Fourth Order Accurate Free Convective Heat Transfer Solutions from a Circular Cylinder

T. V. S. Sekhar, B. Hema Sundar Raju

Abstract—Laminar natural-convective heat transfer from a horizontal cylinder is studied by solving the Navier-Stokes and energy equations using higher order compact scheme in cylindrical polar coordinates. Results are obtained for Rayleigh numbers of 1, 10, 100 and 1000 for a Prandtl number of 0.7. The local Nusselt number and mean Nusselt number are calculated and compared with available experimental and theoretical results. Streamlines, vorticity - lines and isotherms are plotted.

Keywords—Higher order compact scheme, Navier-Stokes equations, Energy equation, Natural convection, Boussinesq's approximation and Mean Nusselt number.

I. INTRODUCTION

THE process of heat transfer by natural convection is frequently occurred in industrial applications such as boilers, digesters, furnaces, etc. Kuehn & Goldstein [1] analyzed numerical and experimental results of natural convective heat transfer from a horizontal cylinder. The theoretical and experimental results of on free convection around horizontal wire is studied by Fujii et.al [2]. The higher order compact schemes (HOCS) are invariably applied for Navier Stokes (N-S) equations in cartesian coordinates [3] - [6] and are applied less to flow problems in curvilinear coordinate systems. Some papers on HOCS in polar coordinates for linear Poisson / quasi-linear Poisson /convection-diffusion equations can be seen in [7] - [9]. An attempt for improved accuracy is achieved by Saitoh et.al [10], in their study of natural convection heat transfer from horizontal circular cylinder by employing traditional fourth order central differences. The present paper is concerned with solving the Navier-Stokes and energy equations using higher order compact scheme for the Laminar natural-convective heat transfer from a horizontal cylinder in cylindrical polar coordinates.

II. BASIC EQUATIONS

The dimensionless equations for steady, laminar natural convection flow can be written in cylindrical polar coordinates by applying the transformation $r = e^{\xi}$ using the Boussinesq approximation as follows:
The velocity components

$$q_r = e^{-\xi}\frac{\partial\psi}{\partial\theta}, \qquad q_\theta = -e^{-\xi}\frac{\partial\psi}{\partial\xi} \qquad (1)$$

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Stream function equation

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\theta^2} = e^{-2\xi}\omega \qquad (2)$$

Vorticity transport equation

$$\frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\theta^2} + e^{\xi}Ra\left(\frac{\partial T}{\partial\xi}\sin\theta + \frac{\partial T}{\partial\theta}\cos\theta\right)$$
$$= \frac{1}{Pr}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial\omega}{\partial\xi} - \frac{\partial\psi}{\partial\xi}\frac{\partial\omega}{\partial\theta}\right) \qquad (3)$$

Energy equation

$$\frac{\partial^2 T}{\partial\xi^2} + \frac{\partial^2 T}{\partial\theta^2} = \left(\frac{\partial\psi}{\partial\theta}\frac{\partial T}{\partial\xi} - \frac{\partial\psi}{\partial\xi}\frac{\partial T}{\partial\theta}\right) \qquad (4)$$

where Ra is the Rayleigh number based on the diameter of the cylinder defined as

$$Ra = \frac{D^3\beta g(T_S - T_\infty)}{\nu\alpha},$$

where D is the diameter of the cylinder, g is the gravitational acceleration, β is the thermal coefficient of volumetric expansion, ν is the kinematic viscosity, T_S is the temperature of the cylinder surface and T_∞ is the temperature of the ambient flow. T is the nondimensionalized temperature, defined by subtracting the ambient flow temperature T_∞ from the temperature and dividing by $T_S - T_\infty$ and Pr is the Prandtl number defined as the ratio between kinematic viscosity (ν) and thermal diffusivity (α).

Equations (1) - (4) are dependent upon the assumption that the only body force operating is that of gravity and that the temperature variations within the fluid are not large, so that Boussinesq's approximation can be applied and the density to be treated as a constant in all terms of the transport equations execept the buoyancy term. Other fluid properties such as the viscosity, specific heat and thermal conductivity are taken to constant. The flow is considered to be symmetric about the vertical line passing through the center of the cylinder so that the flow on only one side needs to be solved. Equations (1) - (4) are subject to the following boundary conditions.
On the surface of the cylinder ($\xi = 0$) :

$$\psi = \frac{\partial\psi}{\partial\xi} = 0, \qquad \omega = -\frac{\partial^2\psi}{\partial\xi^2}, \qquad T = 1,$$

At large distances from the cylinder ($\xi \to \infty$) we follow inflow and outflow condition given below

At the inflow region : $q_\theta = \frac{\partial^2\psi}{\partial\xi^2} = T = 0, \ \omega = -e^{-2\xi}\frac{\partial^2\psi}{\partial\theta^2},$

At the outflow region : $q_\theta = \frac{\partial^2\psi}{\partial\xi^2} = \frac{\partial T}{\partial\xi} = 0, \ \omega = -e^{-2\xi}\frac{\partial^2\psi}{\partial\theta^2},$

Along axis of symmetry ($\theta = 0, \theta = \pi$) : $\psi = \omega = \frac{\partial T}{\partial\theta} = 0.$

III. Fourth order compact scheme

The standard fourth order central difference operator of the first and second order partial derivatives are given by the following equations

$$\frac{\partial \phi}{\partial \xi} = \delta_\xi \phi - \frac{h^2}{6}\frac{\partial^3 \phi}{\partial \xi^3} + O(h^4) \tag{5}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = \delta_\xi^2 \phi - \frac{h^2}{12}\frac{\partial^4 \phi}{\partial \xi^4} + O(h^4) \tag{6}$$

$$\frac{\partial \phi}{\partial \theta} = \delta_\theta \phi - \frac{k^2}{6}\frac{\partial^3 \phi}{\partial \theta^3} + O(k^4) \tag{7}$$

$$\frac{\partial^2 \phi}{\partial \theta^2} = \delta_\theta^2 \phi - \frac{k^2}{12}\frac{\partial^4 \phi}{\partial \theta^4} + O(k^4) \tag{8}$$

Where $\delta_\xi \phi, \delta_\xi^2 \phi, \delta_\theta \phi$ and $\delta_\theta^2 \phi$ are standard second order central discretizations such that

$$\delta_\xi \phi = \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2h},$$

$$\delta_\xi^2 \phi = \frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{h^2},$$

$$\delta_\theta \phi = \frac{\phi_{i,j+1} - \phi_{i,j-1}}{2k},$$

$$\delta_\theta^2 \phi = \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{k^2}.$$

A. Discretization of Stream Function Equation

Using (5) - (8) in (2), we obtain

$$-\delta_\xi^2 \psi_{i,j} - \delta_\theta^2 \psi_{i,j} + p_{i,j} - \gamma_{i,j} = 0 \tag{9}$$

The truncation error of (9) is

$$\gamma_{i,j} = -\left[\frac{h^2}{12}\frac{\partial^4 \psi}{\partial \xi^4} + \frac{k^2}{12}\frac{\partial^4 \psi}{\partial \theta^4}\right]_{i,j} + O(h^4, k^4) \tag{10}$$

and

$$p_{i,j} = -\left(e^{2\xi}\omega\right)_{i,j}.$$

Differentiating partially the stream-function equation (2) twice on both sides with respect to ξ and θ, we obtain the following equations

$$\frac{\partial^3 \psi}{\partial \xi^3} = -\frac{\partial^3 \psi}{\partial \xi \partial \theta^2} + \frac{\partial p}{\partial \xi} \tag{11}$$

$$\frac{\partial^4 \psi}{\partial \xi^4} = -\frac{\partial^4 \psi}{\partial \xi^2 \partial \theta^2} + \frac{\partial^2 p}{\partial \xi^2} \tag{12}$$

$$\frac{\partial^3 \psi}{\partial \theta^3} = -\frac{\partial^3 \psi}{\partial \xi^2 \partial \theta} + \frac{\partial p}{\partial \theta} \tag{13}$$

$$\frac{\partial^4 \psi}{\partial \theta^4} = -\frac{\partial^4 \psi}{\partial \xi^2 \partial \theta^2} + \frac{\partial^2 p}{\partial \theta^2} \tag{14}$$

Using (10) - (14) in (9), we obtain

$$-\delta_\xi^2 \psi_{i,j} - \delta_\theta^2 \psi_{i,j} - \left(\frac{h^2 + k^2}{12}\right)\delta_\xi^2 \delta_\theta^2 \psi_{i,j} + \frac{h^2}{12}\delta_\xi^2 p_{i,j}$$
$$+ \frac{k^2}{12}\delta_\theta^2 p_{i,j} + p_{i,j} = 0 \tag{15}$$

Equation (15) is the fourth order compact discretization of the governing equation (2). The derivatives $\frac{\partial^2 p}{\partial \xi^2}$ and $\frac{\partial^2 p}{\partial \theta^2}$ are calculated analytically and used in (15) in place of difference approximations. Equation (15) is the fourth order compact discretization of the governing equation (2).

B. Discretization of Vorticity Transport Equation

Equation (3) is rewritten as

$$-\frac{\partial^2 \omega}{\partial \xi^2} - \frac{\partial^2 \omega}{\partial \theta^2} + c\frac{\partial \omega}{\partial \xi} + d\frac{\partial \omega}{\partial \theta} + S = 0 \tag{16}$$

where

$$c = \frac{1}{Pr}\frac{\partial \psi}{\partial \theta},$$

$$d = -\frac{1}{Pr}\frac{\partial \psi}{\partial \xi},$$

$$S = -e^\xi Ra\left(\frac{\partial T}{\partial \xi}\sin\theta + \frac{\partial T}{\partial \theta}\cos\theta\right).$$

Once again using (5) - (8) in (16), we obtain

$$-\delta_\xi^2 \omega_{i,j} - \delta_\theta^2 \omega_{i,j} + c_{i,j}\delta_\xi \omega_{i,j} + d_{i,j}\delta_\theta \omega_{i,j}$$
$$-\tau_{i,j} + S_{i,j} = 0 \tag{17}$$

The truncation error of equation (17) is

$$\tau_{i,j} = \left[2\left(\frac{h^2}{12}c\frac{\partial^3 \omega}{\partial \xi^3} + \frac{k^2}{12}d\frac{\partial^3 \omega}{\partial \theta^3}\right) - \left(\frac{h^2}{12}\frac{\partial^4 \omega}{\partial \xi^4} + \frac{k^2}{12}\frac{\partial^4 \omega}{\partial \theta^4}\right)\right]_{i,j} +$$
$$O(h^4, k^4) \tag{18}$$

Differentiating partially the vorticity equation (16) respect to ξ and θ, we obtain

$$\frac{\partial^3 \omega}{\partial \xi^3} = -\frac{\partial^3 \omega}{\partial \xi \partial \theta^2} + c\frac{\partial^2 \omega}{\partial \xi^2} + d\frac{\partial^2 \omega}{\partial \xi \partial \theta} + \frac{\partial c}{\partial \xi}\frac{\partial \omega}{\partial \xi} + \frac{\partial d}{\partial \xi}\frac{\partial \omega}{\partial \theta} + \frac{\partial S}{\partial \xi} \tag{19}$$

$$\frac{\partial^4 \omega}{\partial \xi^4} = -\frac{\partial^4 \omega}{\partial \xi^2 \partial \theta^2} - c\frac{\partial^3 \omega}{\partial \xi \partial \theta^2} + d\frac{\partial^3 \omega}{\partial \xi^2 \partial \theta} + \left(2\frac{\partial c}{\partial \xi} + c^2\right)\frac{\partial^2 \omega}{\partial \xi^2} +$$
$$\left(2\frac{\partial d}{\partial \xi} + cd\right)\frac{\partial^2 \omega}{\partial \xi \partial \theta} + \left(\frac{\partial^2 c}{\partial \xi^2} + c\frac{\partial c}{\partial \xi}\right)\frac{\partial \omega}{\partial \xi} + \left(\frac{\partial^2 d}{\partial \xi^2} + c\frac{\partial d}{\partial \xi}\right)\frac{\partial \omega}{\partial \theta} + \frac{\partial^2 S}{\partial \xi^2} +$$
$$c\frac{\partial S}{\partial \xi} \tag{20}$$

$$\frac{\partial^3 \omega}{\partial \theta^3} = -\frac{\partial^3 \omega}{\partial \xi^2 \partial \theta} + c\frac{\partial^2 \omega}{\partial \xi \partial \theta} + d\frac{\partial^2 \omega}{\partial \theta^2} + \frac{\partial c}{\partial \theta}\frac{\partial \omega}{\partial \xi} + \frac{\partial d}{\partial \theta}\frac{\partial \omega}{\partial \theta} + \frac{\partial S}{\partial \theta} \tag{21}$$

$$\frac{\partial^4 \omega}{\partial \theta^4} = -\frac{\partial^4 \omega}{\partial \xi^2 \partial \theta^2} + c\frac{\partial^3 \omega}{\partial \xi \partial \theta^2} - d\frac{\partial^3 \omega}{\partial \xi^2 \partial \theta} + \left(2\frac{\partial d}{\partial \theta} + d^2\right)\frac{\partial^2 \omega}{\partial \theta^2} +$$
$$\left(2\frac{\partial c}{\partial \theta} + cd\right)\frac{\partial^2 \omega}{\partial \xi \partial \theta} + \left(\frac{\partial^2 c}{\partial \theta^2} + d\frac{\partial c}{\partial \theta}\right)\frac{\partial \omega}{\partial \xi} + \left(\frac{\partial^2 d}{\partial \theta^2} + d\frac{\partial d}{\partial \theta}\right)\frac{\partial \omega}{\partial \theta} +$$
$$\frac{\partial^2 S}{\partial \theta^2} + d\frac{\partial S}{\partial \theta} \tag{22}$$

Substituting (18) - (22) in (17) gives

$$-l_{i,j}\delta_\xi^2 \omega_{i,j} - f_{i,j}\delta_\theta^2 \omega_{i,j} + g_{i,j}\delta_\xi \omega_{i,j} + o_{i,j}\delta_\theta \omega_{i,j} -$$
$$\left(\frac{h^2 + k^2}{12}\right)\left(\delta_\xi^2 \delta_\theta^2 \omega_{i,j} - c_{i,j}\delta_\xi \delta_\theta^2 \omega_{i,j} - d_{i,j}\delta_\xi^2 \delta_\theta \omega_{i,j}\right) + S_{i,j} +$$
$$\frac{h^2}{12}\left(\delta_\xi^2 S_{i,j} - c_{i,j}\delta_\xi S_{i,j}\right) + \frac{k^2}{12}\left(\delta_\theta^2 S_{i,j} - d_{i,j}\delta_\theta S_{i,j}\right) +$$
$$q_{i,j}\delta_\xi \delta_\theta \omega_{i,j} = 0 \tag{23}$$

where the coefficients $l_{i,j}, f_{i,j}, g_{i,j}, o_{i,j}$ and $q_{i,j}$ are given by

$$l_{i,j} = 1 + \frac{h^2}{12}\left(c_{i,j}^2 - 2\delta_\xi c_{i,j}\right),$$

$$f_{i,j} = 1 + \frac{k^2}{12}\left(d_{i,j}^2 - 2\delta_\theta d_{i,j}\right),$$

$$g_{i,j} = c_{i,j} + \frac{h^2}{12}\left(\delta_\xi^2 c_{i,j} - c_{i,j}\delta_\xi c_{i,j}\right) + \frac{k^2}{12}\left(\delta_\theta^2 c_{i,j} - d_{i,j}\delta_\theta c_{i,j}\right),$$

$$o_{i,j} = d_{i,j} + \frac{h^2}{12}\left(\delta_\xi^2 d_{i,j} - c_{i,j}\delta_\xi d_{i,j}\right)$$
$$+ \frac{k^2}{12}\left(\delta_\theta^2 d_{i,j} - d_{i,j}\delta_\theta d_{i,j}\right),$$

$$q_{i,j} = \frac{h^2}{12}\left(2\delta_\xi d_{i,j} - c_{i,j}d_{i,j}\right) + \frac{k^2}{12}\left(2\delta_\theta c_{i,j} - c_{i,j}d_{i,j}\right).$$

Equation (23) is the fourth order compact discretization of (16).

C. Discretization of Energy Equation

Equation (4) is rewritten as

$$-\frac{\partial^2 T}{\partial \xi^2} - \frac{\partial^2 T}{\partial \theta^2} + cc\frac{\partial T}{\partial \xi} + dd\frac{\partial T}{\partial \theta} = 0 \tag{24}$$

where

$$cc = \frac{\partial \psi}{\partial \theta}, \quad dd = -\frac{\partial \psi}{\partial \xi}.$$

Repeating the above discretization process of the vorticity equation (16) to energy equation (24), we obtain

$$-ll_{i,j}\delta_\xi^2 T_{i,j} - ff_{i,j}\delta_\theta^2 T_{i,j} + gg_{i,j}\delta_\xi T_{i,j} + oo_{i,j}\delta_\theta T_{i,j} -$$
$$\left(\frac{h^2+k^2}{12}\right)\left(\delta_\xi^2\delta_\theta^2 T_{i,j} - cc_{i,j}\delta_\xi\delta_\theta^2 T_{i,j} - dd_{i,j}\delta_\xi^2\delta_\theta T_{i,j}\right) +$$
$$qq_{i,j}\delta_\xi\delta_\theta T_{i,j} = 0 \quad (25)$$

where the coefficients $ll_{i,j}, ff_{i,j}, gg_{i,j}, oo_{i,j}$ and $qq_{i,j}$ are given by

$$ll_{i,j} = 1 + \frac{h^2}{12}\left(cc_{i,j}^2 - 2\delta_\xi cc_{i,j}\right),$$

$$ff_{i,j} = 1 + \frac{k^2}{12}\left(dd_{i,j}^2 - 2\delta_\theta dd_{i,j}\right),$$

$$gg_{i,j} = cc_{i,j} + \frac{h^2}{12}\left(\delta_\xi^2 cc_{i,j} - cc_{i,j}\delta_{\xi c} c_{i,j}\right)$$
$$+ \frac{k^2}{12}\left(\delta_\theta^2 cc_{i,j} - dd_{i,j}\delta_\theta cc_{i,j}\right),$$

$$oo_{i,j} = dd_{i,j} + \frac{h^2}{12}\left(\delta_\xi^2 dd_{i,j} - cc_{i,j}\delta_\xi dd_{i,j}\right)$$
$$+ \frac{k^2}{12}\left(\delta_\theta^2 dd_{i,j} - dd_{i,j}\delta_\theta dd_{i,j}\right),$$

$$qq_{i,j} = \frac{h^2}{12}\left(2\delta_\xi dd_{i,j} - cc_{i,j}dd_{i,j}\right)$$
$$+ \frac{k^2}{12}\left(2\delta_\theta cc_{i,j} - cc_{i,j}dd_{i,j}\right).$$

Equation (25) is the fourth order approximation to energy equation (24).

D. Discretization of Boundary Conditions

On the surface of the cylinder, no-slip condition is applied. We now turn to the boundary condition for the vorticity, focusing our discussion on the boundary where i = 1. The vorticity boundary condition at i = 1 is derived using $\psi = \frac{\partial\psi}{\partial\xi} = 0$ in equation (2). Following Briley's procedure [11] we obtain the formula

$$\omega_{1,j} = -\frac{108\psi_{2,j} - 27\psi_{3,j} + 4\psi_{4,j}}{18h^2}$$

For evaluating boundary conditions, along the axis of symmetry, the derivative $\frac{\partial T}{\partial\theta}$ is approximated by fourth order forward difference along $\theta = 0$ (i.e., $j = 1$) and fourth order backward difference along $\theta = \pi$ (or $j = m + 1$) as follows.

$$T(i,1) = \frac{1}{25}[48T(i,2) - 36T(i,3) + 16T(i,4) - 3T(i,5)]$$

$$T(i,m+1) = \frac{1}{25}[48T(i,m) - 36T(i,m-1)$$
$$+ 16T(i,m-2) - 3T(i,m-3)].$$

The algebraic system obtained from the fourth order discretized energy equation (25), vorticity transport equation (23) and stream function equation (15) are solved using line Gauss-Seidel method. The algebraic equations for T, ω and ψ were solved simultaneously and the vorticity boundary condition for ω and temperature boundary conditions on axis of symmetry are updated after every iteration. The iterations are continued until the norm of the dynamic residuals is less than 10^{-5}.

IV. RESULTS AND DISCUSSION

The Laminar natural-convective heat transfer from a horizontal cylinder is analyzed by solving the complete

Navier-Stokes and energy equations using higher order compact scheme on the nine point 2-D stencil. A far field of 24.53 times the radius of the cylinder is considered in all the numerical simulations which are performed in the finest grid of 160 X 80. Numerical investigations were carried out for Rayleigh numbers (Ra) of 1, 10, 100 and 1000 for a Prandtl number (Pr) of 0.7. The local Nusselt number (Nu) at the cylinder surface is given by

$$Nu(\theta) = -2\left(\frac{\partial T}{\partial\xi}\right)_{\xi=0} \quad (27)$$

The average Nusselt number (N_m) is calculated using the formula

$$N_m = \frac{1}{\pi}\int_0^\pi Nu(\theta)d\theta \quad (28)$$

The average Nusselt number (Nm) values obtained for Ra = 1, 10, 100, and 1000 at Pr = 0.7 using different grids is presented in Fig. 1 to show the grid independence. Calculated average Nusselt number values for Ra from 1 to 1000 at Pr = 0.7 is presented in Fig. 2 along with numerical results of Kuehn & Goldstein [1]. The obtained results are in agreement with literature values of Kuehn & Goldstein. The angular variation of local Nusselt number (Nu) for different Ra at Pr = 0.7 along surface of the cylinder are in line with numerical results of Kuehn & Goldstein [1] as shown in Fig. 3. The surface vorticity is also presented in Fig. 4 for different Ra at Pr = 0.7. The pattern of these graphs is in good agreement with those presented by Geoola & Cornish [12].
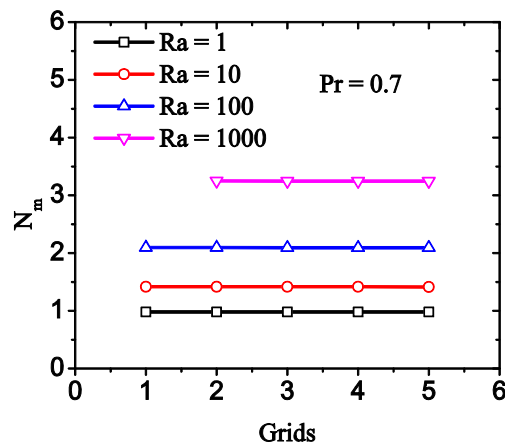

Fig. 1 Effect of grid size on average Nusselt number(Nm) for different Ra at Pr = 0.7. Here the values 1 to 5 in x-axis are the grids 40 X 20, 64 X 32, 80 X 40, 128 X 64 and 160 X 80 respectively

Fig. 5 show the distribution of isotherms around the cylinder at Ra = 1, 100 and 1000 for Pr = 0.7. As is to be expected, an increase in the Ra causes the thickness of the heated layer over the upstream region of the solid cylinder to decrease, while that over the downstream region increases. The pattern of these graphs is in good agreement with those presented by Kuehn & Goldstein [1] and Saitoh et.al [10]. The streamlines for Ra = 1, 100 and 1000 at Pr = 0.7 is presented in Fig. 6. The vorticity contours for Ra = 1, 100 and 1000 at Pr = 0.72 is presented in Fig. 7.

The pattern of these graphs is in agreement with those presented by Kuehn & Goldstein [1] and Saitoh et.al [10].
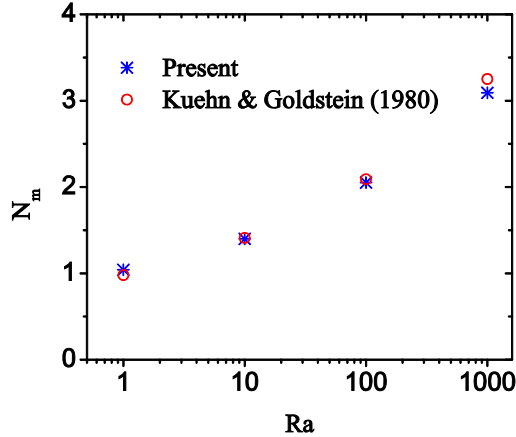

Fig. 2 Comparison of mean Nusselt number N_m values with other numerical results for various Ra at $Pr = 0.7$
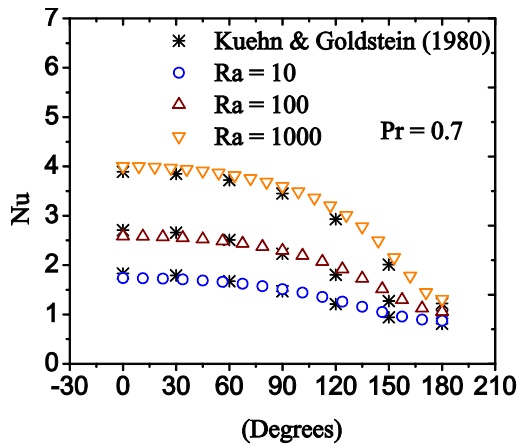

Fig. 3 Comparison of angular variation of local Nusselt number (Nu) values with other numerical results for various Ra at $Pr = 0.7$
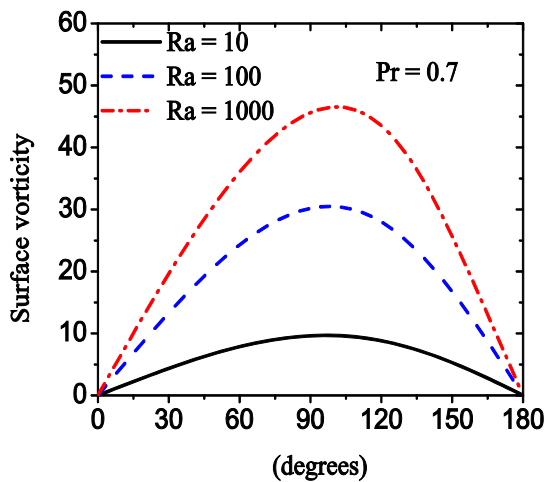

Fig. 4 Angular variation of the surface vorticity for different Ra at $Pr = 0.7$. Here, the surface vorticity values for $Ra = 10$ and 100 are respectively multiplied by 5 and 4
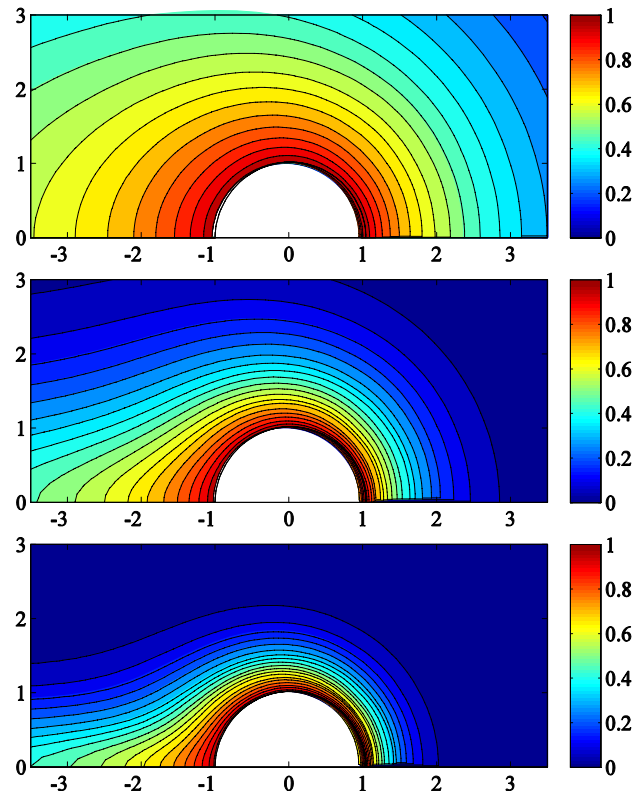

Fig. 5 Isotherms for $Ra = 1$, 100 and 1000 at $Pr = 0.7$ (top to bottom). The isotherms are drawn for values in the range 0(0.05)1
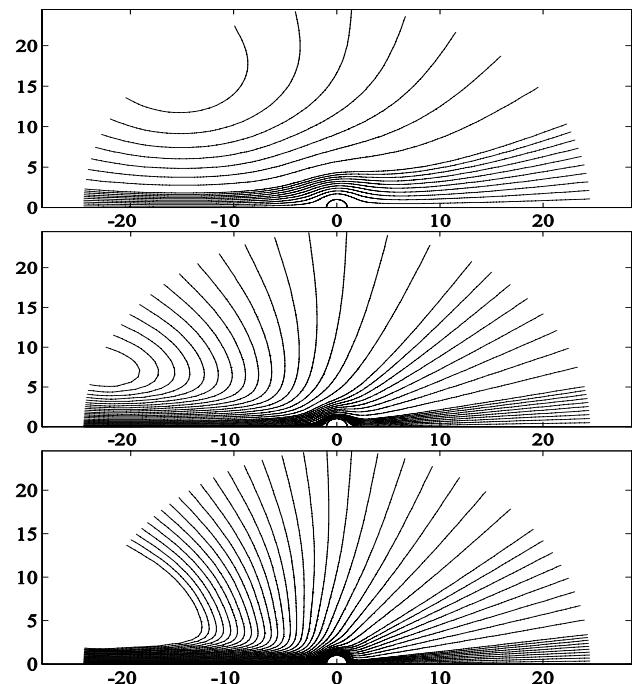

Fig. 6 Streamlines for $Ra = 1$, 100 and 1000 at $Pr = 0.7$(top to bottom). The contours are drawn for values in the range 0(0.1)1(2)40

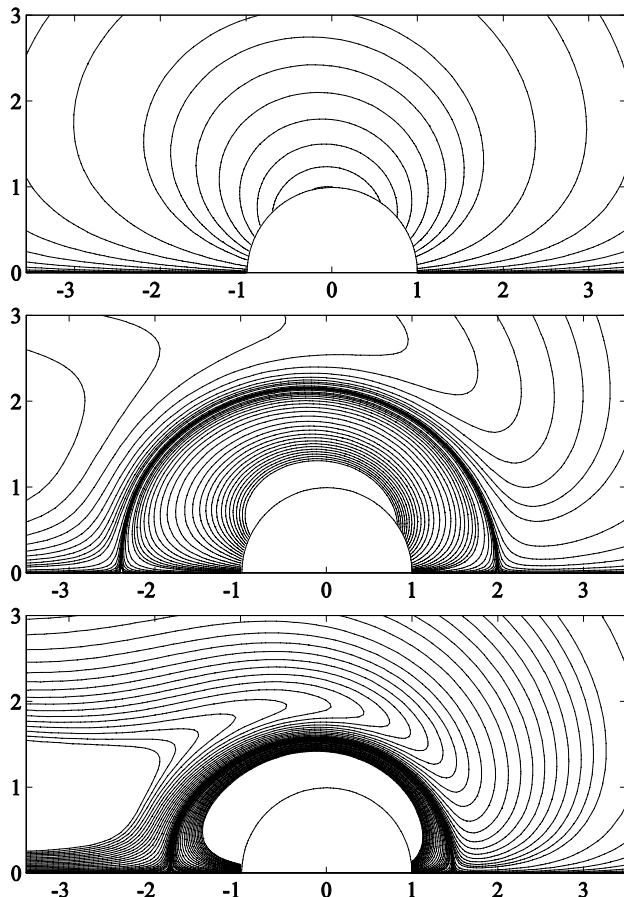[11] F. Geoola, and A. R. H. Cornish, " Numerical solution of steady-state free convective heat transfer from a solid sphere," Int J Heat Mass Transfer, vol. 24, pp. 1369–79, 1981.

Fig. 7 Vorticity contours for $Ra = 1$, 100 and 1000 at $Pr = 0.7$ (top to bottom). The contours are drawn for values in the range -4(0.2)4

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