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Hydrodynamic Modeling of Infinite Reservoir using Finite Element Method

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Abstract**—**In this paper, the dam-reservoir interaction is analyzed using a finite element approach. The fluid is assumed to be incompressible, irrotational and inviscid. The assumed boundary conditions are that the interface of the dam and reservoir is vertical and the bottom of reservoir is rigid and horizontal. The governing equation for these boundary conditions is implemented in the developed finite element code considering the horizontal and vertical earthquake components. The weighted residual standard Galerkin finite element technique with 8-node elements is used to discretize the equation that produces a symmetric matrix equation for the dam-reservoir system. A new boundary condition is proposed for truncating surface of unbounded fluid domain to show the energy dissipation in the reservoir, through radiation in the infinite upstream direction. The Sommerfeld's and perfect damping boundary conditions are also implemented for a truncated boundary to compare with the proposed far end boundary. The results are compared with an analytical solution to demonstrate the accuracy of the proposed formulation and other truncated boundary conditions in modeling the hydrodynamic response of an infinite reservoir.

Keywords**—**Reservoir, finite element, truncated boundary, hydrodynamic pressure

I. Introduction

THE integrity of concrete dams is quite important as there are a large number of them around the world, especially in seismically active areas. With respect to environmental and economical considerations, their safe performance is of vital importance. In many cases, the failure of dams has led to disastrous consequences. An important factor in the design of dams in seismically active regions is the effect of hydrodynamic pressure exerted on the face of dam as a result of earthquake ground motions. The seismic response of a dam depends on different factors such as the effect of dam-reservoir interactions.The exerted hydrodynamic pressure has been recognized as a main loading in the design of dams. Westergaard reported the first analysis of hydrodynamic forces on dam faces during an earthquake [1]. Their results were checked by a simplified analysis. In the following years, many researchers have extensively studied hydrodynamic analysis of dams using various methods. It was found that for an accurate analysis of hydrodynamic pressure on dams with irregular geometries, the reservoir should be treated as an assemblage of finite elements. The finite element method has become more popular in reservoir simulation, partly because of its flexibility in dealing with boundaries. It is not a requirement that the element shape to be square, so the element mesh can handle very complex geometries.In the finite element analysis of dam-reservoir interactions, problems arise due to unbounded reservoir domain. Truncating the infinite reservoir domain at a specific distance from the dam-reservoir interface solves this problem. For accurate analysis, the behavior of outgoing pressure waves at the truncation surface must be precisely represented. The applied truncated boundary at reservoir farfield depends on the geometric configuration. For a finite reservoir, the reflected waves from the truncated farfield are not negligible and may result in a significant increase in induced hydrodynamic pressure in the reservoir. For the case of an infinite reservoir, the location of truncated boundary condition for the outgoing pressure waves in a numerical model with limited length is very important. The proper boundary condition at the truncated reservoir boundary has been the subject of many studies in dynamic analysis of structures.Zienkiewicz et al. studied the dynamic response of submerged structures, assuming incompressible water, using the finite element method [2]. Chopra used the finite element method as a numerical technique for dam-reservoir analysis [3], [4]. He studied the response of the hydrodynamic force on a dam impounding reservoir under horizontal excitation.Zienkiewicz et al. examined the formulation of infinite conditions in the solution of the pressure wave equation in reservoirs [5]. They concluded that Sommerfeld's boundary condition is appropriate for large reservoir models and can be easily incorporated in the finite element discretization of the reservoir domain. Hall and Chopra studied the hydrodynamic effects of the impounded reservoir on the seismic response of gravity dams using one-dimensional boundary conditions for the radiation of waves in a truncated boundary [6]. Sharan proposed a radiation boundary condition for the truncated boundary of the incompressible reservoir model [7], [8]. His proposed boundary condition was based on the analytical solution for the pressure wave equation in the reservoir under a horizontal earthquake component in the frequency domain.Concrete gravity dam-reservoir systems are three-dimensional but they are often idealized as two-dimensional sections in planes normal to the dam axis because of the slowly variation in geometry and material properties of the system as well as the seismic input along the dam axis.The objective of this paper is to present a two-dimensional formulation for dam-reservoir system analysis using the finite element model considering horizontal and vertical components of earthquakes. In the derivation of boundary conditions, it is assumed that the reservoir fluid domain is incompressible. The interface of the dam and reservoir is considered vertical and the bottom of the reservoir is assumed to be rigid and horizontal.

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II. Formulation of unbounded reservoir domain

For an incompressible and inviscid fluid, the hydrodynamic pressure P resulting from the ground motion of a rigid dam (Fig. 1) satisfies the Laplace equation in the following form:

$$\nabla^2 P = 0 \tag{1}$$

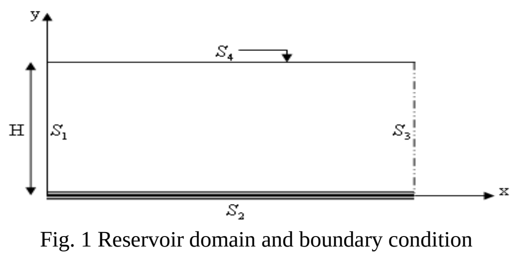

Fig. 1 Reservoir domain and boundary condition

The following boundary conditions are defined by assuming that the effects of surface waves and viscosity of the fluid are negligible.

$$\frac{\partial P}{\partial x} = -\rho a_x \quad \text{on} \quad S_1 \tag{2}$$

$$\frac{\partial P}{\partial y} = -\rho a_y \quad \text{on} \quad S_2 \tag{3}$$

$$P = 0 \quad \text{on} \quad S_3 \tag{4}$$

$$P = 0 \quad \text{on} \quad S_4 \tag{5}$$

In these equations, a_x and a_y are the earthquake acceleration components applied to the dam face and reservoir bottom in the horizontal and vertical directions, respectively.

The analytical solution of equation (1) for the above boundary condition is:

$$P = P_a + P_v = 2\rho a_x H \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\lambda_n^{\ 2}} \exp(-\lambda_n \frac{x}{H}) \cos(\lambda_n \frac{y}{H})$$
$$+ \frac{\rho a_y}{\lambda_n \cot \lambda_n H} (\cos \lambda_n y - \cot \lambda_n H \sin \lambda_n y)$$
(6)

where P_a and P_v are induced hydrodynamic pressure because of the horizontal and vertical components of the earthquake, $\lambda_n = \frac{2n-1}{2}\pi$ and $n = 1,2,3,\ldots$.

III. Proposed Far End Boundary Formulation

The The characteristic of the far boundary condition is one of the most important features in the development of reservoir models. This is because the hydrodynamic pressure on the dam face is very sensitive to the truncated boundary at the far end of the reservoir. Most of the proposed farfield boundary conditions are not exact and it is necessary to take large distances for the truncated surface of the dam. To consider the effect of radiation damping, it is assumed that the magnitude of the hydrodynamic pressure approaches zero at an infinite distance from the dam. If the unbounded reservoir domain is truncated a large distance from the dam, the Sommerfeld's radiation boundary condition is used at the truncated boundary. This usually leads to extra computational effort. The proposed truncated boundary condition is derived using the analytical solution represented in equation (6). The partial derivative of the hydrodynamic pressure with respect to x in equation (6) is given by:

$$\frac{\partial P}{\partial x} = -2\rho a_x \lambda_n \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\lambda_n^{\ 2}} \exp(-\lambda_n \frac{x}{H}) \cos(\lambda_n \frac{y}{H}) + \tag{7}$$
$$\frac{\rho a_y}{\lambda_n \cot \lambda_n H} (\cos \lambda_n y - \cot \lambda_n H \sin \lambda_n y)$$

This equation is used as the truncated boundary condition in developed finite element model.

IV. Finite Element Formulation

Assuming that the hydrodynamic pressure is unknown, the pressure at any point inside an element can be written as:

$$P^{(e)}(x,y) = [N(x,y)]\,\overrightarrow{P^{(e)}} = \sum_{i=1}^{m} N_i(x,y) P_i^{(e)} \tag{8}$$

where $\overrightarrow{P^{(e)}}$ is the vector of pressure at the element nodes and $[N(x,y)]$ is the matrix of interpolation functions [2], [5].

To solve the governing equation with the finite element method, the reservoir domain is divided into elements with *m* nodes. Using the standard Galerkin method, equation (1) can be written in the following form:

$$\iint_{S^{(e)}} N_i \left[\frac{\partial^2 P^{(e)}}{\partial x^2} + \frac{\partial^2 P^{(e)}}{\partial y^2} \right] dS = 0 \qquad i = 1,2,\ldots,m$$
(9)

whereas N_i is the interpolation function.

According to the previously mentioned boundary conditions and using Gauss-Green theorem, the last equation becomes:

$$-\iint_{S^{(e)}} \left[\frac{\partial N_i}{\partial x} \sum_{i=1}^{m} \frac{\partial}{\partial x} (N_i(x,y) P_i^{(e)}) + \frac{\partial N_i}{\partial y} \sum_{i=1}^{m} \frac{\partial}{\partial y} (N_i(x,y) P_i^{(e)}) \right] dS \tag{10}$$
$$+ \int_{C^{(e)}} N_i \rho a_n dC = 0$$

or

$$-\iint_{S^{(e)}} \left(\frac{\partial N_i}{\partial x} \left[\frac{\partial N_1}{\partial x} P_1^{(e)} + \frac{\partial N_2}{\partial x} P_2^{(e)} + \ldots + \frac{\partial N_m}{\partial x} P_m^{(e)} \right] dS \right)$$
$$-\iint_{S^{(e)}} \left(\frac{\partial N_i}{\partial y} \left[\frac{\partial N_1}{\partial y} P_1^{(e)} + \frac{\partial N_2}{\partial y} P_2^{(e)} + \ldots + \frac{\partial N_m}{\partial y} P_m^{(e)} \right] dS \right) \tag{11}$$
$$+ \int_{C^{(e)}} N_i \rho a_n dC = 0$$

The final form of the equation reads as following:

$$\iint_{S^{(e)}} \left[\frac{\partial N_i}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_1}{\partial y} \quad \frac{\partial N_i}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_2}{\partial y} \quad \ldots \quad \frac{\partial N_i}{\partial x}\frac{\partial N_m}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_m}{\partial y} \right]$$
$$\begin{Bmatrix} P_1^{(e)} \\ P_2^{(e)} \\ \vdots \\ P_m^{(e)} \end{Bmatrix} dS - \int_{C^{(e)}} N_i \rho a_n dC = 0$$
(12)

where $i = 1,2,\ldots,m$.

The matrix form of equation (12) is given by:

$$K^{(e)}\,\overrightarrow{P^{(e)}} = \overrightarrow{F^{(e)}} \tag{13}$$

where

$$K^{(e)} = \iint_{S^{(e)}} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_1}{\partial y} \\ \frac{\partial N_2}{\partial x} & \frac{\partial N_2}{\partial y} \\ \vdots & \vdots \\ \frac{\partial N_P}{\partial x} & \frac{\partial N_P}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_P}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_P}{\partial y} \end{bmatrix} dS^{(e)} \quad (14)$$

$$= \iint_{S^{(e)}} [B]^T [B] dS^{(e)}$$

and

$$\vec{F}^e = -\int_{C^{(e)}} \rho a_n \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_m \end{bmatrix} dC^{(e)} \quad (15)$$

In this equation, $C^{(e)}$ contains the boundary condition effects at the dam-reservoir and reservoir-foundation interfaces and S is the reservoir domain.

Using equation (13) for all elements in the entire domain regarding their location gives:

$$[K]\vec{P} = \vec{F} \quad (16)$$

The reservoir response is found solving equation (16) and satisfying the relevant boundary conditions.

V. Case Study

A computer program was developed to solve equation (16) with the aforementioned boundary conditions. Its accuracy is verified by the following example. The 106 meter tall reservoir of the Sefidrud dam in Iran is considered as a case study. Water is assumed to be incompressible with a density of 1000 kg/m^3. The maximum horizontal and vertical acceleration due to the Manjil earthquake (1990) is exerted on the dam. The magnitude of each is $0.356g$ and $0.236g$, respectively.The standard Galerkin method with 8-node elements was used to model the reservoir. Fig. 2 shows the developed finite element model for the reservoir domain.

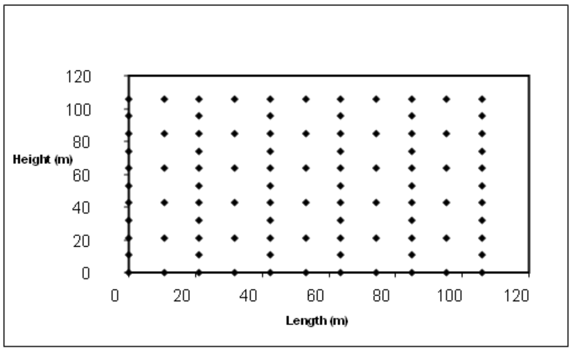

Fig. 2 Developed finite element model for reservoir domain using 8-node elements

Induced hydrodynamic pressure on the dam-reservoir interface is calculated by applying the proposed far end boundary condition using the finite element model. Diagrams 3 to 6 depict the results for different domain length to dam height ratios. The analytical solution for this problem with simple boundary conditions is shown in these figures. There is good agreement with the results of the finite element model for $L/H \geq 0.6$.

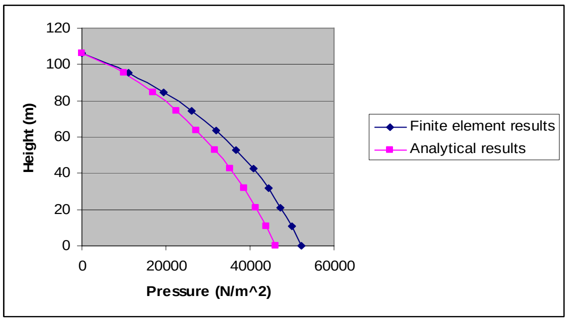

Fig. 3 Hydrodynamic pressure distribution curve by height for dams with $L/H = 0.4$

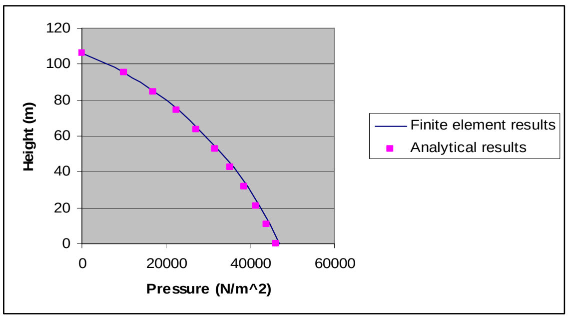

Fig. 4 Hydrodynamic pressure distribution curve by height for dams with $L/H = 0.6$

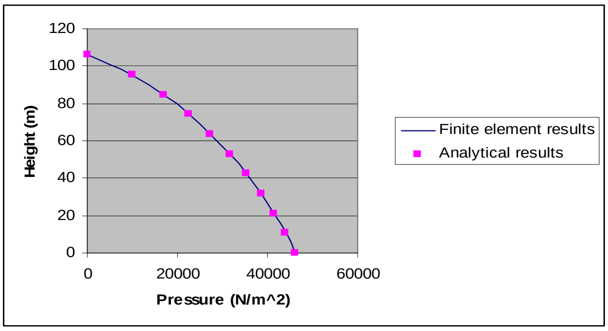

Fig. 5 Hydrodynamic pressure distribution curve by height for dams with $L/H = 0.8$

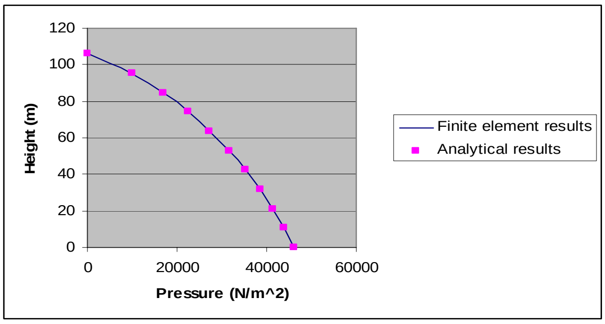

Fig. 6 Hydrodynamic pressure distribution curve by height for dams with $L/H = 1$

The results of the maximum hydrodynamic pressure obtained for different length to height ratios are compared with analytical solutions in table I.

TABLE I
MAXIMUM HYDRODYNAMIC PRESSURE ON THE DAM ($\frac{N}{m^2}$)

$\frac{L}{H}$	Proposed boundary condition at farfield	Analytical solution	Error (%)
0.4	52092.75	46163.13	12.84
0.6	46943.19	46163.13	1.69
0.8	46195.75	46163.13	0.07
1	46162.36	46163.13	0.001

For further assessment of the efficiency of the proposed farfield boundary condition, induced hydrodynamic pressure on the dam-reservoir interface is calculated by applying the perfect damping boundary condition at the truncated reservoir boundary. Diagrams 7 and 8 depict the results for different domain length to dam height ratios for this case. It can be concluded that the presented finite element model with 8-node elements only gives superior results if the perfect damping truncated boundary is located at place at least twice its height.

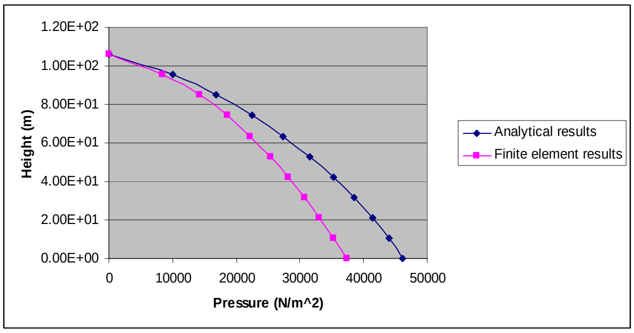

Fig. 7 Hydrodynamic pressure distribution curve by height for dams with $L/H = 1$

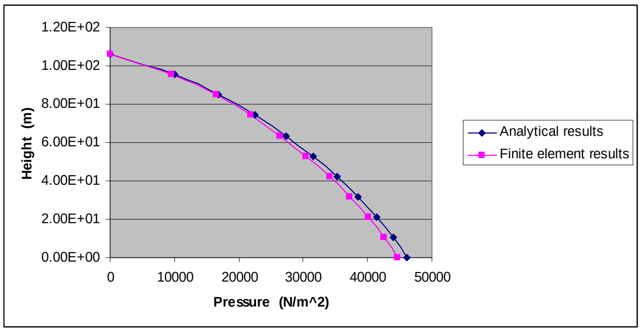

Fig. 8 Hydrodynamic pressure distribution curve by height for dams with $L/H = 2$

In many cases, Sommerfeld's boundary condition is used instead of the perfect damping boundary condition. This is described for incompressible fluid as follows:

$$\frac{\partial P}{\partial X} = 0 \tag{5.17}$$

The aforementioned example was analyzed with Sommerfeld's boundary condition by applying (5.17) to the developed finite element model. Diagrams 9 and 10 depict results for different length to height ratios of dams for this case and the results are compared with analytical solutions. In this case, the truncated boundary should also be located at a position at least twice the dam height.

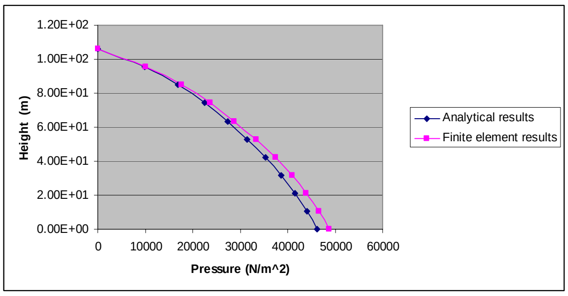

Fig. 9 Hydrodynamic pressure distribution curve by height for dams with $L/H = 1$

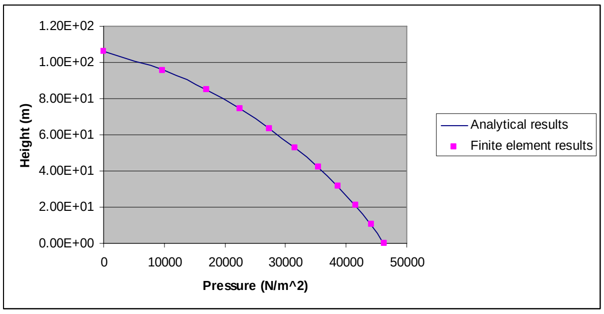

Fig. 10 Hydrodynamic pressure distribution curve by height for dams with $L/H = 2$

The results of the maximum hydrodynamic pressure obtained by applying different boundary conditions and different length to height ratios are presented in table II.

TABLE II
MAXIMUM OF HYDRODYNAMIC PRESSURE ON DAM ($\frac{N}{m^2}$)

Error (%) with $P=0$	Error (%) with $\partial P/\partial x = 0$	Analytical solution	$P=0$ at farfield	$\partial P/\partial x = 0$ at Arfield	$\frac{L}{H}$
19.12	5.620	46163.13	37335.09	48756.09	1
3.243	0.230	46163.13	44666.12	46269.67	2
0.638	0.007	46163.13	45868.34	46166.77	3
0.133	0.002	46163.13	46101.82	46162.35	4

VI. CONCLUSION

The results obtained, regardless of the boundary condition applied to the weighted residual standard Galerkin finite element technique with 8-node elements suggest the developed finite element model is efficient and accurate.

The results for different farfield boundary conditions and the analytical solutions show that the developed model using the proposed truncated boundary condition is also efficient and accurate.

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