

# Measuring Teachers' Beliefs about Mathematics: A Fuzzy Set Approach

M.A. Lazim, and M.T.Abu Osman

**Abstract**—This paper deals with the application of a fuzzy set in measuring teachers' beliefs about mathematics. The vagueness of beliefs was transformed into standard mathematical values using a fuzzy preferences model. The study employed a fuzzy approach questionnaire which consists of six attributes for measuring mathematics teachers' beliefs about mathematics. The fuzzy conjoint analysis approach based on fuzzy set theory was used to analyze the data from twenty three mathematics teachers from four secondary schools in Terengganu, Malaysia. Teachers' beliefs were recorded in form of degrees of similarity and its levels of agreement. The attribute 'Drills and practice is one of the best ways of learning mathematics' scored the highest degree of similarity at 0.79860 with level of 'strongly agree'. The results showed that the teachers' beliefs about mathematics were varied. This is shown by different levels of agreement and degrees of similarity of the measured attributes.

**Keywords**—belief, membership function, degree of similarity, conjoint analysis

## I. INTRODUCTION

THE new advanced mathematics curriculum in Malaysian Secondary Schools Mathematics was implemented throughout the country beginning of the year 2002. One of the main objectives of the curriculum is to ensure students strengthen in their problem solving skills [1]. As the curriculum had been implemented, the school teachers played an important role in ensuring the success of this objective. However the shifting to a problem solving approach is not the same as changing machine operative manuals, rather it requires deep changes in teacher's cognitive and affective domains. Typically, the changes would depend fundamentally on the teacher's system of beliefs. Cobb defined beliefs as an individual's personal assumption about the nature of reality [2]. Pajeras describes beliefs as personal principles, constructed from experiences that an individual employs, often unconsciously, to interpret new experiences and information to guide action [3]. Thus, teachers' beliefs could be included their conception of the nature of mathematics and mental models for learning mathematics.

Teachers' beliefs should be emphasized in several components. Among the key beliefs of mathematics teachers are views on the nature of mathematics, the nature of

mathematics teaching, and the process of learning mathematics [4]. Such views form a basis of mathematical philosophy in the system of beliefs toward teaching and learning mathematics. These views have a great impact on students' ability and beliefs about mathematics.

Undoubtedly the systems of beliefs among mathematics teachers play an important role in the learning process. Teachers' beliefs about mathematics and its teaching play a significant role in shaping teachers' characteristic patterns of instructional behavior [5]. This argument is supported by other, largely case study evidence [6], [7]. The interactive relation between teachers' beliefs and teaching practices has been an interested variable to many mathematics educators. There has been a great deal of research about teachers' beliefs system. Shealy studied how two pre service mathematics teachers shaped their belief about mathematics and how the beliefs affected their teaching of mathematics [8]. However, the contributions of beliefs seem not very often being measured in learning. Yet, to explore the beliefs numerically and rank it in according to their degrees of measurements is not impossible. One approach to be considered in dealing with measurements of beliefs is to look at the possibility of the application of fuzzy sets theory. Fuzzy sets theory provides a mathematical model in which vague conceptual phenomena can be precisely and rigorously studied. Fuzzy sets theory, in other words, provides a means to qualify the subjectivity of beliefs.

The rest of this paper will be organized as follows. For the sake of clarity, a brief definition of fuzzy sets is presented in Section II. Research questions and research design are presented in Section III and IV respectively. Results and discussion of the empirical study is presented in Section V. Finally, the paper is concluded in Section VI.

## II. FUZZINESS IN BELIEFS

The classical (crisp) set  $A$  in the universal set  $X$  is a collection of well-defined objects and is characterized by the function  $\chi_A: X \rightarrow \{0,1\}$  such that  $\chi_A(x) = 1$  if  $x \in A$  and  $\chi_A(x) = 0$  if  $x \notin A$ . However, in the real world, there are abundance of vagueness, impreciseness and uncertainties. Zadeh proposed the idea of a fuzzy set as an expansion of the classical set theory to deal with uncertainties [9]. A fuzzy set  $A$  in  $X$  is characterized by the membership function  $\mu_A: X \rightarrow [0,1]$  such that  $\mu_A(x)$  is the grade of membership of  $x$  with respect to  $A$ . Fuzzy set theory can represent the uncertainty or vagueness inherent in the definition of linguistic variables [10].

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Zimmerman summarized the development of fuzzy sets theory into two different categories [11]. One can see the fuzzy set theory as a formal theory which embraced the classical mathematical areas such as algebra, graph theory, and topology. On the other hand, fuzzy set theory is a powerful modeling language that can cope with a large fraction of uncertainties in real life situations. The nature of its generality, fuzzy set theory can be well-adapted to a variety of circumstances and contexts.

Fuzzy set theory has been applied to many areas such as social science [12], business, finance, management, economics and marketing [13]; [14]. There has been substantial research in cognitive psychology showing that fuzzy sets can be used to represent linguistic variables. One example of the application in assessment of education can be found in Weon and Kim [15]. Analogously, in this study, we apply fuzzy sets theory in education to focus on teachers' beliefs. We attempt to rank teachers' belief about mathematics into numerical form to reflect the strength of beliefs in the range of [0, 1].

### III. RESEARCH QUESTIONS

The main purpose of this research was to measure the mathematics teachers' beliefs about mathematics by using the Fuzzy Conjoint Model. Green & Srinivasan employed conjoint analysis in market research [16] and Turksen & Willson have modified the conjoint approach using fuzzy set to develop a fuzzy preference model for consumer choices and market share [17]. The measurement involved degree of consensus agreement in a preference questionnaire for each selected attribute. There were many attributes constitute in systems of belief. Carter and Norwood outlined teachers' beliefs about mathematics into three categories [18]. First category was beliefs about nature of mathematics, second category was beliefs about teaching of mathematics, and the third category was beliefs about learning mathematics. Without going in-depth for each category, this paper attempts to limit beliefs about mathematics into the first and third categories. Moreover, this paper does not intend to unravel the beliefs system but rather to apply a fuzzy model in describing the subjectivity of teachers' beliefs. There were three attributes sought to be measured in each category. In order to suit with the fuzzy conjoint model, degree of similarity was used rather than degree of consensus. Specifically, the research questions were:

1. To what degree of similarity, mathematics teachers believe (about nature of mathematics) that:
  - i. Mathematics is a field of manipulating numbers and symbols?
  - ii. Mathematics is a difficult subject?
  - iii. Mathematics is important in real life?
2. To what degree of similarity, the mathematics teachers believe (about learning mathematics) that:
  - i. Drills and practice is one of the best ways of learning mathematics?
  - ii. Mathematics is not just about numbers and symbols but a way of thinking using symbols

and equations to represent real world problems in a mathematical model?

- iii. Practicing continuously (persistence) with increases variations can leads to deep understanding of mathematics?

### IV. RESEARCH DESIGN

The respondents consist of 23 mathematics teachers from four secondary schools in the East Coast of Peninsula Malaysia. All the teachers graduated with a first degree in mathematics in addition to holding teaching diploma. All of them had at least three years experience in teaching mathematics at secondary level. The teachers were selected based on a voluntary basis. They volunteered to complete an instrument related to beliefs about nature of mathematics and learning mathematics. The design of the instrument was purely based on the fuzzy preference model. The preference data were then collected and analyzed using the fuzzy conjoint model.

#### A. Fuzzy Preference Model

Preference models were widely used in new product design, marketing management, and market segmentation [16]; [19]. However, there are abundance vague, uncertainty and very subjective nature in the element of preference. The linguistic variable was prevalent in both psychological attributes (for example 'difficult') and the subject preferences (for example 'agree'). The vagueness of the rating 'agree' is inherent rather than being due to a lack of knowledge about the available rating. The fuzzy set for 'strongly agree' would consist of element pairs, each a domain variable and a degree of membership. A membership function maps each value of the domain variable to a degree of membership or belongingness in the set that are ranging from 0 to 1.

A fuzzy preference model requires fuzzy membership function for each of linguistic ratings on the measurement scale. In this research, Likert scale was used for all preference ratings, providing the subject with a balanced selection of 7 linguistic terms to indicate their beliefs about mathematics. This scale has a central neutral evaluation with three positive and three negative evaluations. The underlying theory of fuzzy sets in the preference modeling can be retrieved further from [17]. Additionally, the appeal of using fuzzy sets in preference models comes from representing linguistic variables in a mathematical structure that closely corresponds to actual subject preferences. An overall preference for a statement can be decomposed into a combination of preferences for its constituent parts (attributes), which are combined using a combination function. A combination of preferences becomes the main underlying philosophy in the fuzzy conjoint model.

#### B. Fuzzy Conjoint Model

Turksen & Willson applied fuzzy set to develop an extended conjoint model which was known as The Fuzzy Conjoint Model [17]. A fuzzy set R is formed to represent the hierarchy of all respondents against the specific attributes. This approach gives a degree of consensus agreement for each selected attribute that was used in this study.

The approximate degree of membership for each element,  $y_j$  ( $y = 1, 2, 3, \dots, l$ ) in fuzzy set  $R$  is defined as

$$\mu_R(y_j, M) = \sum_{i=1}^n W_i \cdot \mu_{R_i}(x_j, M) \quad (1)$$

where

$\mu_{R_i}(x_j, M)$  : degree of membership of domain element  $x_j$  in the subject's linguistic rating  $R$  of the  $i$ -th attribute of  $M$  for each element in the fuzzy set  $R_i$ ,  
 $x_j = 1, 2, 3, \dots, l$ .

$R_i \in \{$  very strongly agree ( $L_1$ ), strongly agree ( $L_2$ ), agree ( $L_3$ ), undecided ( $L_4$ ),

Disagree ( $L_5$ ), strongly disagree ( $L_6$ ), very strongly disagree ( $L_7$ ) $\}$  by  $i$ -th respondent,  $i = 1, 2, 3, \dots, 23$  against attribute  $M$

$W_i$  : weight for  $i$ -th respondent and for  $W_i = \frac{w_i}{\sum_{k=1}^n w_k}$ , as  $w_i$

is a score of linguistic values given by  $i$ -th respondent.

$l$  : number of linguistic values used (in this study,  $l = 7$ )

$\mu_R(y_j, M)$  : approximate overall degree of membership of the linguistic value  $R$  for all factor  $M$  attributes based on domain value  $y_j = 1, 2, \dots, 23$ .

$M$  : factor attributes

$n$  : number of respondents

### C. Linguistic Variables

Zadeh introduced the application of linguistic variables [10]. In this study, there were seven linguistic variables that were used and defined as  $L_k = \{$ very strongly agree, strongly agree, agree, undecided, disagree, strongly disagree, very strongly disagree $\}$ . The fuzzy sets represented for each linguistic value,  $L_k$  ( $k = 1, 2, 3, 4, 5, 6, 7$ ) were defined as follow:

- Very strongly agree,  $L_1 = \{1/1, 0.8/2, 0.5/3, 0.2/4, 0/5, 0/6, 0/7\}$
- Strongly agree,  $L_2 = \{0.7/1, 1/2, 0.6/3, 0.4/4, 0/5, 0/6, 0/7\}$
- Agree,  $L_3 = \{0.4/1, 0.6/2, 1/3, 0.6/4, 0.4/5, 0/6, 0/7\}$
- Undecided,  $L_4 = \{0/1, 0.3/2, 0.7/3, 1/4, 0.7/5, 0.3/6, 0/7\}$
- Disagree,  $L_5 = \{0/1, 0.2/2, 0.4/3, 0.6/4, 1/5, 0.6/6, 0.4/7\}$
- Very disagree,  $L_6 = \{0/1, 0/2, 0/3, 0.4/4, 0.6/5, 1/6, 0.7/7\}$

Very strongly disagree,  $L_7 = \{0/1, 0/2, 0/3, 0.2/4, 0.5/5, 0.8/6, 1/7\}$

### D. Degree of Similarity

For each attribute,  $M$ , the calculation of degree of similarity between the fuzzy set representing the whole respondents with every fuzzy set that represented by seven linguistic values ( $L_k$ ,  $k = 1, 2, 3, 4, 5, 6, 7$ ) was executed using a distance formula. The degree of similarity was defined as the Euclidean distance between fuzzy set  $R$  and  $L_k$ . The formula for the similarity of two sets is

$$Si(R, L_k) = \frac{1}{[1 + \sqrt{\sum_{j=1}^7 (\mu_R(j, M) - \mu_{L_k}(j))^2}]} \quad (2)$$

### E. Measurement Procedures

There were six attributes to be considered and analyzed in this study. The calculations were executed based on the procedures as presented in Fig. 1.

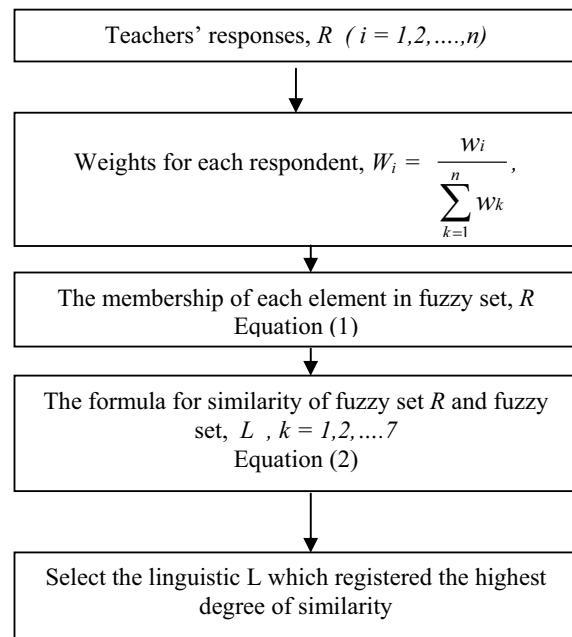


Fig. 1 Measuring procedures based on the fuzzy conjoint model

## V. RESULTS AND DISCUSSION

Measurements and discussions for each attribute were presented in accordance with the respective category.

1. Mathematics teachers believe (about nature of mathematics) that:

i. Mathematics is a field of manipulating numbers and symbols

Measurement outcomes:

$$\begin{aligned} Sim(R, L_1) &= 0.48205; Sim(R, L_2) = 0.52731; \\ Sim(R, L_3) &= 0.70414; Sim(R, L_4) = 0.64198; \\ Sim(R, L_5) &= 0.53350; Sim(R, L_6) = 0.41846; \\ Sim(R, L_7) &= 0.39966. \end{aligned}$$

$$\text{Therefore, } Sim(R, agree)_{max} = 0.70414$$

The degree of agreement about this attribute recorded at 'agree' with a 0.70414 degree of similarity. Teachers appeared to support the general belief of mathematics that relates mathematics with numbers and symbols. This finding could be discussed in line with the definition of mathematics. Despite the controversial debate in defining the nature of mathematics, Borasi revealed that mathematics was defined as a rigid set of formulae (symbols) by both teachers and students [20].

ii. Mathematics is a difficult subject

Measurement outcomes:

$$\begin{aligned} Sim(R, L_1) &= 0.38168; Sim(R, L_2) = 0.38965; \\ Sim(R, L_3) &= 0.43047; Sim(R, L_4) = 0.49123; \\ Sim(R, L_5) &= 0.66163; Sim(R, L_6) = 0.67188; \\ Sim(R, L_7) &= 0.64016. \end{aligned}$$

$$\text{Therefore, } Sim(R, strongly disagree)_{max} = 0.67188$$

To many laymen, mathematics is considered one of the difficult subjects in schools. However, teachers with vast experiences in teaching mathematics appeared not to have shared the same views. Teachers were strongly disagreed that mathematics is one of the difficult subjects. The finding should be viewed as an advantage to the students. The teachers' positive perceptions about the difficulty of mathematics will eventually be transferred to the students along the process of imparting knowledge.

iii. Mathematics is important in real life

Measurement outcomes:

$$\begin{aligned} Sim(R, L_1) &= 0.60252; Sim(R, L_2) = 0.69812; \\ Sim(R, L_3) &= 0.71327; Sim(R, L_4) = 0.64918; \\ Sim(R, L_5) &= 0.50543; Sim(R, L_6) = 0.43442; \\ Sim(R, L_7) &= 0.37278. \end{aligned}$$

$$\text{Therefore, } Sim(R, agree)_{max} = 0.71327$$

The highest and the second highest degree of similarity values were registered at 'agree' and 'strongly agree'. Teachers stand collaboratively on the importance of mathematics in real life and agreed that mathematics provides foundation for seeking knowledge in applied sciences. Reference [21] echoed with this finding. The mathematicians agreed upon the importance of mathematics in real life.

2. Mathematics teachers believe (about learning mathematics) that:

i. 'Drills and practice' is one of the best ways of learning mathematics.

Measurement outcomes:

$$\begin{aligned} Sim(R, L_1) &= 0.67595; Sim(R, L_2) = 0.78960; \\ Sim(R, L_3) &= 0.64889; Sim(R, L_4) = 0.45466; \\ Sim(R, L_5) &= 0.40239; Sim(R, L_6) = 0.35446; \\ Sim(R, L_7) &= 0.35048. \end{aligned}$$

$$\text{Therefore, } Sim(R, strongly agree)_{max} = 0.78960$$

The maximum value of degree of similarity for this attribute registered at the value of 0.78960 with linguistic value in the range of 'strongly agree'. The measurement showed that the respondents strongly agreed that 'drill and practice' is one of the best ways of learning mathematics. Despite of the fact that 'drill and practice' was not clearly defined, most mathematicians agreed that 'drills and practice' is not the same as the rote learning [22]. In mathematics 'drill and practice' can increase the conceptual understanding of a topic. The views of the mathematicians appeared to be shared by this group of teachers. Their beliefs about this attribute recorded at 'strongly agree' followed by 'very strongly agree' of linguistics variable pertaining the 'drill and practice' as one of the ways in learning mathematics.

ii. Mathematics is not just numbers and symbols but a way of thinking using symbols and equations to represent real world problems in a mathematical model.

Measurement outcomes:

$$\begin{aligned} Sim(R, L_1) &= 0.52399; Sim(R, L_2) = 0.55856; \\ Sim(R, L_3) &= 0.52450; Sim(R, L_4) = 0.46621; \\ Sim(R, L_5) &= 0.44040; Sim(R, L_6) = 0.40846; \\ Sim(R, L_7) &= 0.38539. \end{aligned}$$

$$\text{Therefore, } Sim(R, strongly disagree)_{max} = 0.55856$$

As an extension of criterion 1 (i), this criterion again sought to tap teachers' beliefs about the application of mathematics as a mathematical modeling that intentionally represents the real situations. Respondents strongly agreed at 0.55856 degree of similarity that mathematics symbols can be pursued to the mathematical modeling as parts of explaining the real phenomena. This finding supported the instrumentalist view that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end [23]. Also, this finding seemed to echo what Lim reported in her research [22]. She found that among those who reported liking mathematics, in particular, the mathematics teachers and mathematicians tend to hold a problem solving view. They tended to relate to solving problems and mental works.

iii. Practicing continuously (persistence) with increasing variations can lead to deep understanding of mathematics.

Measurement outcomes:

$Sim(R, L_1) = 0.64275$  ;  $Sim(R, L_2) = 0.71016$  ;  
 $Sim(R, L_3) = 0.66899$  ;  $Sim(R, L_4) = 0.48530$  ;  
 $Sim(R, L_5) = 0.43672$  ;  $Sim(R, L_6) = 0.37990$  ;  
 $Sim(R, L_7) = 0.37389$ .

Therefore,  $Sim(R, \text{strongly agree})_{max} = 0.71016$

This criterion taps the teachers' beliefs about the effective ways of learning mathematics. Respondents strongly agreed that persistence in doing mathematics will lead to a better understand of mathematics. This finding could be in line with the views among Malaysian mathematicians [21]. They agreed that mathematics do not require much memorization compared to other sciences. Working with many variations of mathematics problems will enhance understanding in mathematics.

## VI. CONCLUDING REMARKS

Normally, vague entities such as systems of beliefs are very hard to explain in numerical form or rather to measure with precise computation figures. Despite the vagueness of beliefs, the study has provided new insight about measuring the beliefs of mathematics teachers. The use of fuzzy sets theory for the mathematical representation leads to useful quantitative conclusions. There are two conclusive remarks that could be drawn from the study. First, the unbounded and broad spectrum application of fuzzy sets enables the mathematics teachers' belief to be transformed into grades of memberships which reflect the strength of beliefs. The analogous application of fuzzy sets theory has been used on measuring the model of teaching in Euclidean Geometry [24]. Another example of measuring psychological variables can be retrieved in [25]. Second, the degree of similarity offers a significant value that possibly could be conjoined further in their respective category (see [26],[27]). Apart from the degree of agreement in the selected attribute, the numerical conclusions can provide extra information to educators and researchers in regarding the strength of beliefs about mathematics among mathematics teachers.

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