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Simulating the Dynamics of Distribution of Hazardous Substances Emitted by Motor Engines in a Residential Quarter

S. Grishin

***Abstract*—**This article is dedicated to development of mathematical models for determining the dynamics of concentration of hazardous substances in urban turbulent atmosphere. Development of the mathematical models implied taking into account the time-space variability of the fields of meteorological items and such turbulent atmosphere data as vortex nature, nonlinear nature, dissipativity and diffusivity. Knowing the turbulent airflow velocity is not assumed when developing the model. However, a simplified model implies that the turbulent and molecular diffusion ratio is a piecewise constant function that changes depending on vertical distance from the earth surface. Thereby an important assumption of vertical stratification of urban air due to atmospheric accumulation of hazardous substances emitted by motor vehicles is introduced into the mathematical model. The suggested simplified non-linear mathematical model of determining the sought exhaust concentration at a priori unknown turbulent flow velocity through non-degenerate transformation is reduced to the model which is subsequently solved analytically.

***Keywords*—**Urban ecology, time-dependent mathematical model, exhaust concentration, turbulent and molecular diffusion, airflow velocity.

I. Introduction

ANTHROPOGENIC pollution of atmospheric and aquatic urban medium pertains to ecologic problems. Atmospheric air pollution is one of the most vexed ecological problems accompanied by a number of adverse effects. Among anthropogenic emissions into the atmosphere, the following substances are the most dangerous: carbon dioxide, sulfur and nitrogen oxides (nitrogen dioxide, sulfur dioxide), metal dust, carbon monoxide, ozone, aldehydes, lead, phenol etc. The motor transport is ranking first in the list of principal anthropogenic sources of pollution of urban atmospheric air in large cities of the world. A moving motor vehicle emits jet-shaped exhaust gases into the environment. At the expense of gas diffusion, the exhaust gases are mixed up and re-distributed in space. As to the urban atmosphere – it is a complex dynamic system featuring various inherent physic-chemical processes, the intensity of which depends on specific characteristics of the city concerned.

To describe these complex turbulent atmospheric processes, one needs to develop a complex symbolic model that would allow one to solve a broad spectrum of problems in the field of environmental protection. Such a model should include:

1) The model of hydro-thermodynamics of urban and/or regional turbulent atmospheric processes;

2) Models taking into account kinetic processes, condensations, coagulations as well as mass exchange processes on the gas-particle phase;

3) Transfer and diffusion equations for gas-borne particles (and aerosols) in the atmosphere of a city and/or region, - taking into account photochemical and other transformation.

The investigation of ecological problems of such kind does not differ fundamentally from the solution of other tasks pertaining to natural sciences. A specific feature of ecological problems is, first, an extreme uncertainty of problem setting and – secondly – the complex character of the task implying the necessity of taking into account heterogeneous factors as it was shown above. According to engineering practice, the error in estimation and calculation of urban air contamination levels may come up to hundreds per cent.

II. The Problems Of The Existing Symbolic Models

The majority of the modern systems of dynamics and kinetics of atmospheric disperse systems (see [1-9] and the corresponding literature therein) is described by multivariate nonlinear partial differential equations, - and the solution of problems of such class may be obtained only roughly - through approximation of initial differential problem by finite-dimensional model.

In general, the research methodology for urban ecological problems looks as follows:

- Formalization of concepts “environment”, “environmental quality” and other concepts of the specific subject;
- Formulation of environmental criteria;
- Detection, more precise definition, and investigation of the main factors rendering impact on the life of urban population from the ecological standpoint;
- Development of methods of monitoring and integral assessment of environmental quality;
- Development of a well-founded scientific concept of urban environmental quality control and a possibility of a short-term or a long-term prediction.

Currently, the most widely-used models are the ones based on the mathematical statistics apparatus. The usage of

S.Grishin Transport and Telecommunication Institute, Lomonosova 1, Riga, LV-1019, Latvia (phone: +371 29854464; e-mail: ctac@tsi.lv).

the mathematical statistics apparatus as well as other mathematical apparatuses implying the model development based on results of single shots and integral measurements of environmental quality, - in particular, urban atmospheric air, - and determination of the dynamics of concentration of exhaust gases in urban atmospheric air is connected with the problem of measurement accuracy. Namely, single shots of pollutant concentration in urban atmospheric air have a low-accuracy level. If one improves accuracy by increasing measurement rate – the condition of urban atmospheric air can not be inferred after single shots – especially, under the dynamic circumstances of urban environment. Consequently, to estimate the quality of urban atmospheric air, one can not be content with the accuracy of integral criteria, indices, and indicators. Moreover, it seems impossible to place measurement instrumentation at each point of the city; therefore, one has to restore the general picture of atmospheric air contamination, taking as a basis the measurements made at single points, with the help of interpolation models. So far, such interpolation models have not been developed to the extent of a wide practical usage. Therefore, to apply the mathematical statistics apparatus for atmospheric air monitoring, one selects the method of construction of fixed automated control points exercising control over the quality of atmospheric air, -supplementing those by mobile inspection facilities.

III. Formalization Of The Problem Of Determination Of Exhaust Gases Dynamics In The Turbulent Atmosphere Of Urban Air

Let the set vector $X(t)=\{x_1(t),x_2(t),...,x_n(t)\}$ characterize the air medium condition at a given point of three-dimensional space t, where $x_j(t)$; $j=\overline{1,N}$ determines the concentration of *j*-th substance. We assume that, at a certain moment of time t_e the air medium condition can be derived from the functional equation

$X^*(t_e)=F\left[X(t_i),\ i=\overline{1,k};\ Y^*(t_1,...,t_k;t_e);\ Z^*(t_1,...,t_k;t_e)\right]$

, where F – is a composed function. $X(t_1), X(t_2),..., X(t_k)$ – characterizing vectors the condition of air medium at the investigated point at the time points $t_1,t_2,...,t_k$, preceding t_e, т.е. $t_e>t_i$, $i=1,2,...,k$. $Y^*(t_1,...,t_k;t_e)$ – vector of controlled parameters describing the characteristics of external medium affecting airborne pollutant concentration – primarily, the weather conditions (air temperature, wind direction and speed) etc. $Z^*(t_1,...,t_k;t_e)$ – vector of external medium parameters controlled fully or partially, the regulation of which also influences the airborne concentration of substances - like for instance, traffic intensity, vehicle types etc.Generally, the problem of operating management of transportation flows and planning of urban housing and laying new motorways – is set as follows: given the sets as follows: $Y^*(t_1,...,t_k;t_e)$ и $Z^*(t_1,...,t_k;t_e)$ it is required to estimate the airborne harmful substances concentration $X(t_e)$ in the investigated filed of the urban airspace, - and, given the concentration of *i*-th substance exceeds the admissible level $x_i^{\max}$ - one has to change the controlled parameters $Z^*(t_1,...,t_k;t_e)$ so that elements of the set $X^*(t_e)$ would be admissible, - i.e., the concentration levels $x_j(t_e)$ would be within tolerable limits: $x_j(t_e)\le x_j^{\max}$, $j=1,2,...,n$.

One should note that calculation according to the above-stated functional relationship should be performed with respect to different alternatives of the external medium condition $Y^*(t_1,...,t_k;t_e)$; besides, the corresponding managerial solutions will be defined for each alternative. The totality of these solutions allows one to formulate the urban traffic system control strategy and to bring forward proposals concerning its development.

IV. Statement Of Non-Stationary Symbolic Model Of Determination Of Exhaust Gases Concentration In Urban Atmosphere

One has to determine concentrations $C^{\{n\}}(x_1,x_2,x_3,t)$ n-го $\left(n=\overline{1,N}\right)$ of a harmful substance at any spatial point (x_1,x_2,x_3) of the field $[0,l_1]\times[0,l_2]\times[0,l_3]$ at any instant of time $t\in[0,T]$ from the following equation:

$$\frac{\partial C^{\{n\}}(x,t)}{\partial t}=$$
$$=div\left(D\left(\vec{\vartheta}(x,t)\right)\cdot\overrightarrow{grad}\,C^{\{n\}}(x,t)\right)-\vec{\vartheta}(x,t)\cdot\overrightarrow{grad}\,C^{\{n\}}(x,t),$$
$$t\ge 0, x=(x_1,x_2,x_3):\ 0<x_i<l_i\ \left(i=\overline{1,3}\right); \tag{1}$$

from the initial condition

$$C^{\{n\}}(x,t)\Big|_{t=0}=C_0^{\{n\}}(x),\ x=(x_1,x_2,x_3):\ 0\le x_i\le l_i\ \left(i=\overline{1,3}\right); \tag{2}$$

from boundary conditions (at each fixed $j=\overline{0,M-1}$)

$$\gamma_{i,1,j}^{\{n\}}\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_i}\right|_{x_i=a_{i,j}}-\gamma_{i,2,j}^{\{n\}}\cdot C^{\{n\}}(x,t)\Big|_{x_i=a_{i,j}}= \tag{3}$$
$$=C_{i,j}^{\{n\}}\left(x/\{x_i\},t\right),\ a_{i,j}\le x_i\le b_{i,j}\ \left(i=\overline{1,3}\right),\ t\ge 0,$$
$$\gamma_{i,3,j}^{\{n\}}\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_i}\right|_{x_i=b_{i,j}}+\gamma_{i,4,j}^{\{n\}}\cdot C^{\{n\}}(x,t)\Big|_{x_i=b_{i,j}}= \tag{4}$$
$$=C_{i+3,j}^{\{n\}}\left(x/\{x_i\},t\right),\ a_{i,j}\le x_i\le b_{i,j}\ \left(i=\overline{1,3}\right),\ t\ge 0,$$

from conjugating conditions

$$C^{\{n\}}(x,t)\Big|_{x_3=l_3^{\{j\}}-0}=C^{\{n\}}(x,t)\Big|_{x_3=l_3^{\{j\}}+0}, \tag{5}$$
$$j=\overline{1,M-1},\ 0\le x_i\le l_i\ \ (i=1,2),$$
$$D\left(\vec{\vartheta}(x,t)\right)\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_3}\right|_{x_3=l_3^{\{j\}}-0}=D\left(\vec{\vartheta}(x,t)\right)\cdot\left.\frac{\partial C^{\{n\}}(x,t)}{\partial x_3}\right|_{x_3=l_3^{\{j\}}+0}, \tag{6}$$
$$j=\overline{1,M-1},\ \ 0\le x_i\le l_i\ \ (i=1,2).$$

In the non-stationary initial boundary task (1)-(6), the expression $D\left(\vec{\vartheta}(x,t)\right)$ is used to denote the turbulent and molecular diffusion ratio, which is supposed to be a piecewise constant function of the kind

$$0 < d_{\min} \le D\left(\vec{\vartheta}(x,t)\right) \overset{def}{\equiv} \begin{cases} D_1 = const & if \quad 0 = l_3^{\{0\}} \le x_3 \le l_3^{\{1\}}, \\ D_2 = const & if \quad l_3^{\{1\}} \le x_3 \le l_3^{\{2\}}, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ D_M = const & if \quad l_3^{\{M-1\}} \le l_3 = l_3^{\{M\}} \end{cases}$$

for $\forall x_i \in [0, l_i]$, $(i = 1,2)$, where M is a number of stratified media along the vertical axis OX_3 parallel to the plane X_1OX_2; $a_{i,j}$ and $b_{i,j}$ have been used to denote frontier points for record simplification, namely:

$$a_{i,j} \overset{def}{\equiv} \begin{cases} 0 & if \quad i = 1,2, \\ l_3^{\{j\}} & if \quad i = 3; \ j \ne 0, \\ l_3^{\{0\}} & if \quad i = 3; \ j = 0, \end{cases}$$

$$b_{i,j} \overset{def}{\equiv} \begin{cases} l_i & if \quad i = 1,2; \ \forall j, \\ l_3^{\{j+1\}} & if \quad i = 3; \quad \forall j; \end{cases} \tag{7}$$

while the expression $\vec{\vartheta}(x,t) \equiv \vartheta(x_3, t)$ has been used to denote an unknown vector function for the mean velocity of the turbulent atmospheric air which is the solution of the nonlinear problem as follows:

$$\frac{\partial \vartheta(x_3,t)}{\partial t} = \frac{\partial}{\partial x_3}\left(D\left(\vartheta(x_3,t)\right) \cdot \frac{\partial \vartheta(x_3,t)}{\partial x_3} \right) - \vartheta(x_3,t) \cdot \frac{\partial \vartheta(x_3,t)}{\partial x_3} + P(x_3,t;g),$$
$$0 < x_3 < l_3, \ t > 0; \tag{8}$$

$$\vartheta(x_3,t)\big|_{t=0} = \vartheta_{initial}(x_3), \ 0 \le x_3 \le l_3; \tag{9}$$

$$\vartheta(x_3,t)\big|_{x_3 = l_3^{\{i\}}} = \vartheta_i(t), \ t \ge 0, \ i = \overline{0,M}; \tag{10}$$

$$\vartheta_{initial}\left(l_3^{\{i\}}\right) = \vartheta_i(0), \ i = \overline{0,M}; \tag{11}$$

$$\vartheta(x_3,t)\big|_{x_3 = l_3^{\{i\}} - 0} = \vartheta(x_3,t)\big|_{x_3 = l_3^{\{i\}} + 0}, \ i = \overline{1,M-1}, \ t \ge 0; \tag{12}$$

$$D\left(\vartheta(x_3,t)\right) \cdot \left.\frac{\partial \vartheta(x_3,t)}{\partial x_3}\right|_{x_3 = l_3^{\{i\}} - 0} = D\left(\vartheta(x_3,t)\right) \cdot \left.\frac{\partial \vartheta(x_3,t)}{\partial x_3}\right|_{x_3 = l_3^{\{i\}} + 0}, \tag{13}$$
$$i = \overline{1,M-1}, \ t \ge 0.$$

In the problem (1)-(13), the known initial data is the constants $g \approx 9.8 \ m/c^2$ (free fall acceleration), $N \in \mathbb{N}$, $M \in \mathbb{N}$, $T \in \mathbb{R}^1_+$, $D_j \in \mathbb{R}^1_+$ $\left(j = \overline{1,M}\right)$, $l_i^{\{j\}} \in \mathbb{R}^1_+$ $\left(i = \overline{1,3}; \ j = \overline{0,M}\right)$, $\gamma_{i,k,j}^{\{n\}} \in \mathbb{R}^1_+$ $\left(i = 1,2; \ k = \overline{1,4}; \ j = \overline{0,M-1}\right)$, and also, the following takes place for $\forall n = \overline{1,N}$:

$$\gamma_{i,k,j}^{\{n\}} = \begin{cases} 1 & if \ j = 0; \ i = 1; \ k = 1,3, \\ 0 & if \ j = 0; \ i = 1; \ k = 2,4, \\ \gamma_{i,k,j}^{\{n\}} > 0 & if \quad j \ne 0; \ \forall i,k. \end{cases} \tag{14}$$

In addition, the task (8) - (13) assumed to be known functions $\vartheta_{initial}(x_3)$, $\vartheta_i(t)$ $\left(i = \overline{1,M}\right)$, $P(x_3,t;g)$ (the power of external sources), $C_0^{\{n\}}(x)$ $\left(\forall n = \overline{1,N}\right)$, $C_{i,j}^{\{n\}}(\cdot,t)$ $\left(i = \overline{1,6}; \ j = \overline{0,M-1}\right)$, and also, the following takes place for $\forall n = \overline{1,N}$

$$C_{i,j}^{\{n\}}(\cdot,t) = \begin{cases} C_{i,j}^{\{n\}}(\cdot,t) \ if \ \{j = 0\} \wedge \{i = 3,6\}; \\ 0 \ otherwise \ . \end{cases} \tag{15}$$

Finally, it is assumed in the problem (1)-(7), (14)-(15) with respect to concentration that the initial function $C_0^{\{n\}}(x)$ and the boundary function $C_{i,j}^{\{n\}}(\cdot,t)$ fulfill the respective consistency conditions (for instance, [70]) for each $n = \overline{1,N}$ of the exhaust gas.

Thus, in the complete model (1)-(15) the sought functions is the concentration $C^{\{n\}}(x_1,x_2,x_3,t)$ n-го $\left(n = \overline{1,N}\right)$ of the harmful substance and medium scalar velocity $\vartheta(x_3,t)$ of the turbulent flow of atmosphere.

Remark 1. In the formulated model (1)-(15), the axis OX_1 is directed towards the section concerned of the city street with motor traffic, i.e., it is assumed that the length of the section concerned of the city street is located along that axis; the axis OX_2 is directed across the width of that street, while the axis OX_3 is directed along the height of the urban area investigated. The spatial area of the symbolic model (1) – (15) is stratified along the axis OX_3. The model is a parallelepiped where, first, the length/width/height are characterized by axes of reference $OX_1 / OX_2 / OX_3$ respectively, and, secondly, all the layers are parallel to the plane X_1OX_2 (Fig.1).

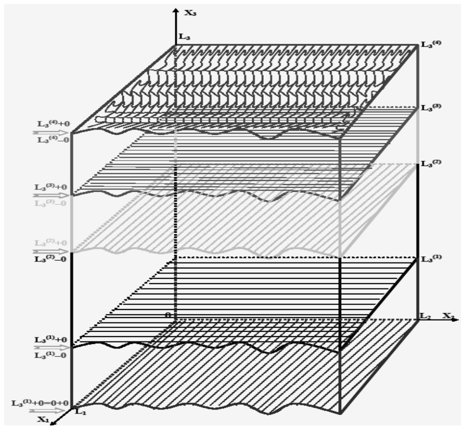

Fig.1. Scheme of stratified area of the section simulated

The stratified field which is taken exactly as a simulated area of the city street section concerned – is schematically depicted on Fig.1 and Fig. 2.

On the Fig 1the first stratum (below) has the altitude 0.2 m from the ground level, in the stratum emission of exhaust motor gases takes place. The second stratum is of 2-metre level, in the stratum, people are moving. Besides, the upper boundary of the stratum most remotely located from the ground surface (i.e., the upper boundary of the last – the fourth stratum) is equal to 24 meters which corresponds to the averaged height of buildings in Riga.

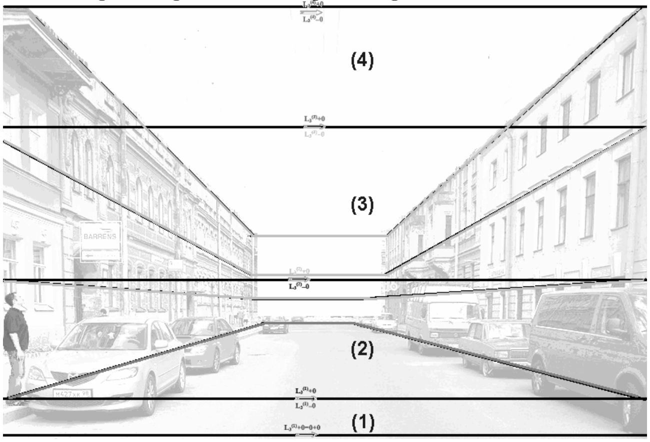

Fig.2 Stratified area of the simulated section of city road with motor traffic

Remark 2. In the formulated model (1)-(15) the boundary conditions (3)-(4) are mixed-type conditions, namely:

- For the first (low-earth) stratum: $0+0=l_3^{\{0\}}+0\le x_3\le l_3^{\{1\}}-0$

– on the parallelepiped wall $\left\{x_1=0+0,\ 0\le x_2\le l_2,\ 0\le x_3\le l_3^{\{1\}}\right\}$, perpendicular to the axis OX_1 at the point $x_1=0+0$ (that is, on the wall simulating the beginning of the section concerned), and

– on the wall $\left\{x_1=l_1-0,\ 0\le x_2\le l_2,\ 0\le x_3\le l_3^{\{1\}}\right\}$ perpendicular to the axis OX_1 at the point $x_1=l_1-0$ (i.e. on the wall simulating the end of the section concerned), - the boundary conditions are set as the third-kind conditions (that is to say, the Newton's conditions);

- On two other walls perpendicular the axis OX_2 at the points $x_2=0+0$ and $x_2=l_2-0$ (i.e., on the walls simulating buildings at the two waysides of the road concerned) – boundary conditions are set as the second-kind conditions (i.e., Neumann conditions);
- For all of the other strata $l_3^{\{1\}}+0\le x_3\le l_3-0$ on all the walls of the parallelepiped, the boundary conditions are set as the first-kind conditions (i.e., Dirichlet conditions).

Depending on what kind of instrumentation and technical facilities are owned by experimentalists (i.e., the experts measuring concentration of harmful substances) to fulfill the initial condition and the boundary conditions in the model suggested, - some other boundary conditions which may be more advantageous from the experimentalist's point of view can be examined instead of the boundary conditions (3) – (4). For example, if, instead of the formulas (7), (14) – (15), we write the following, respectively:

$$a_{i,j}=\begin{cases}0 & if\quad i=1,2;\\ l_3^{(j)} & if\quad i=3;\ j\ne 0;\\ l_3^{(0)}=0 & if\quad i=3;\ j=0,\end{cases}$$

$$b_{i,j}=\begin{cases}l_i & if\quad i=1,2;\\ l_3^{(j+1)} & if\quad i=3,\end{cases}\tag{16}$$

$$\gamma_{i,k,j}^{(n)}=\begin{cases}\gamma_{i,k,j}^{(n)}>0 & if\quad j\ne 0,\ \forall i,k;\\ 0 & if\quad j=0,\ k=1,3,\ \forall i;\\ -1 & if\quad j=0,\ k=2,\ \forall i;\\ 1 & if\quad j=0,\ k=4,\ \forall i,\end{cases}\tag{17}$$

$$C_{i,j}^{\{n\}}(\square,t)=\begin{cases}C_{i,j}^{\{n\}}(\square,t)\ne 0 & if\quad j=0;\\ 0 & if\quad j\ne 0,\ \forall i,\end{cases}\tag{18}$$

- for the first layer (the low-earth level) $0+0=l_3^{\{0\}}+0\le x_3\le l_3^{\{1\}}-0$ on all of the parallelepiped walls, the boundary conditions are set as the first-kind conditions (i.e. Dirichlet conditions);
- for all the rest of the layers $l_3^{\{1\}}+0\le x_3\le l_3-0$ on all of the parallelepiped walls the boundary conditions are set as third-kind conditions (i.e. Newton's conditions).

If, instead of the formulas (7), (14)-(15) we write, respectively,

$$a_{i,j}=\begin{cases}0 & if\quad i=1,2;\\ l_3^{(j)} & if\quad i=3;\ j\ne 0;\\ l_3^{(0)}=0 & if\quad i=3;\ j=0,\end{cases}$$

$$b_{i,j}=\begin{cases}l_i & if\quad i=1,2;\\ l_3^{(j+1)} & if\quad i=3,\end{cases}\tag{19}$$

$$\gamma_{i,k,j}^{(n)}=\begin{cases}\gamma_{i,k,j}^{(n)}>0 & if\quad i\ne 1,\ \forall j,k;\\ 1 & if\quad i=1,\ k=1,3,\ \forall j;\\ 0 & if\quad i=1,\ k=2,4,\ \forall j,\end{cases}\tag{20}$$

$$C_{i,j}^{\{n\}}(\square,t)=\begin{cases}C_{i,j}^{\{n\}}(\square,t)\ne 0 & if\quad \{j=0\}\wedge\{i=2,3,5,6\};\\ 0 & otherwise,\end{cases}\tag{21}$$

Thereby we define the boundary conditions (3)-(4) of another character, namely –

- for all the strata $0=l_3^{\{0\}}+0\le x_3\le l_3$ on the walls, perpendicular to the axis OX_1 at points $x_1=0+0$ (i.e. at the walls simulating the beginning of the section concerned) и $x_1=l_1-0$ (i.e. on the walls simulating the end of the section concerned), the boundary conditions are set as second-kind conditions (i.e. by Neumann condition);

For all the strata $0=l_3^{\{0\}}+0\le x_3\le l_3$ on all the rest of the walls perpendicular to the axis OX_2 at the points $x_2=0+0$ и $x_2=l_2-0$ (i.e. on the walls simulating buildings at the sideways of the road concerned), - the boundary conditions are set as the third-kind conditions (i.e. the Newton conditions);

If, instead of the formulas (7), (14)-(15) we write accordingly:

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$$a_{i,j}=\begin{cases}0 & if \quad j=0,\ i=\overline{1,3};\\ 0 & if \quad j\neq 0,\ i=1,2;\\ l_3^{(j)} & if \quad j\neq 0,\ i=3,\end{cases}$$

$$b_{i,j}=\begin{cases}l_i & if \quad i=1,2,\ \forall j;\\ l_3^{(j+1)} & if \quad i=3,\ \forall j,\end{cases} \tag{22}$$

$$\gamma_{i,k,j}^{(n)}\in\mathbb{R}_+^1 \ \left(i=1,2;\ k=\overline{1,4};\ j=\overline{0,M-1}\right)$$

predetermined numbers; (23)

$$C_0^{\{n\}}(x),\ C_{i,j}^{\{n\}}(\cdot,t)\ \left(i=1,6;\ j=\overline{0,M-1}\right)$$

predetermined functions. (24)

Thereby, we define the boundary conditions (3) – (4) of another character, - namely, with respect to all the strata on all the parallelepiped walls, - the boundary conditions are set only as third-kind conditions (i.e. by Newton conditions). It is obvious that, in this case, the boundary conditions are not mixed-type conditions, and some more complex and more accurate measurements are required to provide for their feasibility.

Some other types of boundary conditions are possible. In this work, the boundary conditions (3) – (4) are set by the formulas (7), (14) – (15); further, however, the obtained results may be quite easily distributed for symbolic models (1) – (6), (8) – (13) as well, - where the boundary conditions with respect to the sought concentration $C^{\{n\}}(x_1,x_2,x_3,t)$ are determined either by formulas (16)-(18) or formulas (19)-(21), or by formulas (22)-(24).

In conclusion of this remark, it is not out of place to remind that in the case where the boundary condition relative to concentration is set as Dirichlet condition, - it is required to measure the concentration proper at the corresponding boundary (on the surface, on the wall); when the boundary condition is set as Neumann condition – one has to measure not the concentration itself but rather “the flow” of concentration at the corresponding boundary of the area; if, however, the boundary condition is set as Neumann condition, it is required to measure concentration taking into account the molecular diffusion transfer to the environment, which is a more complicated measuring process requiring the appropriate engineering means and possibilities.

On finishing nontrivial calculations, the final formula was obtained for a 3D-symbolic model (1) – (15) of unambiguous definition of concentration $C^{\{n\}}(x_1,x_2,x_3,t)$ n-го $\left(n=\overline{1,N}\right)$ of harmful substance in the urban atmospheric air.

V. Example Of Results

Let’s consider the example of calculation of concentration change of harmful substance on a different height above the road area at the initiation fixed time moments 1, 2, 6 and 12 hours, respectively. We consider the road section with the width of 21 m and the length of 165 m (Fig.3). It is supposed that there are multistory buildings in both sides of considered road, at that the average height of buildings is 20 m. Besides, we assume that the number of cars driving through the road section per 12 hours is known and equals to 11000 units, i.e. the traffic flow rate in the considered road is accepted as equally distributed, namely, it equals to 917 carsi/hour approximately. Other initial data are following: depending on the altitude wind velocity on layers changes from 4 m/s till 1 m/s; the coefficient of turbulent diffusion depends on altitude and it changes from 0.13 (highest) to 0.16 (lowest); average concentration of the investigated harmful substance (as investigated material is taken CO_2 particularly) is assumed as 179 g/km; exhaust speed near cars is 60-100 m/s.

Fig. 3 the city block of K. Valdemar Street of Riga (Latvia)

Computations will be performed for an "imaginary vertical column", the foundation of which is exactly in the middle of the considered road and it is determined for the point $\left(x_1=10.5\text{ m};\ x_2=82.5\text{ m}\right)$. Numerical implementation of considered mathematical model has been realized by the packaged MathCAD. The results of calculations for the different moments of time, passing after the beginning of turbulent diffusion process, are presented in Figure 4.

Figure 4*a* shows a change of concentration $C\left(x_1=10.5, x_2=82.5, x_2, t=1\right)$ depending on the variable x_3, i.e. the constructed curve reflects a change of harmful substance concentration depending on a height x_3 in 1 hour after the beginning of process of supervision of harmful substance turbulent diffusion in the fixed point of the road area $\left(x_1=10.5\text{ m};\ x_2=82.5\text{ m}\right)$.

Changes of concentration depending on a height in the same point at the moments 2, 6 and 12 hours after the beginning of turbulent diffusion process are presented in Figure 4*b*, 4*c* and 4*d*, respectively. Note that in Figure 4*a* scale of ordinates is compressed 10^2 times less, and in the other figures scales of ordinates are taken 10 times less, i.e. there is the graph of function $C(x_3)=10^{-2}\cdot C(x_1,x_2,x_3,t)\big|_{x_1=10.5;\,x_2=82.5;\,t=1}$ in Figure 1*a*, and there are the graphs of functions $10^{-1}\cdot C(x_1,x_2,x_3,t)\big|_{x_1=10.5;\,x_2=82.5;\,t=2}$, $10^{-1}\cdot C(x_1,x_2,x_3,t)\big|_{x_1=10.5;\,x_2=82.5;\,t=6}$ and

$10^{-1} \cdot C\left(x_1, x_2, x_3, t\right)\Big|_{x_1=10.5;\, x_2=82.5;\, t=12}$ in Figures 4*b*, 4*c* and 4*d*, respectively[10].

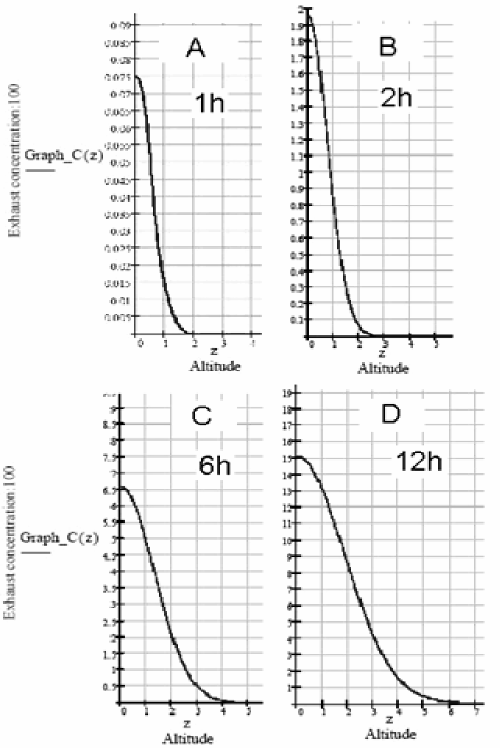

Fig.4 Change of the concentration of harmful matter on a different height above a road area

Let's note that this example considered by authors has illustrative character, because the solving of the offered mathematical model with respect to the considered example has been executed under some simplifying assumptions. For wide application of the offered mathematical model in practical questions it is necessary to develop more complex program using the high-level language.

VI. Conclusion

In this work, a scientific approach to monitoring the urban air pollution brought about by transportation vehicles.

A simplified 3D-symbolic model for determination of the dynamics of harmful substances concentration in stratified turbulent atmosphere of a city is proposed, with the basic requirement that the turbulent atmospheric air flow velocity is not preset a priori.

The results of the investigations performed in this work may be used when solving urgent issues of organizing road traffic within residential blocks in cities.

The models proposed and investigated in this work can be used in process of prospective planning of the city development when designing new blocks as well as constructing new motorways.

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