damped vibration response using the MSKE (Modal Strain and Kinetic Energy) method.

A. Descretized Equation for Internal Gas in Porous Media

First, the sound field of internal air in porous media is discretized using finite elements. Assuming infinitesimal amplitude, the equation of motion of inviscid compressive perfect fluid can be expressed under periodic oscillation as follows [5]-[7].

$$-\nabla P + \rho \omega^2 \{u_f\} = 0 \qquad (1)$$

The equation of continuity is represented by the following equation [5]-[7].

$$P + Ediv\{u_f\} = 0 \qquad (2)$$

P is the pressure. $\{u_f\}$ is the displacement vector of particles. E is the volume elasticity and ρ is the effective density of the internal air in porous media. ω is the angular frequency.

Conventional acoustic analysis often eliminates the particle displacement in (1) and (2), and derives an equation of motion which treats the pressure as unknown. In this study, however, the pressure is eliminated from the two equations and the particle displacement is retained as unknown. An advantage of this technique is that the displacement can be used as a common unknown for solid bodies. This allows the simplified stack of solid bodies and the sound field factors [7], and makes the calculation method more suitable for complex sound problems where solid bodies, porous media and gas scatter. On the other hand, while the unknown for the pressure is a scalar variable, the unknown for the particle displacement is a vector variable, which requires a larger number of calculations.

Relations between $\{u_f\}$ and the particle displacement $\{u_{fe}\}$ at nodal points in the element can be approximated as follows.

$$\{u_f\} = {}^t[N_f]\{u_{fe}\} \qquad (3)$$

${}^t[N_f]$ represents a matrix comprised of appropriate shape functions. Irrotational condition is $rot\{u_f\} = 0$

The kinetic energy $\widetilde{T}_f$, the strain energy $\widetilde{U}_f$, and external work $\widetilde{V}_f$ are obtained from (1), (2), and (3). The following expressions can be derived by applying the minimum energy principle $\delta(\widetilde{U}_f - \widetilde{T}_f - \widetilde{V}_f) = 0$.

$$([K]_{fe} - \omega^2 [M]_{fe})\{u_{fe}\} = \{f_{fe}\} \qquad (4)$$

$$[M]_{fe} = \rho_e [\widetilde{M}]_{fe} \qquad (5)$$

$$[K]_{fe} = E_e [\widetilde{K}]_{fe} \qquad (6)$$

$\{f_{fe}\}$ is the nodal force vector, $[K]_{fe}$ is the element stiffness matrix, and $[M]_{fe}$ is the element mass matrix. ρ_e and E_e are the effective density and the volume elasticity for media in the region of element. $[\widetilde{K}]_{fe}$ and $[\widetilde{M}]_{fe}$ are the matrix consisted of the shape functions and their derivatives.

Equations (4), (5), and (6) are kinetic equations for the element that is linear compressible perfect fluid. These equations can be used as element equations for acoustic problems of gas under undamped conditions.

For expressing the sound in the porous media, a model is proposed which converts the complex effective density and the acoustic velocity or complex volume elasticity, and its effectiveness is confirmed [4], [5], [10]-[12]. Based on this method, the following equations are obtained.

$$\rho_e \Rightarrow \rho_e^* = \rho_{eR} + j\rho_{eI} \qquad (7)$$

$$E_e \Rightarrow E_e^* = E_{eR} + jE_{eI} \qquad (8)$$

This model is mainly applied to textile materials such as glass wool. It ignores the impact of the elastic wave which transmits the frames of porous media, and assumes that the motion of gas is the dominant determiner. The model effectiveness is verified for porous media when their frame materials have adequate flexibility and large damping [4], [5], [9], and automotive sound-proof materials are often the case. On the other hand, when the frame materials of porous media are made of rigid materials such as metal, the elastic wave transmitting through the frames has larger impact than the air wave. In this case other models such as Biot's model will be required [13], [14].

The element mass matrix $[M]_{fe}$ is obtained as follows by substituting (7) into (5).

$$[M]_{fe} = [M_R]_{fe}(1 + j\chi_e) \qquad (9)$$

$$\chi_e = \rho_{eI} / \rho_{eR} \qquad (10)$$

$[M_R]_{fe}$ is the real part of $[M]_{fe}$. The imaginary part of the effective density ρ_{eI} is a term related to the flow resistance of the porous media, and $\chi_e = \rho_{eI} / \rho_{eR}$ corresponds to the material dumping caused by the flow resistance.

In the same way, the element stiffness matrix $[K]_e$ is obtained by substituting (8) into (6).

$$[K]_{fe} = [K_R]_{fe}(1 + j\eta_e) \tag{11}$$

$$E_e = E_{eI}/E_{eR} \tag{12}$$

$[K_R]_{fe}$ is the real part of the $[K]_{fe}$. η_e is the material damping corresponding to the hysteresis in the relationship between the pressure and the volume strain (loss factor; all the damping values below are loss factors).

From the above, among the elements for the sound field in the porous media, the element stiffness matrix $[K]_{fe}$ and the element mass matrix $[M]_{fe}$ are both expressed with complex quantities. Gas such as air can be expressed by lowering their damping parameters, χ_e and η_e. The parameters χ_e, ρ_{eR}, η_e and E_{eR} can be identified by experiments using a impedance tube [5], [10].

Prior to this study, we proposed an analysis method of the field using the fine element method where solid bodies are not included but porous media and air coexist. This method consists of the procedure similar to the above, except that the pressure is treated as unknown instead of the particle displacement. The effectiveness of the method is confirmed regarding the damped response and the modal loss factor [4], [11], [12]. The proposed method in this new study is an enhanced version of the previous method to apply to coupled problems which also include solid bodies.

B. Discretized Equations for Vibration in Damped Solid Bodies

The vibration field of a solid bodies is discretized conventionally with the finite element method [15], using the following equations (13) – (17).

The relationships between the stress ant the strain, and the strain and the displacement are expressed as follows.

$$\{\sigma\} = [D]\{\varepsilon\} \tag{13}$$

$$\{\varepsilon\} = [A]\{u_s\} \tag{14}$$

where, $\{\sigma\}$ is the stress vector, $\{\varepsilon\}$ is the strain vector, and $\{u_s\}$ is the displacement vector of the solid bodies. $[D]$ is the matrix including modulus of elasticity and Poisson's ratio, and $[A]$ is the matrix comprised of differential operators.

By using the matrix comprised of shape functions $[N_s]$, the relationship between the element displacement $\{u_s\}$ and the nodal displacements $\{u_{se}\}$ is approximated as follows.

$$\{u_s\} = {}^t[N_s]\{u_{se}\} \tag{15}$$

The following equation is obtained by obtaining the kinetic energy $\widetilde{T}_s$, the strain energy $\widetilde{U}_s$, and the external work $\widetilde{V}_s$, and applying the minimum energy principle $\delta(\widetilde{U}_s - \widetilde{T}_s - \widetilde{V}_s) = 0$.

$$([K]_{se} - \omega^2[M]_{se})\{u_{se}\} = \{f_{se}\} \tag{16}$$

where, $\{f_{se}\}$ is the nodal force vector in an element e for solid bodies, $[K]_{se}$ and $[M]_{se}$ are the element stiffness matrix and the element mass matrix for solid bodies, respectively.

In order to express the viscoelastic material with hysteresis damping as a finite element, it is necessary to convert the elasticity $[D]$ in (13) into a complex modulus [3], [16]. By doing this, the element stiffness matrix in (16) is also represented by complex quantities as follows.

$$[K]_{se} = [K_R]_{se}(1 + j\eta_e) \tag{17}$$

where, η_e is the material loss factor corresponding to each element e, and $[K_R]_{se}$ is the real part of the element stiffness matrix for solid bodies.

C. Discretized Equation in Global System

At the boundary of solid bodies and gas or a solid bodies and porous media, only the displacement in the normal direction toward the boundary is continuous. By taking this into account and using (4) – (17), all the elements in an intended field (the complex space of gas, porous media and solid bodies) are stacked to obtain the following discrete equation for the global coordinate [7].

$$\sum_{e=1}^{e_{max}}([K_R]_e(1 + j\eta_e) - \omega^2[M_R]_e(1 + j\chi_e))\{u_e\} = \{F\} \tag{18}$$

e_{max} is the total number of elements and $\{F\}$ is the external force vector. $\{u_e\}$ is the nodal displacement vector in global system, which consists of $\{u_{fe}\}$ and $\{u_{se}\}$. Similarly, $[K_R]_e$ consists of $[K_R]_{fe}$ and $[K_R]_{se}$, while $[M_R]_e$ consists of $[M_R]_{fe}$ and $[M_R]_{se}$. In this equation, χ_e of the solid elements must be null.

From the above, for the system where solid bodies, porous media and gas coexist, the stiffness matrix and the mass matrix are both expressed as complex quantities.

D. Approximate Computation of Modal Damping

This section explains the approximate calculation of the mode damping of the global coordinate. The complex eigenvalue problem of (18) is represented by the following equation:

$$\sum_{e=1}^{e_{max}}([K_R]_e(1 + j\eta_e) - (\omega^{(n)})^2(1 + j\eta_{tot}^{(n)})[M_R]_e(1 + j\chi_e))\{\phi^{(n)*}\} = \{0\} \tag{19}$$

$(\omega^{(n)})^2$ is the real part of the *n*'th order complex eigenvalue, $\{\phi^{(n)*}\}$ is the *n*'th order complex eigen mode, and $\eta_{tot}^{(n)}$ is the *n*'th order modal loss factor.

Among the material damping χ_e, η_e ($e=1,2,3,\ldots,e_{max}$), the most largest number is expressed as η_{max}. In addition, the following value is defined and introduced.

$$\beta_{se} = \eta_e / \eta_{max}, \beta_{se} \leq 1, \beta_{ke} = \chi_e / \eta_{max}, \beta_{ke} \leq 1 \qquad (20)$$

Here, by assuming $\eta_{max} \ll 1$ and introducing the small parameter $\mu = j\eta_{max}$, (19) is asymptotically expanded as follows.

$$\{\phi^{(n)*}\} = \{\phi^{(n)}\}_0 + \mu\{\phi^{(n)}\}_1 + \mu^2\{\phi^{(n)}\}_2 + \cdots \qquad (21)$$

$$(\omega^{(n)})^2 = (\omega_0^{(n)})^2 + \mu^2(\omega_2^{(n)})^2 + \mu^4(\omega_4^{(n)})^2 +, \cdots \qquad (22)$$

$$j\eta_{tot}^{(n)} = \mu\eta_1^{(n)} + \mu^3\eta_3^{(n)} + \mu^5\eta_5^{(n)} + \mu^7\eta_7^{(n)} +, \cdots \qquad (23)$$

In these equations, under conditions of $\beta_{ke} \leq 1$, $\beta_{se} \leq 1$ and $\eta_{max} \ll 1$, we can obtain $\eta_{max}\beta_{ke} \ll 1$ and $\eta_{max}\beta_{se} \ll 1$. Thus, both $\mu\beta_{se}$ and $\mu\beta_{ke}$ is regarded as small parameters like μ. In addition, $\{\phi^{(n)}\}_0, \{\phi^{(n)}\}_1, \{\phi^{(n)}\}_2, \cdots, (\omega_0^{(n)})^2, (\omega_2^{(n)})^2, (\omega_4^{(n)})^2, \ldots$ and $\eta_1^{(n)}, \eta_3^{(n)}, \eta_5^{(n)}, \cdots$ are real quantities.

Then by substituting (21) – (23) into (19), the orders μ^0 and μ^1 are respectively combined as the following equations:

μ^0 order:

$$\sum_{e=1}^{e\,max} \left([K_R]_e - (\omega_0^{(n)})^2 [M_R]_e \right)\{\phi^{(n)}\}_0 = \{0\} \qquad (24)$$

μ^1 order:

$$\sum_{e=1}^{e_{mav}} \left(\mu\beta_{se}[K_R]_e - \mu\eta_1^{(n)}(\omega_0^{(n)})^2[M_R]_e - \mu\beta_{ke}(\omega_0^{(n)})^2[M_R]_e \right)\{\phi^{(n)}\}_0$$
$$+ \sum_{e=1}^{e_{max}} \left(\mu[K_R]_e - \mu(\omega_0^{(n)})^2[M_R]_e \right)\{\phi^{(n)}\}_1 = \{0\} \qquad (25)$$

Furthermore, by arranging (24) and (25), (26) is obtained.

$$\eta_{tot}^{(n)} = \eta_{ke}^{(n)} - \eta_{se}^{(n)} \qquad (26)$$

$$\eta_{ke}^{(n)} = \sum_{e=1}^{e_{max}} (\eta_e S_{ke}^{(n)}), \quad \eta_{se}^{(n)} = \sum_{e=1}^{e_{max}} (\chi_e S_{se}^{(n)})$$

$$S_{ke}^{(n)} = \frac{\{\phi^{(n)}\}_0^T [K_R]_e \{\phi^{(n)}\}_0}{\sum_{e=1}^{e_{max}} \{\phi^{(n)}\}_0^T [K_R]_e \{\phi^{(n)}\}_0}$$

$$S_{se}^{(n)} = \frac{\{\phi^{(n)}\}_0^T [M_R]_e \{\phi^{(n)}\}_0}{\sum_{e=1}^{e_{max}} \{\phi^{(n)}\}_0^T [M_R]_e \{\phi^{(n)}\}_0}$$

According to these expressions, modal loss factor $\eta_{tot}^{(n)}$ can be approximately calculated using material loss factors η_e of each element *e* concerning elasticity, share $S_{se}^{(n)}$ of strain energy of each element to total strain energy, material loss factors χ_e of each element *e* concerning effective density and share $S_{ke}^{(n)}$ of kinetic energy of each element to total kinetic energy. The eigen modes in (26) are real, which is easily obtained by solving (24), which is obtained by ignoring all the damping terms, as real eigenvalue problem. Equation (26) is an extended method of the MSE method, which calculates the modal loss factor of a structure where an elastic and a viscoelastic coexist, and Modal Strain and Kinetic Energy Method (MSKE method), which calculated the modal loss factor of a sound field where porous media and gas coexist [4], [11], [12].

E. Damped Vibration Response Using MSKE Method

The acceleration response that uses the modal loss factor obtained from (26) and the modal parameter obtained from real eigenvalue analysis is represented by the following equation:

$$\{A\} = \sum_{n=1}^{max} \frac{-\omega^2\{\phi^{(n)}\}^T\{F\}\{\phi^{(n)}\}}{m^{(n)}\left[(\omega^{(n)})^2 - \omega^2 + j(\omega^{(n)})^2\eta_{se}^{(n)} - j\omega^2\eta_{ke}^{(n)} \right]} \qquad (27)$$

$\{A\}$ is the acceleration vector at the response, $\{F\}$ is the external force vector at the excitation point, $\{\phi^{(n)}\}$ is the *n*'th order mode vector at the excitation point, and $m^{(n)}$ is *n*'th order modal mass.

III. ANALYSIS RESULTS AND TEST VERIFICATION

A vibration analysis was performed with finite elements model. A base panel, porous media (felt) and a upper panel (steel) were laminated, as shown Fig. 3. When we made this model, we use HyperMesh (Altair Engineering Inc.) ver.11.0 at meshing. With a similar test piece, as shown Fig. 4, the vibration response (hereafter "response" means acceleration response) was also measured. The base panel was made of 1.6mm thick steel sheet and constrained by edging its frame with a jig and bolting. The felt was 25mm thick. Around the felt was the wall of the jig, which prevented the leakage of air in the felt and closed boundary condition could be assumed. The boundary condition of the upper panel (1.6mm thick steel) was free. Although the base panel and the felt, the felt and upper panel were not adhered, there was no clearance between them. The finite element model was solid elements with the mesh pitch of 5mm (the thickness direction is excluded). For the boundary condition of the base panel, springs were installed in the rectilinear X, Y, and Z directions to account for the support stiffness of the jig. For the boundary condition of the beaded panel, springs were installed in the rectilinear X, Y, and Z directions to account for the support stiffness of the jig. The boundary conditions of the sides of the felt were rigid wall in the normal direction and free in the tangential direction. The

upper panel displacement and the particle displacement in the internal air are continuous only in the normal direction towards the boundary surface. This continuous condition is applied to the particle displacement in the air in the felt and the base panel displacement.

First of all, the vibration response of only the base panel was measured and the spring constants in the X, Y and Z directions which were set on the boundary of the finite element model were tuned so that the resonance frequency and the acceleration response (= inertance) agree with the measurement results. The lower left end of the base panel was made into the origin of coordinate (0.0, 0.0, 0.0). The point of coordinate (50.0, 50.0, 0.0) was inputted with the hammer, and measured the point of coordinate (75.0, 25.0, 0.0) with the laser doppler vibrometer. The unit of coordinate was [mm]. The inertance level shows $0[dB] = 20\log(1)$. Fig. 5 shows the tuning results. The blue line shows the calculation results, the red line shows the experimental results. The calculated value agrees well with the experimental value overall, far to 500 Hz.
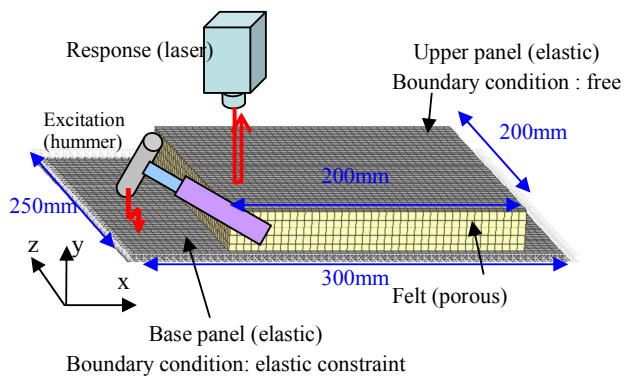

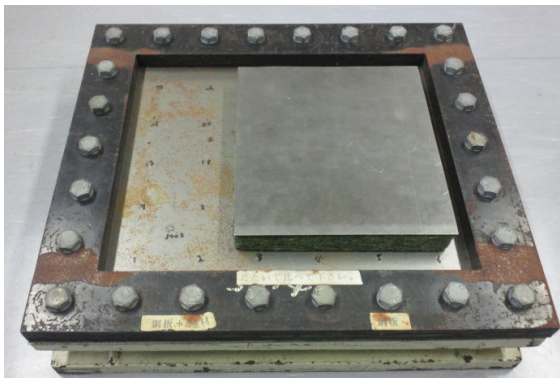Fig. 3 Finite element model of test piece
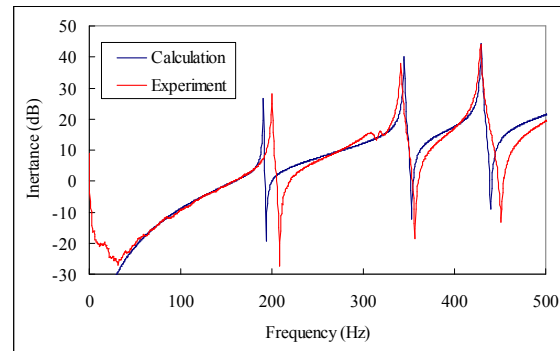

Fig. 4 Test piece for experiment

Fig. 5 Response level (base panel only)

Secondly, analysis and measurement were performed for the laminated model shown in the Fig. 3. The material data of the air in the felt was identified by improved two-cavity method [8]. Specifically, the real part of the effective density $\rho_{eR} = 1.98 \times 10 \text{kg/m}^3$, the imaginary part $\chi_e = -1.33$, the real part of the volume elasticity $E_{eR} = 1.19 \text{N/m}^2$, the imaginary part $\eta_e = 0.151$. The excitation point was the same position as the case of only the base panel. The measured point of response was coordinate (45.0, 95.0, 26.6). Fig. 5 shows response measurement results of only the base panel (blue line in Fig. 6) and the panel which was laminated with upper panel and felt (red line). Fig. 7 shows the calculation results of the same conditions. Compared Fig. 6 with Fig. 7, it was confirmed that resonance frequencies and response levels were enough calculation precision. Also, an effect of the porous media was reproduced fairly well.
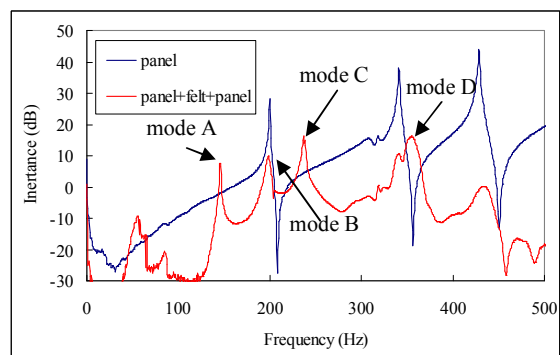

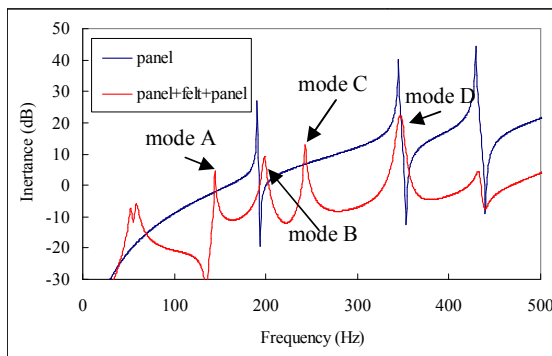Fig. 6 Response level (experimental results)

Fig. 7 Response level (calculation results)

Figs. 8 and 9 show the four vibration modes resulting from experimental modal analysis and eigenvalue analysis. The experimental modal analysis calculated vibration modes from 25 points of frequency response functions measurement results on the upper panel by excitation of the point that mentioned above. The sufficient calculation accuracy for mode shapes, resonance frequencies and modal loss factors was confirmed.
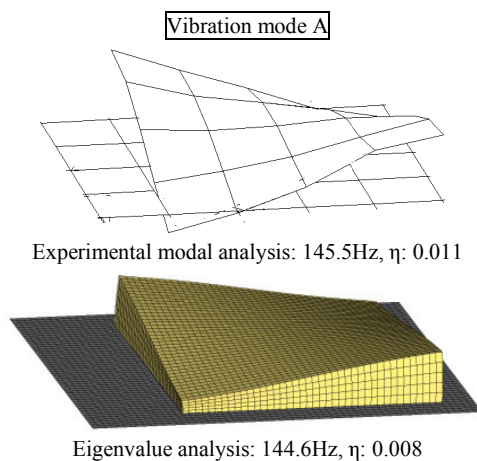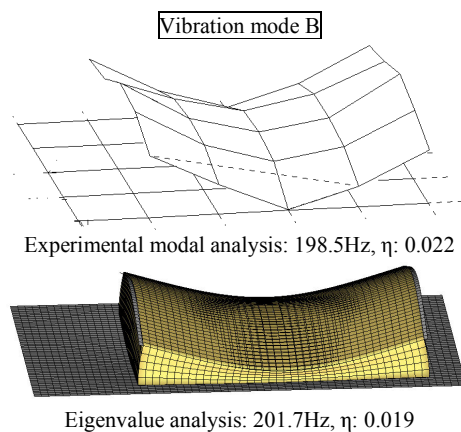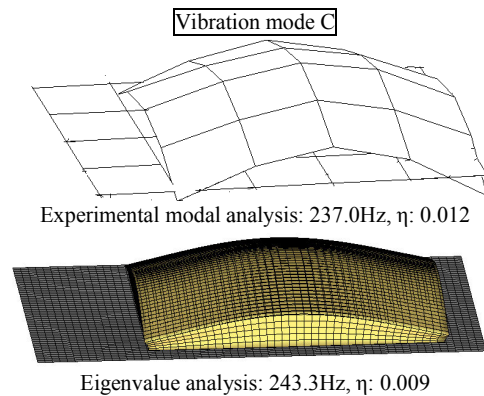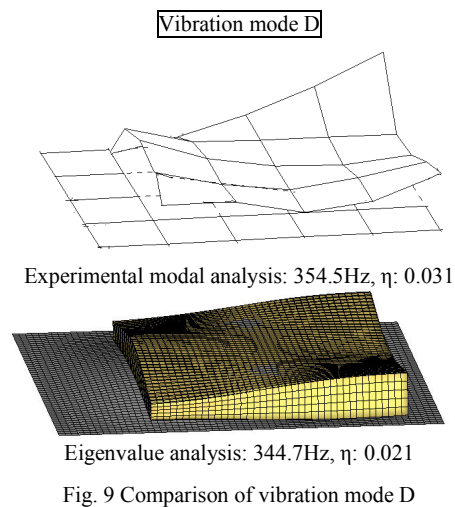
Vibration mode A

Experimental modal analysis: 145.5Hz, η: 0.011

Eigenvalue analysis: 144.6Hz, η: 0.008

Vibration mode B

Experimental modal analysis: 198.5Hz, η: 0.022

Eigenvalue analysis: 201.7Hz, η: 0.019

Vibration mode C

Experimental modal analysis: 237.0Hz, η: 0.012

Eigenvalue analysis: 243.3Hz, η: 0.009

Fig. 8 Comparison of vibration mode A,B,C

Vibration mode D

Experimental modal analysis: 354.5Hz, η: 0.031

Eigenvalue analysis: 344.7Hz, η: 0.021

Fig. 9 Comparison of vibration mode D

IV. CONCLUSION

An analysis method based on the finite element method was proposed for the analyzing the vibration characteristics of the structure in which an elastic, a viscoelastic, porous media, and gas coexist in order to analyze by CAE the vibration damping problem of complex sound-proof structures used for automotive panels. The obtained results are summarized as follows:

1) Porous media was formulated by the model which expressed the sound field of the internal air with complex effective density and the complex volume elasticity, and discretized with the element which treats particle displacement as unknown. Elastic and viscoelastic materials were discretized and formulated by the element which treats displacement as unknown. By combining these, the coupled problem where an elastic, a viscoelastic, porous media and gas coexist in any form was modeled with finite elements and formulated with displacement as common unknown.

2) A Modal Strain and Kinetic Energy Method (MSKE method) was developed to apply the approximate calculation of the modal loss factor by based on the asymptotic method (MSE method) to complex

sound-proof structures. With this method the modal loss factor was obtained from the results of real eigenvalue analysis and the number of calculations required was considerably reduced.

3) A calculation method of the vibration was developed based on the modal method which uses the modal loss factor obtained by the above MSKE method. An experimental modal analysis were performed with the test pieces laminated with porous media, and the sufficient calculation accuracy for mode shapes, modal loss factors and inertance was confirmed.

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