An Efficient Energy Adaptive Hybrid Error Correction Technique for Underwater Wireless Sensor Networks

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***Abstract*—**Variable channel conditions in underwater networks, and variable distances between sensors due to water current, leads to variable bit error rate (BER). This variability in BER has great effects on energy efficiency of error correction techniques used. In this paper an efficient energy adaptive hybrid error correction technique (AHECT) is proposed. AHECT adaptively changes error technique from pure retransmission (ARQ) in a low BER case to a hybrid technique with variable encoding rates (ARQ & FEC) in a high BER cases. An adaptation algorithm depends on a pre-calculated packet acceptance rate (PAR) look-up table, current BER, packet size and error correction technique used is proposed. Based on this adaptation algorithm a periodically 3-bit feedback is added to the acknowledgment packet to state which error correction technique is suitable for the current channel conditions and distance. Comparative studies were done between this technique and other techniques, and the results show that AHECT is more energy efficient and has high probability of success than all those techniques.

***Keywords*—**Underwater communication, wireless sensor networks, error correction technique, energy efficiency

I. INTRODUCTION

UNDERWATER wireless sensor networks find many applications in oceanographic data collection, environmental monitoring, disaster prevention and oil exploration [1]-[2]-[3]. For such applications a reliable and efficient communication data transport is demanded [4]. ARQ and FEC are the two main error correction techniques that guarantee reliability in underwater environment [5]. Energy is the most important efficiency issue for underwater wireless sensors due to the difficulty in recharging or replacing batteries in most aquatic medium [4]. In this paper an efficient energy adaptive hybrid error correction technique (AHECT) is proposed. This technique depends on a proposed adaptation algorithm which is calculated based on a pre-calculated packet acceptance rate (PAR) look-up table, current bit error rate (BER), packet size, and last error correction technique used. Based on this adaptation algorithm output a periodically 3-bit feedback is added to the acknowledgement packet to state which error correction technique is suitable for the current channel conditions and distance. The error correction is chosen from a pure ARQ in a good channel conditions and short distances to an ARQ with variable encoding rates in bad channel conditions and long distances.In [6], we have performed an energy efficiency analysis to ARQ and FEC in underwater environment. Energy efficiency of both techniques depends on channel conditions, transmission power, distance, and packet size. ARQ is found to be more energy efficient in some cases, while FEC is more efficient in others. In [7] a propagation model to calculate the signal to noise ratio for underwater acoustic channel was designed and implemented. In [8] modulation and encoding techniques for underwater communication system were studied. It was found that 8-PSK is the best modulation for underwater systems, whereas convolution coding is found to achieve better coding gain so it is the modulation and encoding techniques used in this work. In [9] an optimization metric for energy efficiency was proposed, and it was used in [6, 10] for energy efficiency calculations. In [10] Tian et el. have proven that energy efficiency of ARQ techniques is independent of retransmission attempts,; they compared ARQ and FEC techniques for terrestrials wireless sensor networks in terms of energy efficiency.In [11] ARQ is proposed for multi-hop underwater communication channel, the acknowledged can be achieved by explicitly transmitting the ack., or implicitly by hearing that packet transmitted forward to the next hop.In [12] an efficient ARQ is proposed by utilizing the sharing properties of underwater channel (scheduling packets transmission to achieve collision free transmission). In [13] juggling concepts enable a continuous ARQ irrespective of half-duplex properties of acoustic. This leads to high throughput, but not affecting energy efficiency. In [14] an opportunistic multi-hop ARQ is implemented in real system. This provides improvements in terms of data delivery ratio, but end to end delay increases due to queuing and retransmission. In [15], network coding technique is proposed to make use of the broadcast nature of acoustic channel. It is good in error recovery, but at the cost of energy efficiency. In [16], ARRTP (Adaptive Redundancy Reliable Transport Protocol), previously known as ADELIN [17] (an ADaptive rELiable traNsport protocol) is proposed. Three schemes which combine forward error correction (FEC) mechanism at the bit and/or packet level in non-cooperative and cooperative scenarios were proposed. ARRTP uses different schemes for

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different distances depending on trade-off between reliability and energy consumption. In this paper an efficient energy AHECT for underwater communication system is proposed. A comparison between this technique and the techniques that use pure ARQ and FEC is done. AHECT is compared with ARRTP after an energy efficiency analysis for scheme 1 and 3 were done. ARRTP scheme 2 is neglected as it is found to be inefficient [16]. It is also compared with the system when variable power supply is used as adaptation factor.Due to space limitation, we are not going to repeat the underwater propagation model and the mathematical energy efficiency analysis for ARQ and FEC which is found in [6].

Our contribution can be summarized as follows:

- Energy efficient Adaptive Hybrid Error Correction Technique is proposed for UWSN.
- An adaptation algorithm which depends on a Pre-calculated PAR ranges look-up table, current BER, packet length, and current error correction technique is proposed, this algorithm adapt to the variation in both channel conditions and distances.

In section 2 we present the main idea of our proposed AHECT. The adaptation algorithm will be presented in section 3. In section 4 we will present how to calculate the look-up table. Adaptation using variable power supply will be presented in section 5. Energy efficiency analysis for ARRTP will be presented in section 6. Results and analysis in section 7, and in section 8 the paper is concluded.

II. AHECT Mechanism

The results of the analysis in [6] state that energy efficiency of error correction techniques varies with the variation in transmission distances and channel conditions. In some cases, one technique is better than the other, and vice versa. With this in mind we propose AHECT which achieves high energy efficiency in a varying distance, variable channel condition cases by adaptively changes the error correction technique used.The technique works like this: for variable distances and variable channel conditions, AHECT always search for the technique with the highest energy efficiency, and since reliability is one part in energy efficiency calculation, it will also be a reliable technique. The technique depends on an adaptation algorithm which based on the current packet acceptance rate (PAR), current error correction technique used, and a pre-calculated PAR ranges look-up table to determine which error correction technique is suitable for the current distance and current channel conditions. In AHECT, only modulation technique (i.e. ARQ) is used in good channel conditions and short distances, which means low BER. Selective repeat ARQ will be the most suitable type of ARQ for two reasons:

- As the BER is very low, no acknowledgement is needed for every packet, so either Go-Back-N or selective repeat is most suitable.
- In Go-back-N, the error packet and all the subsequent packets will be retransmitted, which results in a waste of energy; whereas in selective repeat only the error packet will be retransmitted.

In bad channel conditions and long distances variable code rates convolutional coding are used. Convolutional code is used for two reasons also:

- It is the best encoding technique for underwater communications as stated by [8].
- With convolutional coding, we can easily use puncturing technique to obtain variable code rates, which is needed in our AHECT.

Variable code rates are obtained using puncturing technique by deleting part of the bits of low-rate convolution code [18] as in the Table 1, and it is represented in MATLAB using systematic puncturing convolution codes with the parameters obtained from [18] as shown in Table II.:

III. AHECT Adaptation Algorithm

Table I
Puncturing matrix

Code rate	Puncturing Matrix
2/3	[1 1 0 1]
3/4	[1 1 0 1 1 0]
4/5	[1 1 0 1 1 0 1 0]
5/6	[1 1 0 1 1 0 1 0 1 0]
6/7	[1 1 0 1 1 0 1 0 1 0 1 0]

AHECT adaptation algorithm can be described as follows:

Using error detection technique in the receiver, BER is periodically calculated, and from which PAR is calculated using the packet length n as:

$$PAR = (1 - BER)^n \tag{1}$$

Then the suitable error correction technique is calculated from the function:

$$J = f(PAR, I, PARMAX(I,J), PARMIN(I,J)) \tag{2}$$

Where J is the suitable error correction technique required, PAR is the current packet acceptance rate, I is the current error correction technique used, and $PARMAX(I,J), PARMIN(I,J)$ are the maximum and minimum values in the pre- calculated PAR lookup table ranges.

We can mathematically model this function as in the following formula:

$$J = \sum_{n=1}^{6} n \times I_{A_n^I}(PAR) \tag{3}$$

Where A_n^I is the values in the look-up table taken from the energy efficiency analysis of six error correction techniques (One ARQ and five varying code rate FEC) [6], and

$$I_B(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

From the value of J obtained, a 3-bit feedback is added to the acknowledgement to state which error correction technique to use as in table (3)

TABLE III
ERROR CORRECTION TECHNIQUES DETAILS

Error correction technique	Consists of	FEC Code Rate	Feedback
1	Pure ARQ		000
2	Hybrid ARQ& FEC	6/7	001
3	Hybrid ARQ& FEC	5/6	010
4	Hybrid ARQ& FEC	4/5	011
5	Hybrid ARQ& FEC	3/4	100
6	Hybrid ARQ& FEC	2/3	101

The adaptation algorithm can be written as in algorithm Fig. 1 below:

Adaptation Algorithm

Feed Back ($BER_{current}$ (current BER), n (packet length), *PARMAX(I,J), PARMIN(I,J), I* (current error correction technique))
1 let J= 1
2 $PAR_{current} = (1 - BER_{current})^n$
3 If *PARMIN(I,J)* < $PAR_{current}$ < *PARMIN(I,J)*
4 Suitable Error Correction Technique = J
5 Go to 8
6 else J=J+1
7 Go back 3
8 If J = 1, then Feed Back = 000
9 Else If J=2, then Feed Back = 001
10 Else If J = 3, then Feed Back = 010
11 Else If J = 4, then Feed Back = 100
12 Else If J =5, then Feed Back. = 101
13 Else Feed Back. = 110
14 end
Return (Feed Back)

Fig. 1 Adaptation algorithm

IV. PRE-CALCULATED LOOKUP TABLE CALCULATIONS

The pre-calculated lookup table is calculated as follows:
1. Energy efficiencies and PARs of the six error correction techniques (ARQ plus five variable code rates FECs) for variable values of SNR are found as in [6], (SNR are taken as a measure for distance and channel conditions variations).
2. Starting with the SNR values which gives PAR values equal to 1 for all the techniques; at this SNR ARQ will have the maximum energy efficiency compared to the others, so the PAR for all those technique at this point is the maximum values in the ranges which makes the suitable technique is technique 1 (pure ARQ).

This means $PARMAX_{J,1}$ = 1, i.e. if the current technique is J and the current PAR is in the range that has 1 as the maximum value, then technique one is the most energy efficient technique.

3. Then decreasing SNR value until the energy efficiency of the first technique is less than the energy efficiency of the second technique; at this SNR the PAR for all techniques will be the minimum values in the ranges which makes the suitable technique is technique 1 (pure ARQ).

This means the PAR of any technique J at this point = $PARMIN_{J,1}$ i.e. if the PAR of the current technique J is in between $PARMIN_{J,1}$ and $PARMAX_{J,1}$, then technique 1 is the most energy efficient technique.
As the minimum values in the first range equal the maximum values in the second range, then:

$$PARMAX_{J,2} = PARMIN_{J,1}$$

4. Repeat step three above and increasing one to the error correction technique in each time until we come to the last technique as in the algorithm Fig. 2 below:

Look-up table calculation algorithm

PARMAX(I,J), PARMIN(I,J)(E.Ef (J, SNR) from [1], (ARQ Energy Efficiency calculation), and (FEC Energy Efficiency calculations))
1 for J = 1:6;
2 $PARMAX_{J,1}$ =1; $PARMIN_{J,6}$ = 0;
3 SNR = SNRMAX;
4 For I= 1:5;
5 If E.Ef (I, SNR)<=E.EF (I+1, SNR);
6 then $PARMIN_{J,I}$ = *PAR (J, SNR);*
7 $PARMAX_{J,I+1}$ = *PAR (J, SNR);*
8 SNR = SNR -1;
9 else go to 5;
10 end;
11 end;
12 return ($PARMIN_{J,I}$, $PARMAX_{J,I}$)//The maximum and minimum values in the lookup table from error correction techniques 1 to error correction techniques 6//

Fig. 2 Look-up table calculation algorithm

V. ADAPTIVE VARIABLE POWER SUPPLY (AVPS)

Adaptivity can also be achieved using variable power supply. For different channel conditions and different distances between sensor nodes variable transmit power values can be used to achieve the highest energy efficiencies using the same idea of adaptation algorithm. When using variable power supply as adaptation, ARQ with six different power supply values as in Table 4 is used instead of the six error correction techniques used in AHECT to calculate the pre-calculated look-up table. The energy efficiency in case of variable power supply can be calculated using the following formula:

$$Eff_{vps-ARQ} = \frac{E_{ARQ}^{eff}}{E_{ARQ}^{tot}}(1 - PER_{ARQ})\frac{P_{ref}}{P_t}$$

$$= \frac{l}{l + \alpha + \tau + ack}(1 - PER_{ARQ})\frac{P_{ref}}{Pt} \qquad (5)$$

Where $Eff_{vps-ARQ}$ is the energy efficiency for ARQ when using variable power supply, P_{ref} is a reference transmit power, or the designed power, P_t is the variable transmit power, and ($PER_{ARQ}, l, \alpha, \tau, ack$) from [6].

From the pre-calculated lookup table, current PAR, and current power supply value used, the suitable power supply value which will gives the most energy efficient transmission can be calculated.

TABLE IV
AVPS ERROR CORRECTION TECHNIQUES DETAILS

Error Correction technique	Transmit power (Watt)	Feedback
1	2.0	000
2	2.5	001
3	3.0	010
4	3.5	011
5	4.0	100
6	4.5	101

VI. INTERNODES DISTANCE-BASED REDUNDANCY RELIABLE TRANSPORT PROTOCOL (ARRTP) ENERGY EFFICIENCY ANALYSIS

Energy Efficiencies for the different ARRTP schemes, [16], Fig. 3, previously known as ADELIN [17] is found as follows:
For non-cooperative scheme-1,

$$PAR_{scheme-1} = \sum_{i=0}^{t} \binom{n}{i} (1-P_b)^{n-i} (P_b)^i \qquad (6)$$

Where $PAR_{scheme-1}$ is the packet acceptance rate, t is the correctability factor, n is the packet length.

$$Eff_{scheme-1} = \frac{E^{eff}_{scheme-1}}{E^{tot}_{scheme-1}} (PAR_{scheme-1}) = \frac{n-\alpha-\phi}{n} (PAR_{scheme-1}) \qquad (7)$$

For non-cooperative scheme-3, since reconstructing k original data packets needs receiving any k packets out of (k+s) packets, the probability of successfully transmission of k packets is given by:

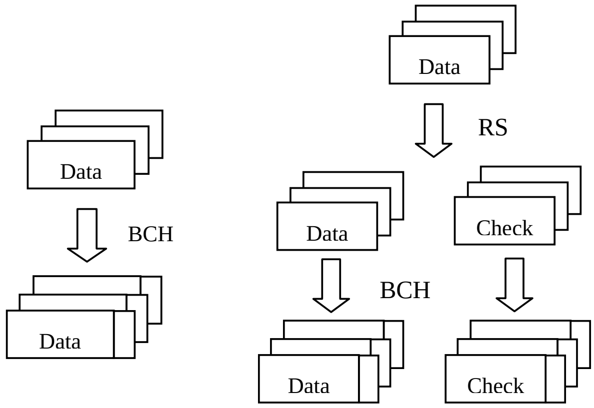
$$PAR_{scheme-3} = \sum_{i=k}^{k+s} \binom{k+s}{i} (1-P_s)^{K+s-i} (P_s)^i$$

(8)

Where P_s is the probability of successfully transmitting one packet over one hob with BCH coding for scheme-3 which is the same as equation (2).
For scheme-3, energy efficiency can be given by:

$$Eff_{scheme-3} = \frac{E^{eff}_{scheme-3}}{E^{tot}_{scheme-3}} \left(\sqrt[k]{(PAR_{scheme-3})}\right) = \frac{k-\alpha}{k+s+\phi_1+\phi_2} \sqrt[k]{(PAR_{scheme-3})} \qquad (9)$$

Where ϕ_1 is overhead in the data packet due to BCH coding ϕ_2 is the overhead in the check packet due to BCH coding. Scheme-2 is ignored as it is inefficient compared with scheme-1 and scheme-3 [16].



(a) Scheme-1 (BCH) (b) Scheme-3 (BCH-RS)
Fig. 3 Different ARRTP schemes

VII. RESULTS AND DISCUSSION

A. AHECT Versus ARQ and FEC Probability of Success and Energy Efficiency

From Fig. 4 below; it is clear that AHECT has higher probability of success (PAR) compared with both ARQ and FEC, except for a short distance from 2200 m to 2400 m; where FEC has a higher probability of success than AHECT. This differences which is around 2-3 % has no noticeable effect on the system since both techniques have more than 90 % probability of success.

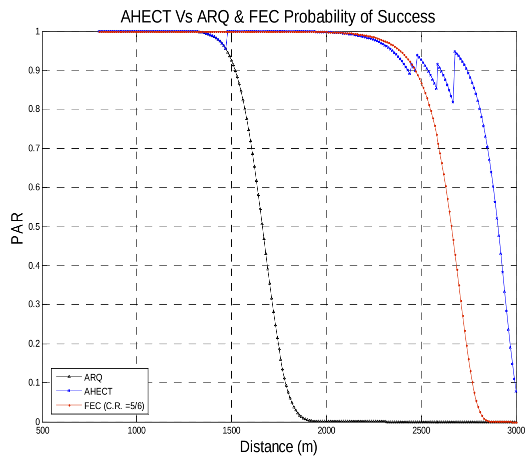


Fig. 4 AHECT Vs ARQ & FEC Probability of Success

Fig. 5 gives a comparison between the energy efficiency of AHECT and pure ARQ and FEC for varying distances. From this figure it is clear that AHECT is more energy efficient than both ARQ and FEC in variable distances situation.

Compared with the pure ARQ, AHECT achieves 10 % increase in saving energy when the distance is around 1500 m to more than 60 % when the distance increases above 1700 m. When compared with FEC, it achieves around 10 % increase in energy saving when the distance is below 1500 m, and around 7 % saving when the distance goes above 1500 m. between 2400 and 2600 m, both AHECT and FEC have the same energy efficiencies as AHECT uses the same code rate of this FEC.

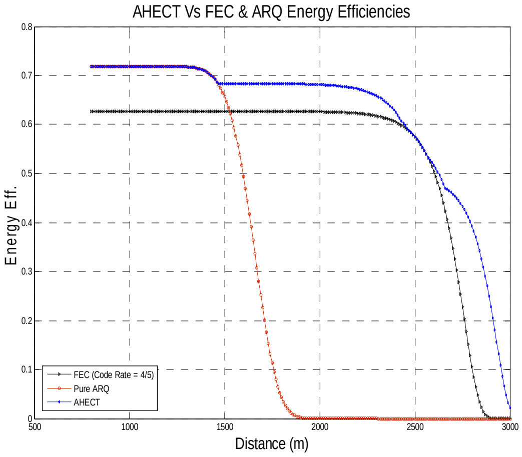


Fig. 5 AHECT Vs ARQ & FEC Energy Efficiency (Variable Distances Case)

In Fig. 6 variable wind speed is taken as a measure for the variation in channel conditions. From this figure it is clear that AHECT is more energy efficient than both ARQ and FEC for variable wind speed (i.e. variable channel conditions). Compared with the pure ARQ, and when the transmission distance is 1500 m, AHECT achieves 5 % increase in energy saving when there is wind of 0.5 m/s speed, more than 60 % energy saving when wind speed increases to 1 m/s. When compared with FEC, AHECT achieves around 7 % increase in energy saving when there is no wind, and around 5 % when the speed is greater than 0.5 m/s.

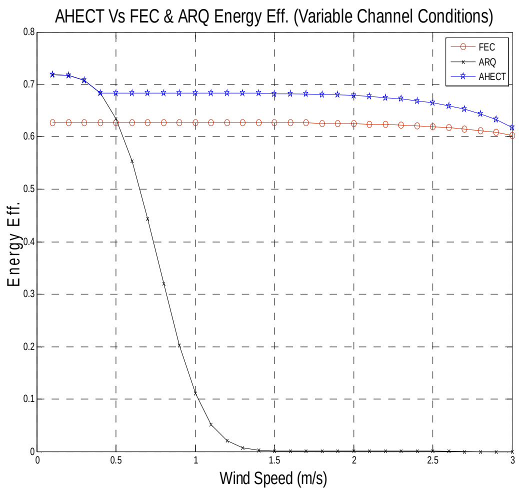


Fig. 6 AHECT Vs ARQ & FEC Energy Efficiency (Variable Channel conditions)

B. AHECT Versus AVPS Probability of Success and Energy Efficiency

From Fig. 7, it is clear that AHECT has higher probability of success than AVPS. This differences range from 10 % in short distances (around 1000 m) to more than 80 % in long distances (around 2400 m).

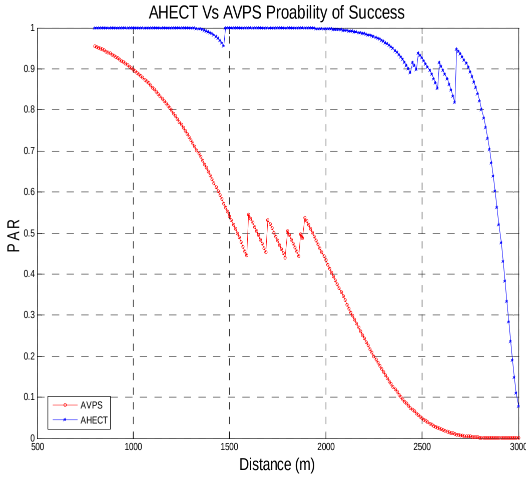


Fig. 7 AHECT Vs AVPS probability of Success

Fig 8 below gives a comparison between energy efficiency of AHECT and AVPS for varying distances. From this figure it is clear that AHECT is more energy efficient than AVPS in variable distances situation.

Compared with AVPS, AHECT achieves 20 % increase in saving energy when the distance is around 1700 m, and more than 60 % when the distance increase above 2200 m.

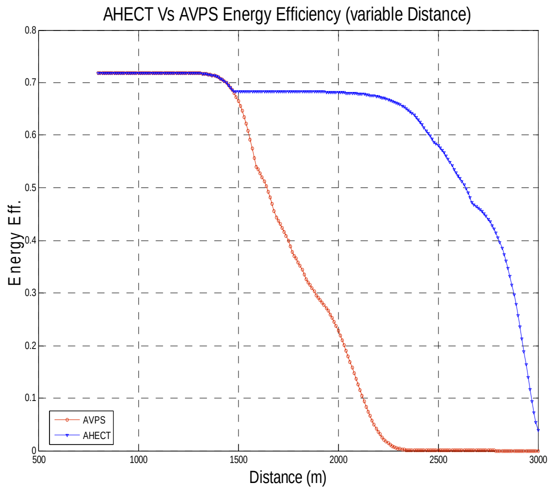


Fig. 8 AHECT Vs AVPS Energy efficiency (Variable Distance Case)

In Fig. 9 variable wind speed is taken as a measure for the variation in channel conditions. From this figure it is clear that AHECT is more energy efficient than AVPS for variable wind speed (i.e. variable channel conditions). Compared with AVPS when the transmission distance is 1500 m, AHECT achieves 5 % increase in saving energy when there is wind of speed 0.5 m/s, more than 30 % when wind speed increases to 1 m/s, and more than 60 % when wind speed is greater than 2 m/s.

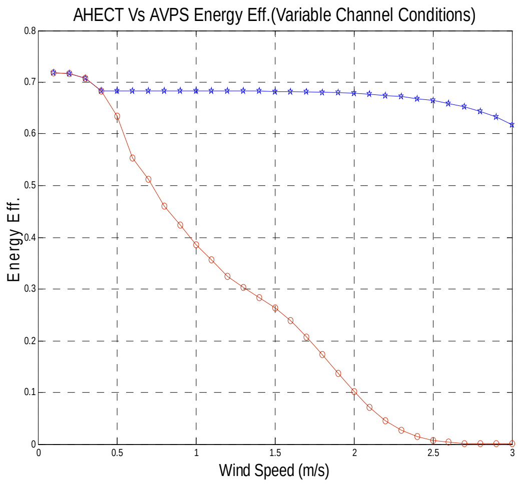


Fig. 9 AHECT Vs AVPS Energy Efficiency (Variable Channel Conditions)

C. AHECT Versus ARRTP Probability of Success and Energy Efficiency

From Fig. 8 below; it is clear that AHECT has higher probability of success than ARRTP. This differences range from around 20 % when the transmission distance is around 2000 m to more than 90 % when the transmission reach 2500 m. it is also clear that ARRTP with both BCH and CS has higher probability of success than when only BCH coding is used.

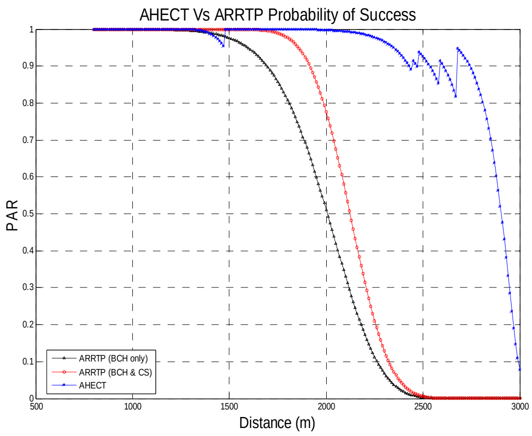


Fig. 10 AHECT Vs ARRTP probability of Success

Fig. 11 below gives a comparison between the energy efficiency of AHECT and ARRTP for varying distances. From this figure it is clear that AHECT is more energy efficient than both types of ARRTP in variable distances situation. Compared with ARRTP (Only BCH), AHECT achieves 5 % increase in energy saving when the distance is below 1500 m, and more than 50 % when the distance increase above 2200 m. when compared with ARRTP (BCH and CS), AHECT achieves around 25 % saving in energy when the distance is less than 1700 m and more than 50 % when the distance is more than 2200 m.

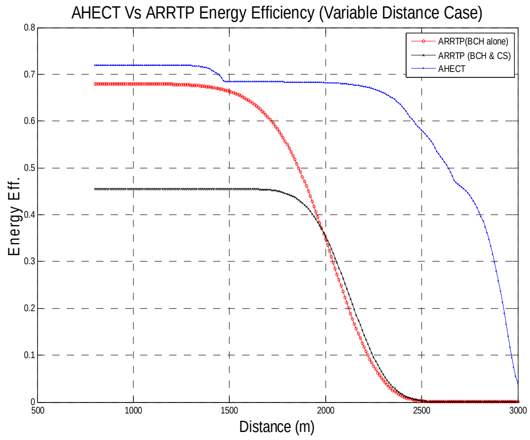


Fig. 11 AHECT Vs ARRTP Energy Efficiency (Variable Distances Case)

Regarding variable channel conditions, ARRTP don't adapt to the variations in channel conditions, as it uses approximate formula for calculating noises, which ignore the effect of wind speed and shipping factors.

VIII. Conclusions

In this paper, we have presented our AHECT idea, and how the adaptation occurs. The adaptation algorithm is based on:

- A pre-calculated PAR look-up table which is calculated from the energy efficiency analysis we have done on [6].
- Current BER, which can be easily determined using any error detection techniques, packet length, and current error correction techniques.

Based on the results of the adaptation algorithm the receiver sends 3-bit feedback with the acknowledgement telling the sender which error correction technique is most suitable for the current distance and current channel conditions.

The results show that our AHECT has more probability of success and more energy efficient than all the other error correction techniques.

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TABLE II
MINIMUM HAMMING DISTANCES (d_{free}) AND WEIGHT DISTRIBUTION (w_{dfree}) FOR VARIABLE RATE CONVOLUTIONAL CODES

R_c	d_{free}	w_{dfree}	$w_{dfree+1}$	$w_{dfree+2}$	$w_{dfree+3}$	$w_{dfree+4}$	$w_{dfree+5}$	$w_{dfree+6}$	$w_{dfree+7}$	$w_{dfree+8}$
2/3	3	1	10	54	226	853	3038	10432	34836	114197
3/4	3	15	104	540	2520	11048	46516	190448	763944	3016844
4/5	2	1	36	309	2058	12031	65754	344656	1755310	8754128
5/6	2	2	111	974	6815	43598	263671	1536563	8724988	46801477
6/7	2	5	186	1942	16423	124469	887512	6088910	40664781	266250132