International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:1, 2010

Forward Kinematics Analysis of a 3-PRS Parallel Manipulator

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Abstract**—**In this article the homotopy continuation method (HCM) to solve the forward kinematic problem of the 3-PRS parallel manipulator is used. Since there are many difficulties in solving the system of nonlinear equations in kinematics of manipulators, the numerical solutions like Newton-Raphson are inevitably used. When dealing with any numerical solution, there are two troublesome problems. One is that good initial guesses are not easy to detect and another is related to whether the used method will converge to useful solutions. Results of this paper reveal that the homotopy continuation method can alleviate the drawbacks of traditional numerical techniques.

Keywords**—**Forward kinematics; Homotopy continuation method; Parallel manipulators; Rotation matrix

I. Introduction

QUITE recently, parallel manipulators have received a great deal of attention from many researchers [1]. This popularity is a result of the fact that the parallel manipulators have more advantages in comparison to their serial counterparts in many aspects, such as stiffness in mechanical structure, high position accuracy, high speed and high load carrying capacity.

Parallel manipulators generally perform the task of controlling the moving platform with respect to the base frame. To achieve this goal, the position analysis of parallel manipulators, the forward and inverse kinematics problems, is a mandatory step. The forward problem, which is the problem of finding the poses (positions and orientations) of the top platform when every actuator displacement is given, is challenging. In contrast to this, the inverse kinematics problem, which consists in finding the set of joint variables to achieve a desired configuration of the top platform, is easy in contrast to serial chain manipulators where the opposite is true. Many researchers have studied the forward displacement analysis of parallel manipulators [2], [9]-[11], [22], [23].

Six degrees of freedom (DOF) parallel manipulators have many advantages mentioned above and many literatures have introduced them; however, 6-DOF is not always required for many applications. Recently, parallel manipulators with less than 6-DOF have attracted various researchers. For example, many 3-DOF parallel manipulators have been designed and investigated for relevant applications [3]-[10]. The 3-RPS parallel manipulators are a group of these 3-DOF manipulators. The 3-PRS architecture parallel manipulators were already well known in the mechanism community, and several 3-PRS parallel manipulators were designed and analyzed separately [7]-[10]. Here the notation of R, P, U, C, and S denotes the revolute, prismatic, universal, cylindrical, and spherical joint, respectively.

In most cases of direct kinematic analysis of parallel manipulators, the solution of a system of nonlinear coupled algebraic equations leads to the variables describing the platform posture and there may be many solutions [11]. Except in a limited number of these problems, there are difficulties in finding exact analytical solutions. So these nonlinear simultaneous equations should be solved using other methods. Recently, numerical calculation methods were used to achieve this; but as the numerical calculation methods improved, semi-exact analytical methods did, too. Most scientists believe that the combination of numerical and semi-exact analytical methods can also end with useful results [12], [13].

To date, there exist many different methods that can deal with simultaneous non-linear equations, such as the Newton–Raphson method which is very efficient in the convergence speed [14]-[16]. However, there always needs to guess the initial value in the iteration process. Good initial guess value can solve the equations quickly; while bad initial guess value usually will yield divergence. Homotopy continuation method (HCM) is a type of perturbation and homotopy method [14]-[17]. It can guarantee the answer by a certain path, if the auxiliary homotopy function is chosen well. It does not share the drawbacks of traditional numerical techniques, specifically the acquirement of good initial guess values, the problem of convergence and computing time [14]-[16]. This method, known as early as in the 1930s [20]-[22], was used by kinematicians in the 1960s for solving mechanism synthesis problems. The latest development was done by Morgan [18], [19], Garcia [20] and Allgower [21]. Wu [14]-[16] presented some techniques by combining Newton's and homotopy methods to avoid divergence in solving nonlinear equations. Also Wu [14] and Varedi [22] et al. used this technique in kinematics analysis of robots. In this paper homotopy

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continuation method is used to solve the forward kinematic problem of a 3-PRS parallel manipulator.

II. THE HOMOTOPY CONTINUATION METHOD

In connection with any numerical problem, e.g. the Newton–Raphson method, there are two troublesome questions. One is that good initial guesses are not easy to detect and another is whether the used method will converge to useful solutions. The homotopy continuation method can eliminate these shortcomings [14].

Let us consider the following system of nonlinear equations:

$$F(X)=0 \quad i.e. \begin{cases} f(x,y,...,z)=0 \\ g(x,y,...,z)=0 \\ . \\ . \\ . \\ h(x,y,...,z)=0 \end{cases} \tag{1}$$

The numerical iteration formula of Newton's method for solving these equations is given as:

$$\begin{bmatrix} \frac{\partial f(x_n,y_n,...)}{\partial x} & \frac{\partial f(x_n,y_n,...)}{\partial y} & . & . & . \\ \frac{\partial g(x_n,y_n,...)}{\partial x} & \frac{\partial g(x_n,y_n,...)}{\partial y} & . & . & . \\ . & . & . & & \\ . & . & & . & \\ . & . & & & . \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} -f(x_n,y_n,...) \\ -g(x_n,y_n,...) \\ . \\ . \\ . \end{bmatrix} \tag{2}$$

Given a system of equations in n variables x_1, x_2,…, x_n, the equations are modified by omitting some of the terms and adding new ones until a new system of equations, the solutions of which may be easily guessed/given/known, is obtained. Then coefficients of the new system are deformed into the coefficients of the original system by a series of small increments to obtain the solutions. This is called homotopy continuation technique. To find the solution for (1), a new simple start system or called auxiliary homotopy function [18]-[20] is chosen as:

$$G(x)=0 \tag{3}$$

G(X) must be known or controllable and easy to solve. Then the homotopy continuation function can be written as follows:

$$H(X,t)\equiv tF(X)+(1-t)G(X)=0 \tag{4}$$

Where t is an arbitrary parameter and varies from 0 to 1, i.e. $t\in[0,1]$. Therefore, the following two boundary conditions [14]-[16] exist.

$$\begin{aligned} H(X,0)&=G(X) \\ H(X,1)&=F(X) \end{aligned} \tag{5}$$

The goal is to solve the $H(X, t) = 0$ instead of $F(X) = 0$ by varying parameter t from 0 to 1 and avoid divergence. Hence Eq. (2) is rewritten [16] as:

$$\begin{bmatrix} \frac{\partial H_1(x_n,y_n,...)}{\partial x} & \frac{\partial H_1(x_n,y_n,...)}{\partial y} & . & . & . \\ \frac{\partial H_2(x_n,y_n,...)}{\partial x} & \frac{\partial H_2(x_n,y_n,...)}{\partial y} & . & . & . \\ . & . & . & & \\ . & . & & . & \\ . & . & & & . \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ y_{n+1}-y_n \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} -H_1(x_n,y_n,...) \\ -H_2(x_n,y_n,...) \\ . \\ . \\ . \end{bmatrix} \tag{6}$$

To avoid divergence, Wu [15] provided some useful choices for the auxiliary homotopy function. They are polynomial, harmonic, exponential or any combinations of them. By appropriate choosing/adjusting the auxiliary homotopy function, the solutions of (1) can be obtained [16].

III. DESCRIPTION AND MOBILITY ANALYSIS OF THE 3-PRS PARALLEL MANIPULATOR

A 3-PRS parallel manipulator, as shown in Fig. 1, is studied by Li and Xu [23]. This manipulator is composed of a moving platform, a fixed base, and three supporting limbs with identical kinematic structure. Each limb connects the fixed base to the moving platform by a P joint, an R joint, and an S joint in sequence, where the P joint is actuated by a linear actuator. Thus, the moving platform is attached to the base by three identical PRS linkages.

The vectors and reference frames are described in Figs. 2 and 3. For the sake of analysis, as shown in Fig. 2, a fixed Cartesian reference coordinate frame $O\{x, y, z\}$ is attached at the centered point O of the fixed triangle base platform $\Delta A_1A_2A_3$. Moreover, a moving coordinate frame P$\{u, v, w\}$ is attached on the moving platform at point P which is the centered point of triangle $\Delta B_1B_2B_3$. For simplicity and without losing the generality, let the x-axis points in the direction of vector OA_1 and the u-axis points along vector PB_1. The three rails D_iE_i for i = 1, 2, and 3 intersect each other at the vertex N of the cone and intersect the x–y plane at points A_1, A_2, and A_3 that lie on a circle of radius a. The three links C_iB_i for i = 1, 2, and 3 with the length of l intersect the u–v plane at points B_1, B_2, and B_3 which lie on a circle of radius b. The sliders of the P joints are restricted to move along the rails between D_i and E_i. Angle α is measured from the fixed base to rails D_iE_i and is defined as the actuator layout angle. Angle β is defined from the x-axis to OA_2 in the fixed frame, and also from the u-axis to PB_2 in the moving frame. Similarly, the angle γ is measured from the x-axis to OA_3 in the fixed frame and from the u-axis to PB_3 in the moving frame. In the following discussions, β = 120° and γ = 240° are assigned for simplicity. Moreover, this assignment also results in a symmetric workspace [23].

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:1, 2010

Fig. 1 The 3-PRS parallel manipulator

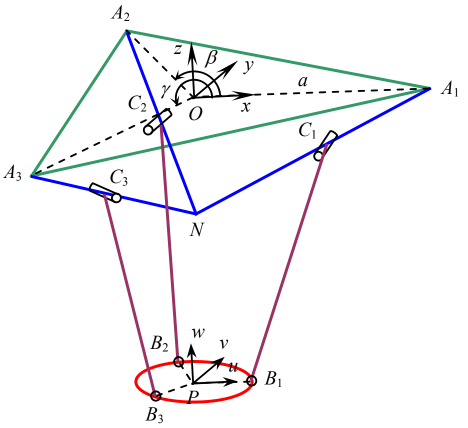


Fig. 2 Schematic representation of the 3-PRS parallel manipulator

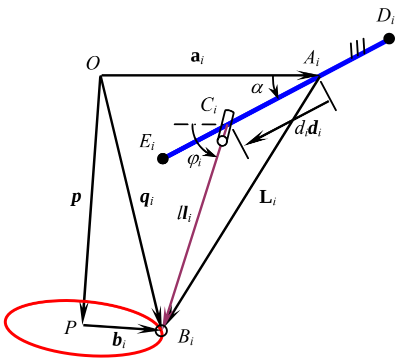


Fig. 3 Geometry of one typical kinematic chain

Let $\mathbf{d} = [d_1\ d_2\ d_3]^{\mathrm{T}}$ be the vector of the three actuated joint variables where d_i denotes the distance between A_i and C_i and is taken positive if C_i is located above the xy plane of the reference coordinate frame $O\{x, y, z\}$. Moreover, $\mathbf{x} = [p_x\ p_y\ p_z\ \psi\ \theta\ \varphi]^{\mathrm{T}}$ is considered as the vector of Cartesian variables (constrained and unconstrained) which describes the position and orientation of the moving platform. Three Euler angles φ, ψ, and θ rotating about the z-, x-, and y-axes of the fixed reference frame respectively are defined. The transformation from the moving frame to the fixed frame can be described by a position vector $\mathbf{p} = [p_x\ p_y\ p_z]^{\mathrm{T}}$ and a 3 × 3 rotation matrix ${}^{O}\mathbf{R}_p$, which can be expressed as follows:

$$
{}^{O}\mathbf{R}_p = \mathbf{R}_y(\theta)\mathbf{R}_x(\psi)\mathbf{R}_z(\varphi) =
\begin{bmatrix} c\theta c\varphi + s\psi s\theta s\varphi & -c\theta s\varphi + s\psi s\theta c\varphi & c\psi s\theta \\ c\psi s\varphi & c\psi c\varphi & -s\psi \\ -s\theta c\varphi + s\psi c\theta s\varphi & s\theta s\varphi + s\psi c\theta c\varphi & c\psi c\theta \end{bmatrix} \tag{7}
$$

Where c and s stand for cosine and sine, respectively.

The general Grübler–Kutzbach criterion is of importance in mobility analysis of many parallel mechanisms. The number of DOF of a 3-PRS parallel manipulator given by the general Grübler–Kutzbach criterion is:

$$
F = \lambda(n - g - 1) + \sum_{i=1}^{g} f_i = 6 \times (8 - 9 - 1) + 15 = 3 \tag{8}
$$

Where λ represents the order of task space, n is the number of links, g is the number of joints, and f_i denotes the DOF of joint i.

IV. CONSTRAINT CONDITIONS

The position vectors of points A_i and B_i with respect to frames O and P can be written as ${}^{O}\mathbf{a}_i$ and ${}^{P}\mathbf{b}_i$, respectively, where a leading superscript indicates the coordinate frame with respect to which a vector is expressed. For brevity, the leading superscript will be omitted whenever the coordinate frame is the fixed frame, e.g., ${}^{O}\mathbf{a}_i = \mathbf{a}_i$. The vectors of $\mathbf{a}_i$ and ${}^{P}\mathbf{b}_i$ can be expressed as follows, respectively.

$$
\mathbf{a}_1 = [a \quad 0 \quad 0]^{\mathrm{T}} \tag{9a}
$$

$$
\mathbf{a}_2 = \left[-a/2 \quad \sqrt{3}a/2 \quad 0\right]^{\mathrm{T}} \tag{9b}
$$

$$
\mathbf{a}_3 = \left[-a/2 \quad -\sqrt{3}a/2 \quad 0\right]^{\mathrm{T}} \tag{9c}
$$

$$
{}^{P}\mathbf{b}_1 = [b \quad 0 \quad 0]^{\mathrm{T}} \tag{10a}
$$

$$
{}^{P}\mathbf{b}_2 = \left[\frac{-b}{2} \quad \frac{\sqrt{3}b}{2} \quad 0\right]^{\mathrm{T}} \tag{10b}
$$

$$p_{\mathbf{b}_3} = \begin{bmatrix} \frac{-b}{2} & -\frac{\sqrt{3}b}{2} & 0 \end{bmatrix}^{\mathrm{T}} \tag{10c}$$

Let **u**, **v**, and **w** be three unit vectors defined along the u-, v-, and w-axes of the moving frame P. Then the rotation matrix can be expressed in terms of the direction cosines of **u**, **v**, and **w** as

$$^{O}\mathbf{R}_{p} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} \tag{11}$$

The position vector $\mathbf{q}_i$ pointing from O to the ith S joint, B_i, can be expressed by:

$$\mathbf{q}_i = \mathbf{p} + \mathbf{b}_i \tag{12}$$

Where

$$\mathbf{b}_i = {}^{O}\mathbf{R}_{p}\,{}^{p}\mathbf{b}_i \tag{13}$$

The vector $\mathbf{b}_i$ can be reached by substituting (10) and (11) into (13), and according to (12) the vector $\mathbf{q}_i$ can be obtained as below:

$$\mathbf{q}_1 = \begin{bmatrix} p_x + bu_x \\ p_y + bu_y \\ p_z + bu_z \end{bmatrix} \tag{14a}$$

$$\mathbf{q}_2 = \begin{bmatrix} p_x - \frac{bu_x}{2} + \frac{\sqrt{3}bv_x}{2} \\ p_y - \frac{bu_y}{2} + \frac{\sqrt{3}bv_y}{2} \\ p_z - \frac{bu_z}{2} + \frac{\sqrt{3}bv_z}{2} \end{bmatrix} \tag{14b}$$

$$\mathbf{q}_3 = \begin{bmatrix} p_x - \frac{bu_x}{2} - \frac{\sqrt{3}bv_x}{2} \\ p_y - \frac{bu_y}{2} - \frac{\sqrt{3}bv_y}{2} \\ p_z - \frac{bu_z}{2} - \frac{\sqrt{3}bv_z}{2} \end{bmatrix} \tag{14c}$$

Considering the mechanical constraints imposed by the R joint, the S joint B_i can only move in the plane defined by the ith linear actuator and ith link C_iB_i. Therefore, the following three equations hold:

$$q_{1y} = 0 \tag{15}$$

$$q_{2y} = -\sqrt{3}q_{2x} \tag{16}$$

$$q_{3y} = -\sqrt{3}q_{3x} \tag{17}$$

Substituting the components of $\mathbf{q}_i$ from (14) into (15)–(17) yields:

$$p_y + bu_y = 0 \tag{18}$$

$$p_y - \frac{1}{2}bu_y + \frac{\sqrt{3}}{2}bv_y = -\sqrt{3}(p_x - \frac{1}{2}bu_x + \frac{\sqrt{3}}{2}bv_x) \tag{19}$$

$$p_y - \frac{1}{2}bu_y - \frac{\sqrt{3}}{2}bv_y = \sqrt{3}(p_x - \frac{1}{2}bu_x - \frac{\sqrt{3}}{2}bv_x) \tag{20}$$

Taking 2×(18) – (19) – (20) yields

$$v_x = u_y \tag{21}$$

Subtracting (19) from (20) leads to

$$p_x = \frac{b}{2}(u_x - v_y) \tag{22}$$

Hence (18), (21), and (22) impose three constraints on the motion of the moving platform.

V. FORWARD POSITION KINEMATICS MODELING

The aim of forward position kinematics is obtaining the position and orientation of the moving platform by a given set of actuated inputs. So the input in forward position kinematics is the vector of the three actuated joint ($\mathbf{d} = [d_1\ d_2\ d_3]^{\mathrm{T}}$) and output is the vector of Cartesian variables ($\mathbf{x} = [p_x\ p_y\ p_z\ \psi\ \theta\ \varphi]^{\mathrm{T}}$) (constrained and unconstrained) which describes the position and orientation of the moving platform.
Referring to Fig. 3, the following vectorial relation can be written

$$\mathbf{L}_i = d_i\mathbf{d}_i + l\mathbf{l}_i \tag{23}$$

Where $\mathbf{L}_i$ is the vector pointing from point A_i to B_i, $\mathbf{l}_i$ is the unit vector along C_iB_i, d_i represents the linear displacement of the ith actuator, and $\mathbf{d}_i$ is the corresponding unit vector directed along D_iE_i for i = 1, 2, and 3, which can be expressed as follows:

$$\mathbf{d}_1 = \begin{bmatrix} -c\alpha & 0 & -s\alpha \end{bmatrix}^{\mathrm{T}} \tag{24a}$$

$$\mathbf{d}_2 = \begin{bmatrix} \dfrac{c\alpha}{2} & -\dfrac{\sqrt{3}c\alpha}{2} & -s\alpha \end{bmatrix}^{\mathrm{T}} \tag{24b}$$

$$\mathbf{d}_3 = \begin{bmatrix} \dfrac{c\alpha}{2} & \dfrac{\sqrt{3}c\alpha}{2} & -s\alpha \end{bmatrix}^{\mathrm{T}} \tag{24c}$$

Additionally

$$\mathbf{L}_i = \mathbf{q}_i - \mathbf{a}_i \tag{25}$$

Where $\mathbf{q}_i$ is expressed by (12). Equation (23) yields:

$$\mathbf{L}_i - d_i \mathbf{d}_i = l\mathbf{l}_i \tag{26}$$

Squaring both sides of (26) and rearranging the items yields

$$d_i^{\,2} - 2d_i \mathbf{L}_i \cdot \mathbf{d}_i + \mathbf{L}_i \cdot \mathbf{L}_i - l^2 = 0 \tag{27}$$

Which leads to three equations with assuming $i = 1, 2$ and 3. Components of the unit vectors $\mathbf{u}$ and $\mathbf{v}$ can be obtained through the rotation matrix, expressed in (7); substituting these components into the three constraint (18), (21), and (22) yields:

$$-bc\psi\, s\varphi \;\; = p_y \tag{28}$$

$$-c\theta\, s\varphi + s\psi\, s\theta\, c\varphi\, c\varphi = c\psi\, s\varphi\, s\varphi \tag{29}$$

$$\frac{b}{2}(c\theta\, c\varphi + s\psi\, s\theta\, s\varphi - c\psi\, c\varphi) = p_x \tag{30}$$

Due to the physical constraints introduced by cone angle limits of the S joints, the moving platform cannot rotate about the x- and y-axes unlimitedly, i.e., $\psi > -\pi/2$, $\theta < \pi/2$, then $c\psi + c\theta \neq 0$. Solving Eq. (29) with respect to φ leads to:

$$\varphi = \tan^{-1} 2(s\psi s\theta, c\psi + c\theta) \tag{31}$$

Substituting φ from this equation into (28) and (30) allows the generation of p_y and p_x, respectively. So there is only need to calculate ψ, θ and p_z, then the other variables φ, p_y and p_x can be obtained respectively.

For solving this problem, equation (27) is used. This equation makes a system of nonlinear equations which is described below:

$$f = d_1^2 - 2d_1 \mathbf{L}_1 \cdot \mathbf{d}_1 + \mathbf{L}_1 \cdot \mathbf{L}_1 - l^2 = 0 \tag{32a}$$

$$g = d_2^2 - 2d_2 \mathbf{L}_2 \cdot \mathbf{d}_2 + \mathbf{L}_2 \cdot \mathbf{L}_2 - l^2 = 0 \tag{32b}$$

$$h = d_3^2 - 2d_3 \mathbf{L}_3 \cdot \mathbf{d}_3 + \mathbf{L}_3 \cdot \mathbf{L}_3 - l^2 = 0 \tag{32c}$$

Where $\mathbf{d}_i$ and $\mathbf{L}_i$ ($i = 1, 2$ & 3) can be calculated.

For solving this system of nonlinear equations, the homotopy continuation method is used as follows:

$$H_1 = t \times (d_1^2 - 2d_1 \mathbf{L}_1 \cdot \mathbf{d}_1 + \mathbf{L}_1 \cdot \mathbf{L}_1 - l^2) + (1-t) \times G_1 = 0 \tag{33a}$$

$$H_2 = t \times (d_2^2 - 2d_2 \mathbf{L}_2 \cdot \mathbf{d}_2 + \mathbf{L}_2 \cdot \mathbf{L}_2 - l^2) + (1-t) \times G_2 = 0 \tag{33b}$$

$$H_3 = t \times (d_3^2 - 2d_3 \mathbf{L}_3 \cdot \mathbf{d}_3 + \mathbf{L}_3 \cdot \mathbf{L}_3 - l^2) + (1-t) \times G_3 = 0 \tag{33c}$$

Equations (33a)–(33c) can be solved by the Newton–Raphson method while the homotopy parameter t changes from 0 to 1.

VI. CASE STUDY

To show the efficiency of the proposed method, the direct kinematics of problem of the manipulator under study is solved with assumption of geometric parameters d_1= 101.4888, d_2= 91.8057, d_3= -80.5667, a = 400, b=200, l=550, $\alpha = \pi/6$ (all the lengths are in mm and angles are in radians). The initial guesses of unknown parameters are chosen as: (p_z, ψ, θ) = (20, 2, 1).

Equations (33a)–(33c) are solved by the Newton–Raphson method and various auxiliary homotopy functions (G_1, G_2, G_3) are used to obtain the result. The auxiliary homotopy functions and the results of these nonlinear equations are given in Table 1 and depicted in Fig. 4.

Although all possible configurations of the moving platform can be derived by using the Silvester elimination method, it is a very time-consuming work and many solutions are meaningless [23]. In this case, solving the problem by Silvester elimination method results in eight real configurations that two of them are meaningless. In comparison with the Silvester elimination method, the homotopy continuation method converges to all six accurate results.

It is noteworthy that, changing the initial guesses of unknown parameters doesn't have sensible effect on the result. Table 2 shows this property in which G_1, G_2 and G_3 are chosen as $p_z + \cos(\psi)$, $\sin(\psi) + \cos(\psi)$ and $p_z \sin(\psi)$, respectively and different initial guesses are used for the unknown parameters.

VII. CONCLUSIONS

In this paper the homotopy continuation method is applied on the forward kinematic problem of the 3-PRS parallel manipulator. Some advantages of homotopy continuation method over the conventional methods are its fast convergence and leading to its final values even with bad initial guesses, while the Newton–Raphson method would easily become divergent. Also, the algorithm is very straightforward.

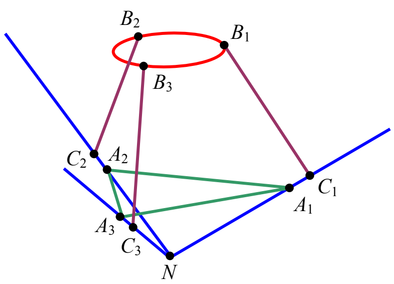
TABLE I
THE AUXILIARY HOMOTOPY FUNCTIONS AND THEIR RESULTS

Result No.	G_1	G_2	G_3	*Result* (p_z, ψ, θ)
1	$2p_z$+8sin(θ)	5cos(ψ)-$6p_z$	7sin(θ)+ 10sin(ψ)	(510.924, 0.074,-0.042)
2	$3p^2_z$+ 8sin(θ^2)	5cos(ψ^2)- $6p_z$sin(θ^3)	7sin(θ)+10 sin(ψ^2)cos(θ)	(-411.604, 0.274, 5.0363)
3	p^4_zsin(θ)+sin(ψ^2)cos(θ)	sin(ψ)	sin(θ)	(405.858, -0.979, -1.243)
4	$2p^2_z$+ 8sin(θ)	5cos(ψ^2)+ $6p_z$	7sin(θ)+ 10sin(ψ)	(-405.169, 1.235, -5.538)
5	p_z+sin(ψ)	cos(ψ)	sin(θ)	(406.586, 0.0436, 1.365)
6	p_z+cos(ψ)	sin(ψ)+ cos(ψ)	$p_z$sin(ψ)	(-470.00, 0.400, -0.300)

TABLE II
SOME INITIAL GUESSES WHICH RESULT IN THE SAME ANSWER

(p_{z0}, ψ_0, θ_0)	*Result* (p_z, ψ, θ)
(20, 2, 1)	(-470.0000, 0.4000, -0.3000)
(-20, -2, -1)	(-470.0000, 0.4000, -0.3000)
(30, 5, 1)	(-470.0000, 0.4000, -0.3000)
(-25, -2, 1)	(-470.0000, 0.4000, -0.3000)
(30, -2, 1)	(-470.0000, 0.4000, -0.3000)
(-25, -2, -1)	(-470.0000, 0.4000, -0.3000)

a



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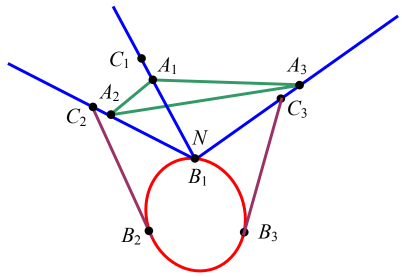

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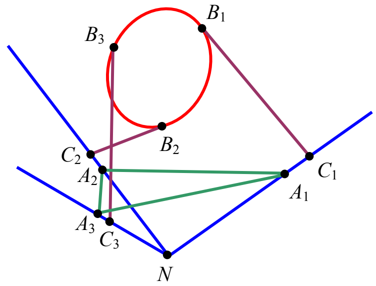

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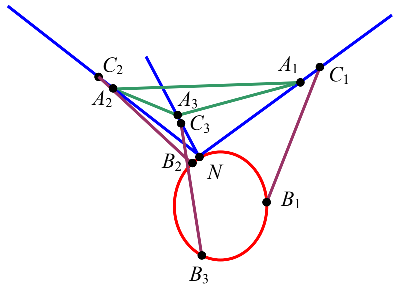

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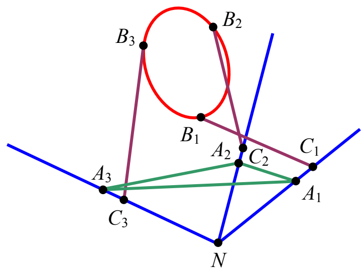

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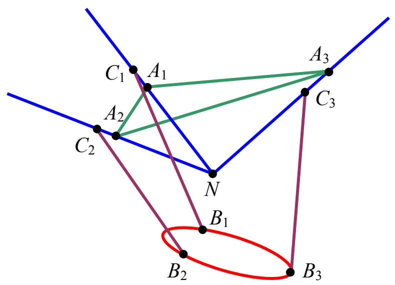

Fig. 4 Six solutions of the forward kinematics problem corresponding to assembly modes of the manipulator

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