Nonlinear Dynamics of Cracked RC Beams under Harmonic Excitation

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Abstract—Nonlinear response behaviour of a cracked RC beam under harmonic excitation is analysed to investigate various instability phenomena like, bifurcation, jump phenomena etc. The nonlinearity of the system arises due to opening and closing of the cracks in the RC beam and is modelled as a cubic polynomial. In order to trace different branches at the bifurcation point on the response curve (amplitude versus frequency of excitation plot), an arc length continuation technique along with the incremental harmonic balance (IHBC) method is employed. The stability of the solution is investigated by the Floquet theory using Hsu's scheme. The periodic solutions obtained by the IHBC method are compared with these obtained by the numerical integration of the equation of motion. Characteristics of solutions fold bifurcation, jump phenomena and from stable to unstable zones are identified.

Keywords—Incremental harmonic balance; Arc-length continuation; Bifurcation; Jump phenomena.

I. INTRODUCTION

CIVIL engineering structures are generally subjected to primarily natural loads such as Earthquake, Wind Blast and man-made loads impact, vehicle movement, operating equipments etc. The study is mainly focused on the inelastic structural behaviour caused by yielding of reinforcement, crushing of concrete, debonding, etc. [1,2]. (Davenne et al. 2003; Mo 1994). Generally, the dynamic analysis concrete structures are carried out considering they are linear elastic and no tension cracks develop at working loads. In reality, since concrete has low tensile resistance, the cracks develop even at servicing loading level. The damages become more pronounced, if the applied loads are further increased and as a result behaviour of concrete beams is best studied considering them nonlinear and inelastic. The concrete structures may also exhibit quasi static behaviour under the action of applied loads. Such concrete structures may also be treated as nonlinear elastic assuming material linearity and small displacements. If no further crack occurs under loads, the concrete structures with cracks resemble to mechanical systems due to opening and closing of the cracks. As a result, damaged structures may often show nonlinear characteristics under dynamic loads and using these characteristics of nonlinear dynamics, procedures for damage identification in the RC concrete structures have been developed. [3,4]. In the present study, nonlinear response behaviour of a cracked RC beam under harmonic excitation is analysed to investigate

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various instability phenomena like, bifurcation, jump phenomena etc. The cracked RC beam is modeled as a SDOF system. The nonlinearity of the system arises due to opening and closing of the cracks in the RC beam and is modelled as a cubic polynomial.

The dynamic behaviour of such cracked concrete structures has not yet been thoroughly studied. To study the dynamic behaviour, in the past, as in the problems resembling the nonlinear mechanical systems, either the problem is linearized and analysed to obtain highly approximate response diagram [5] or the harmonic balance method [6] was used to obtain the response diagram, but it was observed that it is highly approximate while obtaining the response assuming one or two dominant frequencies of the solutions. The main difficulty of the method is that a set of nonlinear algebraic equations in terms of the Fourier coefficients need to be reformulated each time the number of harmonics is changed. In recent years, this drawback was greatly removed by applying arc-length continuation technique along with incremental harmonic balance method [7] for such type of problems. With this back ground, nonlinear response behaviour of a cracked RC beam under harmonic excitation is analysed to investigate various instability phenomena that may exist in the system using incremental harmonic balance method with arc length continuation technique (IHBC). The cracked RC beam has stiffness nonlinearity and therefore, expected to show a variety of instability phenomena. The nonlinearity of the restoring force is represented by a cubic polynomial. The forcing function on the cracked RC beam is monoharmonic excitation and stability of the periodic motion is investigated by Floquet theory. The present study is motivated by the need for a better semianalytical prediction of complex periodic via IHB, as previous theoretical analysis focused on weakly nonlinear regimes (via both multiple-scales asymptotics and a straight forward harmonic balance analysis). Further, although there have been a few applications of the incremental harmonic balance method with arc-length continuation technique for studying the stability problems, the efficiency of the method has not been thoroughly investigated in identifying different types of instability phenomena that exist in nonlinear mechanical systems such as cracked RC beam. Nevertheless, the objective of the study is to demonstrate how efficiently IHBC can be used to treat nonlinearity present in the system and also to investigate latent instability phenomena present in the problem under study.

II. MATHEMATICAL MODEL

The dynamical system considered is a simplified model of the cracked RC beam (Fig.1) system as considered by Chen et al.[6] The cracked RC beam is simply supported and modelled

as a single degree of freedom nonlinear system under monoharmonic excitation at mid span only. The equation of motion of the system is written as

$$my'' + cy' + ky - \beta y^3 = P_0 \sin(\omega t) \tag{1}$$

where $y' = dy/dt$ and $y'' = d^2y/dt^2$

where $m = \rho L/2$ is the modal mass of the fundamental mode, $c = 2\xi\omega_n m$, $k = \omega_n^2 m$ and β is the parameter representing the nonlinearity of the cracked RC beam.
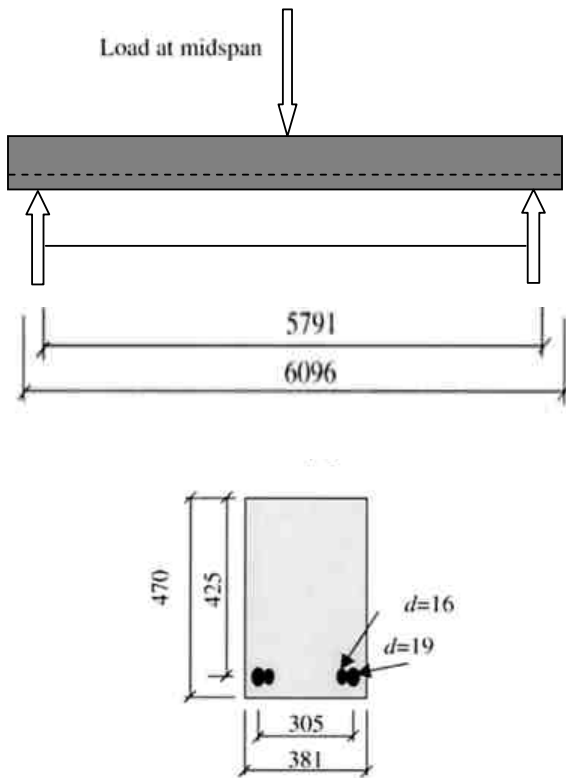

Fig. 1 RC Beam under midspan dynamic load (Dimensions in mm)

III. RESPONSE ANALYSIS IN FREQUENCY DOMAIN USING IHB

Oscillations of nonlinear dynamical systems with N degrees of freedom, subjected to periodic excitation can be described by a system of nonlinear ordinary differential equations

$$\omega^2 \ddot{x}_i + f_i(x_1,...,x_N; \dot{x}_i,...,\dot{x}_N; \omega, \lambda) = q_i(\tau), \quad (i = 1,...,N) \tag{2}$$

where dimensionless time $\tau = \omega t$ is introduced as an independent variable and $\dot{x} = dx/d\tau$, $\ddot{x} = d^2x/d\tau^2$. $f_i(x_1,...,x_N; \dot{x}_i,...,\dot{x}_N; \omega, \lambda)$ denotes nonlinear functions of dependent variables xi and corresponding first derivatives

$$\dot{x}_i = \frac{1}{\omega}\frac{dx_i}{dt}, (i = 1,...,N),$$

the exciting frequency ω and the variable parameter λ. Nonlinear system can be excited by general functions, expressed in terms of truncated Fourier series

$$q_i(\tau) = \frac{1}{2}g_{i_0} + \sum_{n=1}^{K}\left[g_{i,n}\cos(n\tau) + h_{i,n}\sin(n\tau) \right]$$

$$= \frac{1}{2}g_{i_0} + \sum_{n=1}^{K}\left[g_{i,n}\cos(n\omega t) + h_{i,n}\sin(n\omega t) \right] \tag{3}$$

Clearly, when the calculation of the resonance curve is of interest, only one term representation is assumed. Therefore,

$$q_i(\tau) = g_i \sin(m\tau) \tag{4}$$

is chosen in Eq. (2), where m is an integer, $m \in \{1, 2,...\}$. Note that for subharmonic resonances of order m, value of m is appropriately adjusted [8].

As first step in the IHB method, the Newton-Raphson iterative process is introduced. Assuming that some initial solution or initial guess determining the initial state $x_0, \dot{x}_0, \omega_0, \lambda_0, g_{i_0}$; $(i = 1,...,N)$ is known and try to get a neighbouring solution

$$x_i = x_{i_0} + \Delta x_i \quad \dot{x}_i = \dot{x}_{i_0} + \Delta \dot{x}_i \quad \omega = \omega_0 + \Delta\omega \quad \lambda = \lambda_0 + \Delta\lambda \quad g_i = g_{i_0} + \Delta g_i \quad (i = 1,...,N) \tag{5}$$

by adding small increments $\Delta x_i, \Delta \dot{x}_i, \Delta\omega, \Delta\lambda, \Delta g_i$ to the initial solution.

Substituting these terms in Eq. (4) and expanding it by Taylor's series about the initial state, the linearized incremental equations are obtained as

$$\omega_0^2 \Delta\ddot{x}_i + \sum_{j=1}^{N}\left(\frac{\partial f_i}{\partial \dot{x}_j}\right)_0 \Delta\dot{x}_j + \sum_{j=1}^{N}\left(\frac{\partial f_i}{\partial x_j}\right)_0 \Delta x_j = \left[g_{i_0}\sin(m\tau) - \omega_0^2 \ddot{x}_{i_0} - f_{i_0} \right]$$

$$- \left[\left(\frac{\partial f_i}{\partial\omega}\right)_0 + 2\omega_0\ddot{x}_{i_0}\right]\Delta\omega - \left(\frac{\partial f_i}{\partial\lambda}\right)_0\Delta\lambda + \sin(m\tau)\Delta g_i \tag{6}$$

neglecting all higher order terms of small increments.

By collecting initial values and increments in vector forms

$$\mathbf{X_0} = \left[x_{1_0},..., x_{N_0} \right]^T \quad \Delta\mathbf{X} = \left[\Delta x_1,..., \Delta x_N \right]^T \tag{7a}$$

$$\mathbf{G_0} = \left[g_{1_0},..., g_N \right]^T \quad \Delta\mathbf{G} = \left[\Delta g_1,..., \Delta g_N \right]^T \tag{7b}$$

$$\mathbf{F_0} = \left[f_{1_0},..., f_{N_0} \right]^T \tag{7c}$$

Defining matrices C and K with corresponding elements

$$\mathbf{C}_{ij} = \left(\frac{\partial f_i}{\partial \dot{x}_j} \right)_0 \tag{8a}$$

$$\mathbf{K}_{ij} = \left(\frac{\partial f_i}{\partial x_j} \right)_0 \tag{8b}$$

and vectors $\mathbf{Q}$, $\mathbf{P}$ and $\mathbf{R}$ with appropriate components

$$\mathbf{Q}_i = \left(\frac{\partial f_i}{\partial \omega} \right)_0 \tag{9a}$$

$$\mathbf{P}_i = \left(\frac{\partial f_i}{\partial \lambda} \right)_0 \tag{9b}$$

$$\mathbf{R}_i = g_{i_0} \sin(m\tau) - (\omega_0^2 \ddot{x}_{i_0} + f_{i_0}) \tag{9c}$$

linearized incremental equations can be written in the convenient matrix form as

$$\omega_0^2 \mathbf{\Delta \ddot{X}} + \mathbf{C \Delta \dot{X}} + \mathbf{K \Delta X} = \mathbf{R} - \left(2\omega_0 \mathbf{\ddot{X}}_0 + \mathbf{Q} \right) \Delta \omega$$
$$- \mathbf{P} \Delta \lambda + \sin(m\tau) \mathbf{\Delta G} \tag{10}$$

Matrix Eq. (10) is linear with variable coefficients and can be solved by Galerkin procedure, which is the second step in the IHB method. The steady-state solution of Eq. (10) is periodic and can be represented by

$$x_{i_0} = \sum_{n=0}^{K} \left(a_{i,n}^0 \cos(n\tau) + b_{i,n}^0 \sin(n\tau) \right) \tag{11a}$$

$$\Delta x_i = \sum_{n=0}^{K} \left(\Delta a_{i,n} \cos(n\tau) + \Delta b_{i,n} \text{in}(n\tau) \right) \quad (i = 1,...,N) \tag{11b}$$

Eqs. (11a), and (11b) can be written in the matrix form as

$$x_{i_0} = \mathbf{T a}_{i_0} \tag{12a}$$

$$\Delta x_i = \mathbf{T \Delta a}_i, \quad (i = 1, ..., N) \tag{12b}$$

Where

$$\mathbf{T} = \left[1, \cos\tau, ..., \cos(K\tau); \ 0, \sin\tau, ..., \sin(K\tau) \right] \tag{13a}$$

$$\mathbf{a}_{i_0} = \left[a_{i,0}^0, a_{i,1}^0, ..., a_{i,K}^0; \ b_{i,0}^0, b_{i,1}^0, ..., b_{1,K}^0 \right]^T \tag{13b}$$

$$\mathbf{\Delta a_i} = \left[\Delta a_{i,0}, \Delta a_{i,1}, ..., \Delta a_{i,K}; \ \Delta b_{i,0}, \Delta b_{i,1}, ..., \Delta b_{1,K} \right]^T \tag{13c}$$

Additionally, the matrix $\mathbf{Y}$ and vectors $\mathbf{A}_0$, $\mathbf{\Delta A}$ are

$$\mathbf{Y} = \begin{bmatrix} \mathbf{T} & 0 & ... & 0 \\ 0 & \mathbf{T} & & 0 \\ ... & ... & ... & ... \\ 0 & 0 & & \mathbf{T} \end{bmatrix} \tag{14a}$$

$$\mathbf{A}_0 = \begin{bmatrix} \mathbf{a}_{1_0} \\ \mathbf{a}_{2_0} \\ ... \\ \mathbf{a}_{n_0} \end{bmatrix} \tag{14b}$$

$$\mathbf{\Delta A} = \begin{bmatrix} \mathbf{\Delta a}_1 \\ \mathbf{\Delta a}_2 \\ ... \\ \mathbf{\Delta a}_N \end{bmatrix} \tag{14c}$$

which are used in the interpretation of vectors $\mathbf{X}_0$ and $\mathbf{\Delta X}$ as

$$\mathbf{X}_0 = \mathbf{Y A}_0 \tag{15a}$$

$$\mathbf{\Delta X} = \mathbf{Y \Delta A} \tag{15b}$$

Applying the Galerkin procedure for one period

$$\int_0^{2\pi} \delta \mathbf{\Delta A}^T \{ \mathbf{Y}^T [\omega_0^2 \mathbf{\ddot{Y}} + \mathbf{C \dot{Y}} + \mathbf{KY}] \mathbf{\Delta A} \} d\tau = \int_0^{2\pi} \delta \mathbf{\Delta A}^T \{ \mathbf{Y}^T [\mathbf{R} - (\mathbf{Q} + 2\omega_0 \mathbf{\ddot{Y} A}_0) \Delta \omega - \mathbf{P} \Delta \lambda$$
$$+ \mathbf{\Delta G} \sin(m\tau)] \} d\tau \tag{16}$$

the linear matrix equation for unknown vector of harmonic increments $\mathbf{\Delta A}$ is obtained

$$\mathbf{k \Delta A} = \mathbf{r} + \mathbf{q} \Delta \omega + \mathbf{p} \Delta \lambda + \mathbf{s \Delta G} \tag{17}$$

with matrix $\mathbf{k}$, $\mathbf{s}$ and vectors $\mathbf{r}$, $\mathbf{q}$ and $\mathbf{p}$ are given

$$\mathbf{k} = \int_0^{2\pi} \mathbf{Y}^T [\omega_0^2 \mathbf{\ddot{Y}} + \mathbf{C \dot{Y}} + \mathbf{KY}] d\tau \tag{18a}$$

$$\mathbf{r} = \int_0^{2\pi} [\omega_0^2 \mathbf{\dot{Y}}^T \mathbf{\dot{Y} A}_0 - \mathbf{Y}^T \mathbf{F}_0 + \mathbf{Y}^T \mathbf{G}_0 \sin(m\tau)] d\tau \tag{18b}$$

$$\mathbf{q} = \int_0^{2\pi} [2\omega_0 \mathbf{\dot{Y}}^T \mathbf{\dot{Y} A}_0 - \mathbf{Y}^T \mathbf{Q}] d\tau \tag{18c}$$

$$\mathbf{p} = - \int_0^{2\pi} \mathbf{Y}^T \mathbf{P} d\tau \tag{18d}$$

$$\mathbf{s} = \int_0^{2\pi} \mathbf{Y}^T \sin(m\tau) d\tau$$

(18e)

where k (Eq. 17) is the Jacobian matrix obtained from a set of $N \times (2K+1)$ equations for the unknown coefficients vector $\mathbf{\Delta A}$.

In the above equations, the vector double and triple product of displacements are obtained from the expressions given in the appendix [20] and matrices $\mathbf{k}$ and $\mathbf{s}$ and vectors $\mathbf{r}$, $\mathbf{q}$ and $\mathbf{p}$ are calculated using Eqs. (20a), (20e) and (20b), (20c) and (20d) respectively.

Finally, the matrix Eq. (17) is solved at each time step using Newton-Raphson iterative process. The IHB method with a variable parameter is ideally suited to parametric continuation for obtaining the response diagrams of nonlinear systems (IHBC). After obtaining the solution for the particular value of a parameter, the solution for another value of parameter perturbed from the old one can be obtained by iterations using the previous solution as an approximation. The main aim of the path following and parametric continuation is to effectively trace the bifurcation sequence as a parameter of the system is varied. In this study, an arc-length procedure [7] is adopted for the parametric continuation.

Application of IHBC for response analysis of two point mooring system

The simplified model of the cracked RC beam is considered and the equation of motion of the nonlinear system is written as

$$mx'' + cx' + kx - \beta x^3 = P_0 \sin(\omega t)$$

(19)

where $x' = dx/dt$ and $x'' = d^2x/dt^2$ forcing function $P_0\sin(\omega t)$ has period $T = 2\pi/\omega$. The Eq. (19) is rewritten as

$$\ddot{x}(t') + 2\xi\dot{x}(t') + x(t') - \mu x^3(t') = X_{st}\sin(rt')$$

(20)

where

$$\dot{x} = dx/dt', \ddot{x} = d^2x/dt'^2, \mu = \beta/k, r = \omega/\omega_n, t' = \omega_n t$$

and $X_{st} = P_0/k$. The quantity X_{st} denotes the pseudo-static displacement under the peak load P_0.

The first step for application of IHB method is to transform the equation of motion into non-dimensional form. Considering Eq. (19) and introducing non-dimensional form $\tau = rt$, the equation of motion is transformed to

$$r^2\ddot{x}(\tau) + 2\xi r\dot{x}(\tau) + x(\tau) - \mu x^3(\tau) = X_{st}\sin(\tau)$$

(21)

where $\dot{x} = dx/d\tau$ and $\ddot{x} = d^2x/d\tau^2$.

The application of the IHBC method in this case starts from linearizing the Eq. (21), which is rewritten in a compact form

$$f(\ddot{x}, \dot{x}, x, X_{st}, r, \tau) = 0$$

(22)

The periodic solution of this equation is assumed in the form of a truncated Fourier series as explained in the preceding section. The assumed solution x0 being approximate $x_0 + \Delta x$ would be more accurate solution of Eq. (22). Expanding Eq. (22) in Taylor series about the initial state, the linearized equations in terms of the increments can be written corresponding to Eq. (22).

$$\frac{\partial f}{\partial \ddot{x}}\left[\sum_{n=0}^{K}\left(-n^2\Delta\mathbf{a}_n\cos n\tau - n^2\Delta\mathbf{b}_n\sin n\tau\right)\right] + \frac{\partial f}{\partial \dot{x}}\left[\sum_{n=0}^{K}\left(-n\Delta\mathbf{a}_n\sin n\tau + n\Delta\mathbf{b}_n\cos n\tau\right)\right] + \frac{\partial f}{\partial x}\left[\sum_{n=0}^{K}\left(\Delta\mathbf{a}_n\cos n\tau + \Delta\mathbf{b}_n\sin n\tau\right)\right] = \mathbf{R} - \mathbf{Q}\Delta r + \Delta X_{st}\sin\tau$$

(23)

Where R is the correction term, which will vanish when x0(τ) is an exact solution and Q is the unbalanced force term due to unit frequency shifting. Since, the frequency response behaviour of the equation of motion is of interest, ΔX_{st} are set to zero. For the present problem, R is expressed as

$$\mathbf{R} = -[r_0^2\ddot{x}_0 + c_1 r_0\dot{x}_0 + k_1 x_0 + k_2 x_0^3 - X_{st_0}\sin(\tau)]$$

(24)

and the variable coefficients $\dfrac{\partial f}{\partial \ddot{x}}, \dfrac{\partial f}{\partial \dot{x}}, \dfrac{\partial f}{\partial x}$ and $\mathbf{Q}$ are obtained as

$$\frac{\partial f}{\partial \ddot{x}} = r^2$$

(25a)

$$\frac{\partial f}{\partial \dot{x}} = 2\xi r$$

(25b)

$$\frac{\partial f}{\partial x} = 1 - 3\mu x^2$$

(25c)

$$\mathbf{Q} = \frac{\partial f}{\partial \omega} = 2r\ddot{x} + 2\xi\dot{x}$$

(25d)

Substituting Eqs. (24, 25a-25b) in Eq. (23), Galerkin's procedure is performed with Δans and Δbns as generalised coordinates. Typical elements of matrices k and s and vectors r, q and p are obtained using Eqs. (18a), (18e) and (18b), (18c) and (18d) by applying the knowledge of trigonometric sum, product rule and their integrals. Solutions of Eq. (21) are used to provide the amplitude vs frequency plots. The continuation technique is used to obtain response plot automatically over the frequency range of interest from a known solution at the starting frequency. The accuracy (or goodness of the method) is tested by comparing the results of IHBC with those of NI wherever possible. A tolerance limit for the residual given by Eq. (24) for the converged solution is specified (generally taken as 10-4 times the amplitude of excitation).

IV. PATH FOLLOWING AND PARAMETRIC CONTINUATION

The IHB method with a variable parameter is ideally suited to parametric continuation for obtaining the response

diagrams of nonlinear systems. After obtaining the solution for the particular value of a parameter, the solution for another value of parameter perturbed from the old one can be obtained by iterations using the previous solution as an approximation. The main aim of the path following and parametric continuation is to effectively trace the bifurcation sequence as a parameter of the system is varied. In this study, an arc-length procedure [7] is adopted for the parametric continuation.

V. STABILITY ANALYSIS OF PERIODIC SOLUTIONS

When the steady state solution is computed by using IHB method, usually the stability of the obtained solution is of great interest. The stability of the periodic solutions is investigated by the Floquet Theory. This is done by perturbing the state variables about the steady state solution, which results in a system of linearized equations with periodically varying coefficients. The stability of the periodic solutions of the original system of nonlinear equations is determined by the eigenvalues of the monodromy matrix which transforms the state vector of the linearized equations at one instant of time to another instant one period ahead. If the absolute values of all eigenvalues of the monodromy matrix are less than unity then the periodic solution is stable. If at least one of the eigenvalues has a magnitude greater than one, then the periodic solution is unstable. The way the eigenvalues leave the unit circle determines the nature of bifurcations. The monodromy matrix is obtained by a matrix exponentiation procedure outlined by Friedmann et al. [10].

VI. NUMERICAL STUDY

The simplified model of the cracked RC beam in Fig. 1 is considered and the equation of motion of the nonlinear system, defined by equation (19) is solved for different frequency ratio r with different parameters taken as μ =0.6299,1.2598 and 1.8896, X_{st}=0.0126 mm ξ=2%.The amplitude vs frequency plot of the system for the frequencies ratio r range of higher end of 1.2 rad/sec to lower end of 0.8 rad/sec. It is seen from the figures that IHBC traces all stable and unstable period one solutions (instability of the solutions is verified by Floquet's theory) and show a number of interesting phenomena. Tthe nondimensional ratio (X/ X_{st}) amplitude of responses goes up to 25.58 (Fig. 2) and then, decreases as the frequency ratio is varied from to 0.8 rad/sec. Within this frequency ranges, the solutions obtained are only period one stable solutions. The figure shows that the effect of nonlineary does not influence the deflection curve for a value of μ =0.6299 and the deflection curve resemble as if the system is predominantly linear. .Fig. 3 shows that nondimensional ratio (X/ X_{st}) amplitude of responses goes up to 26.20 (Fig. 3) and then, decreases as the frequency ratio is varied from to 0.8 rad/sec. Here in this frequency ranges, the solutions obtained are both period one stable and unstable solutions. It is evident that as the nonlinearity increases, the unstable solutions appear from the peak value of 26.20 and continues up to 17.45. Beyond this, branch of stable solution continues upto frequency ratio 0.8 rad /sec. For other values μ the deflection curve is more skewed to the right and sharp bent in the solution branch is observed in the Fig. 4 and Fig. 5

as the value of μ resulting stronger nonlinearity due to strong nonlinearly. The skewed deflection curves with bent in the solution branches are accompanied by fold bifurcation and jump phenomena. Also another characteristic of nonlinear system is that there exist multiple solutions in the same frequency ratio level. The unstable solution branch is sandwiched between two stable branches. This is important for the fact that even with low amplitude of excitation, at a frequency ratio a cracked RC beam may suddenly vibrate with higher amplitude ratio and sometimes may lead to collapse. For the same value of μ=1.8896, the sensitivity analysis is carried out (Fig. 6) for different damping ratios (ξ=1.5%, 2%, 3% and 4%). and it may be observed that as the damping ratio increases, sharpness in the bent of the solution branches decreases as damping suppresses amplitude ratio.

VII. CONCLUSION

Nonlinear response behaviour of a cracked RC beam is investigated under harmonic excitation. The nonlinearity of the system is characterized by a cubic polynomial and is due to opening and closing of the cracks in the RC beam. An incremental harmonic balance method with arc length continuation technique (IHBC) is employed to trace different types of solutions. The stability of the solutions is examined by Floquet theory. From the numerical solutions, following conclusions are drawn.

IHBC is capable of tracing all types of period one etc together with folds and bifurcations in amplitude Vs frequency plots.

The nonlinear cracked RC beam shows the co-existence of a variety of solutions for response like period 1 stable and unstable solutions at the same frequency.

As the nonlinearity increases, the deflection curves skewed more and more to the left with sharp bent of the solution branches.

The jump phenomena are also observed for strongly nonlinear cracked RC beam characterized by cubic polynomial.

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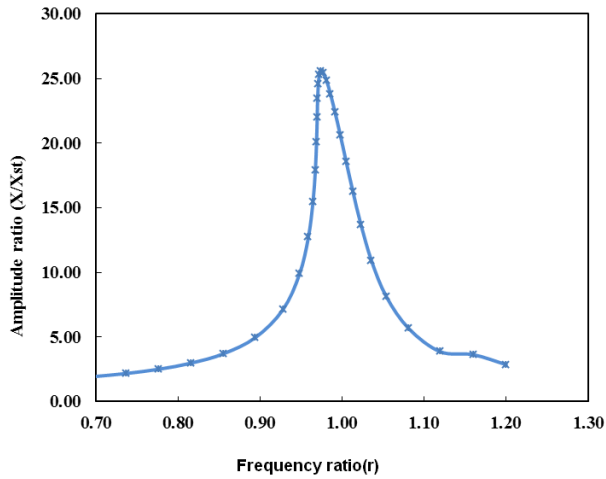
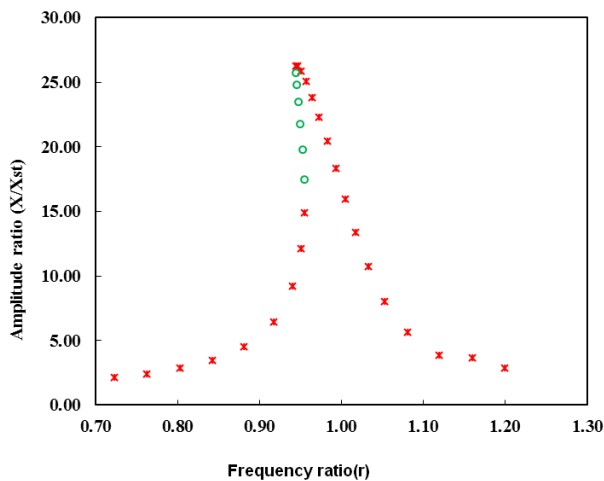Fig. 2 Variation of Amplitude Ratio with Frequency Ratio
(μ=0.6299)

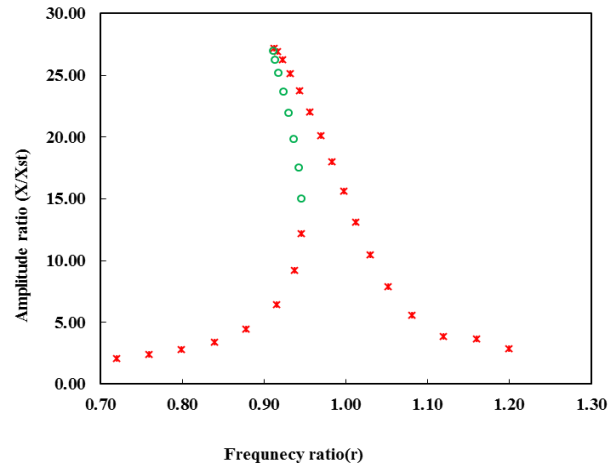Fig. 3 Variation of Amplitude Ratio with Frequency Ratio
(μ=1.2598)

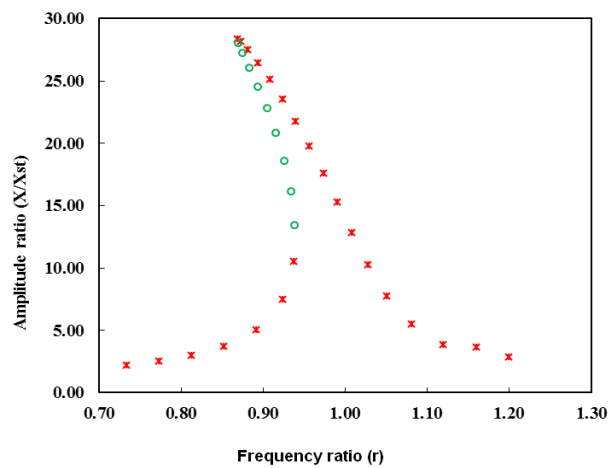Fig. 4 Variation of Amplitude Ratio with Frequency Ratio
(μ=1.8896)

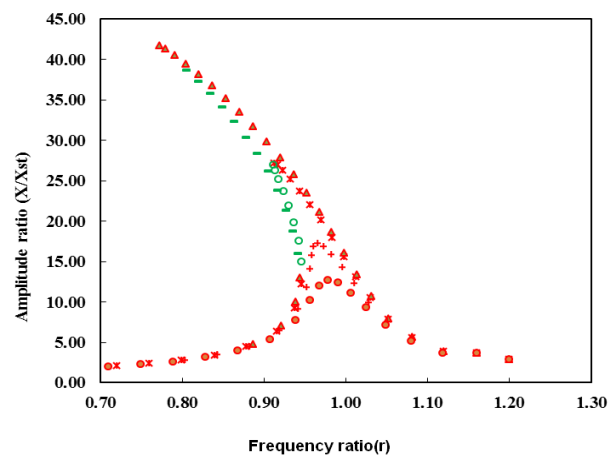Fig. 5 Variation of Amplitude Ratio with Frequency Ratio
(μ=2.5195)

Fig. 6 Sensitivity Analysis (Amplitute ratio v/s Frequency
ratio, μ=1.8896 and ξ = 1.5, 2%,3%,4%)