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Trajectory Control of a Robotic Manipulator Utilizing an Adaptive Fuzzy Sliding Mode

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***Abstract*—**In this paper, a novel adaptive fuzzy sliding mode control method is proposed for the robust tracking control of robotic manipulators. The proposed controller possesses the advantages of adaptive control, fuzzy control, and sliding mode control. First, system stability and robustness are guaranteed based on the sliding mode control. Further, fuzzy rules are developed incorporating with adaptation law to alleviate the input chattering effectively. Stability of the control system is proven by using the Lyapunov method. An application to a three-degree-of-freedom robotic manipulator is carried out. Accurate trajectory tracking as well as robustness is achieved. Input chattering is greatly eliminated.

***Keywords*—**Fuzzy control, sliding mode control, robotic manipulator, adaptive control.

I. Introduction

ROBOTIC manipulators are complicated nonlinear dynamical systems with inherent unmodeled dynamics and unstructured uncertainties. These dynamical uncertainties make the controller design for manipulators a difficult task in the framework of classical control method. Therefore, the development of intelligent control for robotic manipulators has received considerable interest. Many control techniques have been proposed for robotic manipulators, such as the adaptive control, the sliding mode control, the fuzzy control, etc [1-6].

In general, the adaptive control method has a fixed structure, and is very effective in coping with structured uncertainties and maintaining a uniformly good performance over a limited range. It could not solve the problem of unstructured uncertainties [5]. The sliding mode control method is a nonlinear control scheme that is effective in dealing with the parameter variations and external disturbances [6-9]. However, the input chattering problem is a major drawback. The boundary layer technique can be used to avoid chattering phenomena. However, the cost of this technique is a reduction of the accuracy of the tracking performance [10, 11].

One of the most popular intelligent control approaches is the fuzzy control (FC). The merit of the fuzzy control is that it can explicitly use human knowledge and experience in its control strategy. Fuzzy control using linguistic information possesses several advantages such as robustness, model free, universal approximation theorem, and rule-based algorithm [12]. Thus, FC methods have attracted more attention to deal with the complex control problem such as robots, chaotic systems, servo systems, and so on [13–15]. Fuzzy logic (FL), which was first proposed by Zadeh [16], has proven to be a valid method for controlling ill-defined or parameter-variant plants. Very often, the approximation capabilities of a fuzzy logic system (FLS) are used [17] for compensating the unknown dynamics such as model uncertainties [18], and an influence of the friction and payload variation [19]. The drawback of FC is that there is no rigorous analysis of stability for the general fuzzy control systems [4].

To overcome the demerits and to take advantage of the attractive features of conventional control and intelligent control, this paper proposes an adaptive fuzzy sliding mode controller (AFSMC) for the robust tracking control of robotic manipulators. Besides the advantage of stability and robustness of sliding mode control is maintained, the proposed method suppresses the input chattering when the control system is in the sliding mode. Further, an adaptive tuning algorithm is developed so that the control parameters can be adjusted online. Therefore, better control result could be obtained. The stability and the convergence of the tracking error are guaranteed by using the Lyapunov method. An application to a three-degree-of-freedom SCARA robotic manipulator is carried out. Simulation results demonstrate that tracking error is eliminated rapidly. Satisfactory tracking performance and robustness are achieved effectively.

II. Control Methodology

A. Dynamic Model

The dynamic model of an *n*-link robotic manipulator can be expressed as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) + \mathbf{F}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{u}(\mathbf{t}), \tag{1}$$

where $\mathbf{q} \in R^n$ is the joint position, $\dot{\mathbf{q}} \in R^n$ is the joint velocity vector, $\ddot{\mathbf{q}} \in R^n$ is the joint acceleration vector; $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \in R^{n\times n}$ stands for the Coriolis and centrifugal torques, $\mathbf{M}(\mathbf{q}) \in R^{n\times n}$ is the inertia matrix, $\mathbf{G}(\mathbf{q}) \in R^n$ is the gravity vector; $\mathbf{F}(\mathbf{q},\dot{\mathbf{q}}) \in R^{n\times n}$ is the unstructured uncertainties of the dynamics including other disturbances and exterior friction; and $\mathbf{u}(\mathbf{t}) \in R^{n\times n}$ is the control input vector representing the torque exerting on joints.

The following properties for the dynamics of a robotic manipulator are required [20]:

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Property 1: The inertia matrix $\mathbf{M}(\mathbf{q})$ is symmetric and positive definite. It is also bounded as a function of $\mathbf{q}$: $m_1 \le \|\mathbf{M}(\mathbf{q})\| \le m_2$, $\forall \mathbf{q} \in R^n$, where m_1 and m_2 are positive constants.

Property 2: $\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is a skew symmetric matrix, and satisfies

$$\mathbf{x}^T \left[\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\right]\mathbf{x} = 0 , \forall \mathbf{x} \in R^n , \tag{2}$$

where $\mathbf{x}$ is an $n \times 1$ nonzero vector.

Property 3: The gravity vector $\mathbf{G}(\mathbf{q})$ is bounded, and

$$\|\mathbf{G}(\mathbf{q})\| \le g_b , \forall \mathbf{q} \in R^n , \tag{3}$$

where g_b is a known positive constant.

B. Sliding Mode Control

Consider the sliding mode control system as shown in Fig. 1. The objective is to drive the joint position vector $\mathbf{q}$ to track the desired position vector $\mathbf{q_d}$. Let the tracking error vector be

$$\mathbf{e} = \mathbf{q} - \mathbf{q_d} , \tag{4}$$

where $\mathbf{e} = [e_1, e_2, \cdots, e_n]^T$. Define the sliding surface function as

$$\mathbf{s} = \dot{\mathbf{e}} + \mathbf{\Lambda e} , \tag{5}$$

where $\mathbf{\Lambda} = diag[\lambda_1, \lambda_2, \cdots, \lambda_m]$, λ_i , $i = 1, 2, \cdots, m$, are positive constants. Differentiating (5) with respect to time, one can obtain

$$\dot{\mathbf{s}} = \ddot{\mathbf{q}} - \ddot{\mathbf{q}}_\mathbf{d} + \mathbf{\Lambda}\dot{\mathbf{e}} . \tag{6}$$

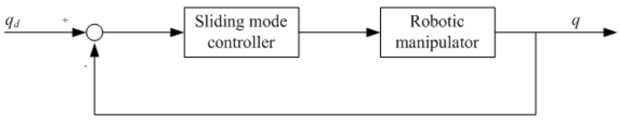

Fig. 1 Block diagram of the control system with SMC

Let the control input be

$$\mathbf{u}(t) = \mathbf{u}_0 + \mathbf{u}_s , \tag{7}$$

$$\mathbf{u}_0 = \hat{\mathbf{M}}(\ddot{\mathbf{q}}_\mathbf{d} - \mathbf{\Lambda}\dot{\mathbf{e}}) + \hat{\mathbf{C}}(\dot{\mathbf{q}}_\mathbf{d} - \mathbf{\Lambda e}) + \hat{\mathbf{G}} - \mathbf{As} , \tag{8}$$

$$\mathbf{u}_s = -\mathbf{K}\,\mathrm{sgn}(\mathbf{s}) , \tag{9}$$

where $\mathbf{A} = diag[a_1, a_2, \cdots, a_n]$ is a diagonal positive definite matrix, and $a_i > 1$, $i = 1, 2, \cdots, n$. In (8), $\hat{\mathbf{M}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{G}}$ are the estimation of $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{G}$, respectively. Let $\mathbf{M} = \hat{\mathbf{M}} + \Delta\mathbf{M}$, $\mathbf{C} = \hat{\mathbf{C}} + \Delta\mathbf{C}$, and $\mathbf{G} = \hat{\mathbf{G}} + \Delta\mathbf{G}$, where $\Delta(.)$ stands for the estimation error. In order to simplify the equation, we define $\Delta\mathbf{f}$ as a lumped function with all the uncertainties of the *n*-link robot manipulator (1), i.e., $\Delta\mathbf{f} = -(\Delta\mathbf{M}(\ddot{\mathbf{q}}_\mathbf{d} - \mathbf{\Lambda}\dot{\mathbf{e}}) + \Delta\mathbf{C}(\dot{\mathbf{q}}_\mathbf{d} - \mathbf{\Lambda e}) + \Delta\mathbf{G} + \mathbf{F})$. Let Δf_i be the *i*th row of $\Delta\mathbf{f}$. Suppose that Δf_i is bounded, i.e., $|\Delta f_i| < |\Delta f_i|_{bound}$. Choose $\mathbf{K} = diag[k_1, k_2, \cdots, k_n]$, where $k_i > |\Delta f_i|_{bound}$, $i = 1, 2, \cdots, n$.

Define a Lyapunov function candidate as

$$V = \frac{1}{2}\mathbf{s}^\mathbf{T}\mathbf{Ms} . \tag{10}$$

Differentiating (10) yields

$$\dot{V} = \mathbf{s}^\mathbf{T}\mathbf{M}\dot{\mathbf{s}} + \frac{1}{2}\mathbf{s}^\mathbf{T}\dot{\mathbf{M}}\mathbf{s} . \tag{11}$$

Applying (1), (4) and (6) to (11), together with Property 2 in (2), implies

$$\begin{aligned}\dot{V} &= \mathbf{s}^\mathbf{T}\mathbf{M}(\ddot{\mathbf{q}} - \ddot{\mathbf{q}}_\mathbf{d} + \mathbf{\Lambda}\dot{\mathbf{e}}) + \frac{1}{2}\mathbf{s}^\mathbf{T}(2\mathbf{C})\mathbf{s} \\ &= \mathbf{s}^\mathbf{T}(\mathbf{u} - \mathbf{M}(\ddot{\mathbf{q}}_\mathbf{d} - \mathbf{\Lambda}\dot{\mathbf{e}})) - \mathbf{s}^\mathbf{T}(\mathbf{C}(\dot{\mathbf{q}}_\mathbf{d} - \mathbf{\Lambda e}) + \mathbf{G} + \mathbf{F}) + \mathbf{s}^\mathbf{T}\mathbf{Cs} .\end{aligned} \tag{12}$$

Substituting (7), (8) and (9) into (12), one obtains

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n} s_i[\Delta f_i - k_i\,\mathrm{sgn}(s_i)] - \mathbf{s}^\mathbf{T}\mathbf{As} \\ &< \sum_{i=1}^{n} |s_i|(|\Delta f_i| - k_i) - \mathbf{s}^\mathbf{T}\mathbf{As} .\end{aligned} \tag{13}$$

Because $\mathbf{A}$ is a positive definite matrix and $-\mathbf{s}^\mathbf{T}\mathbf{As} < 0$, thus,

$$\dot{V} < 0 . \tag{14}$$

Equation (14) promises the decay of the energy of $\mathbf{s}$ as long as $\mathbf{s} \neq 0$. Thus, the tracking control is guaranteed.

C. Adaptive Fuzzy Sliding Mode Control

In (9), it is not easy to obtain the value of $\mathbf{K}$ because the determination of $\mathbf{K}$ depends on the bound of uncertainties. Although a large value of $\mathbf{K}$ can overcome the effect of uncertainties, it causes input chattering. In order to deal with this problem, as shown in Fig. 2, an adaptive fuzzy sliding mode control is proposed. A fuzzy control gain is to replace the switching control input gain. In this paper, the fuzzy controller consists of fuzzifier, fuzzy rule base, fuzzy inference engine, and defuzzifier. The output of the fuzzy system can be described as

$$y = \frac{\sum_{j=1}^{l} \theta^j \prod_{i=1}^{n} \mu_{A_i^j}(x_i^*)}{\sum_{j=1}^{l} \prod_{i=1}^{n} \mu_{A_i^j}(x_i^*)} , \tag{15}$$

where $\theta^j = [\theta^1, \theta^2, \cdots, \theta^l]^T$ is the vector of the centers of output fuzzy variable and $\mu_{A_i^j}(x_i^*)$ is the weighting of the membership function. Let $\Psi(x) = [\psi^1(x), \psi^2(x), \cdots, \psi^l(x)]^T$, where $\psi^l(x) = \dfrac{\prod_{i=1}^{n} \mu_{A_i^j}(x_i^*)}{\sum_{j=1}^{l} \prod_{i=1}^{n} \mu_{A_i^j}(x_i^*)}$, and l be the number of fuzzy rules. Note that the input chattering is induced from the discontinuous function $\mathrm{sgn}(\mathbf{s})$ and the constant value of $\mathbf{K}$. Now let the fuzzy gain vector $\mathbf{K}_f$ replace the control gain $\mathbf{K}$. The fuzzy gain vector $\mathbf{K}_f$ is defined as

$\mathbf{K}_f = [k_{f1}, k_{f2}, \cdots, k_{fn}]^T$, where k_{fi} is the output of the fuzzy controller. A shifted sigmoid function [21] is used to further enhance the elimination of the chattering, i.e.

$$h(s_i) = \frac{2}{1+e^{-s_i}} - 1 . \tag{16}$$

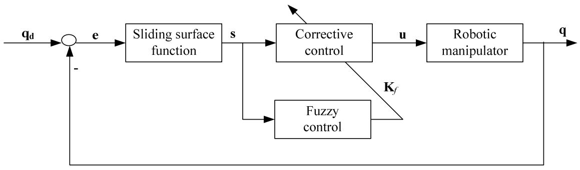

Fig. 2 Block diagram of the control system with AFSMC

Hence, the control input becomes

$$\mathbf{u(t)} = \hat{\mathbf{M}}\ddot{\mathbf{q}} + \hat{\mathbf{C}}\dot{\mathbf{q}} + \hat{\mathbf{G}} - \mathbf{As} - \mathbf{K}_f h(\mathbf{s}) . \tag{17}$$

The fuzzy rules are in the following simple format:

IF s_i is A_i^l , THEN k_{fi} is B_i^l

where A_i^l and B_i^l are fuzzy sets. Define the membership functions as NB, NM, NS, ZE, PS, PM, PB, where N represents negative, P represents positive, B represents big, M represents medium, S represents small, and ZE represents zero. Table I shows the fuzzy rules.

TABLE I
FUZZY RULE TABLE

s_i	NB	NM	NS	ZE	PS	PM	PB
k_{fi}	NB	NM	NS	ZE	PS	PM	PB

The membership functions are chosen to be Gaussian functions, i.e.

$$\mu_{A_i^l}(s_i) = \exp\left(-\left(\frac{s_i - \alpha_i}{\sigma_i}\right)^2\right), \tag{18}$$

where σ_i is the width and α_i is the center. The parameters of the membership function of s_i are pre-defined. The value of k_{fi} is on-line updated. Therefore, the controller is an adaptive controller. The defuzzification k_{fi} can be described as

$$k_{fi} = \frac{\sum_{j=1}^{l} \theta_{k_i}^j \mu_{A^j}(s_i)}{\sum_{j=1}^{l} \mu_{A^j}(s_i)} = \Theta_{k_i}^T \Psi_{k_i}(s_i) , \tag{19}$$

where $\mathbf{\Psi}_{k_i}(s_i) = [\psi_{k_i}^1(s_i), \psi_{k_i}^2(s_i), \cdots, \psi_{k_i}^l(s_i)]^T$, $\psi_{k_i}^j(s_i) = \frac{\mu_{A^j}(s_i)}{\sum_{j=1}^{l}\mu_{A^j}(s_i)}$, and $\Theta_{k_i} = [\theta_{k_i}^1, \theta_{k_i}^2, \cdots, \theta_{k_i}^l]^T$. Define $\Theta_{k_{id}}^T$ so that $k_{fi} = \Theta_{k_{id}}^T \Psi_{k_i}(s_i)$ is the optimal compensation for Δf_i. According to the Wang's theorem [12, 22], there exists $\omega_i > 0$ satisfying

$$\left|\Delta f_i - \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i)\right| \le \omega_i , \tag{20}$$

where ω_i can be as small as possible. Now define the estimation error as

$$\tilde{\Theta}_{k_i} = \Theta_{k_i} - \Theta_{k_{id}} . \tag{21}$$

Equation (19) can be rewritten as

$$k_{fi} = \tilde{\Theta}_{k_i}^T \mathbf{\psi}_{k_i}(s_i) + \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i) . \tag{22}$$

Consequently, let the adaptive law to be

$$\dot{\Theta}_{k_i} = s_i \mathbf{\psi}_{k_i}(s_i) . \tag{23}$$

Next, we verify the control law. Let the Lyapunov function candidate be

$$V_L = \frac{1}{2}(\mathbf{s^T M s} + \sum_{i=1}^{n} \tilde{\Theta}_{k_i}^T \tilde{\Theta}_{k_i})^2 . \tag{24}$$

Differentiating the equation (24) yields

$$\dot{V}_L = \frac{1}{2}\left[\dot{\mathbf{s}}^\mathbf{T}\mathbf{Ms} + \mathbf{s^T}\dot{\mathbf{M}}\mathbf{s} + \mathbf{s^T M}\dot{\mathbf{s}} + \sum_{i=1}^{n}\left(\dot{\tilde{\Theta}}_{k_i}^T \tilde{\Theta}_{k_i} + \tilde{\Theta}_{k_i}^T \dot{\tilde{\Theta}}_{k_i}\right)\right]$$

$$= \mathbf{s^T M}\dot{\mathbf{s}} + \sum_{i=1}^{n} \tilde{\Theta}_{k_i}^T \dot{\tilde{\Theta}}_{k_i}$$

$$= \mathbf{s^T}[-\mathbf{As} + \Delta\mathbf{f} - \mathbf{K}_f] + \sum_{i=1}^{n} \tilde{\Theta}_{k_i}^T \dot{\tilde{\Theta}}_{k_i}$$

$$= -\mathbf{s^T As} + \sum_{i=1}^{n}(s_i[\Delta f_i - k_{fi}]) + \sum_{i=1}^{n} \tilde{\Theta}_{k_i}^T \dot{\tilde{\Theta}}_{k_i} \tag{25}$$

Submitting (22) to (25) yields

$$\dot{V}_L = -\mathbf{s^T As} + \sum_{i=1}^{n}(s_i[\Delta f_i - \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i)])$$

$$+ \sum_{i=1}^{n} \tilde{\Theta}_{k_i}^T(-s_i \mathbf{\psi}_{k_i}(s_i) + \dot{\tilde{\Theta}}_{k_i}) . \tag{26}$$

Applying $\dot{\Theta}_{k_i} = \dot{\tilde{\Theta}}_{k_i}$ and adaptive law equation (23) to (26), the derivative of Lyapunov function becomes

$$\dot{V}_L = -\mathbf{s^T As} + \sum_{i=1}^{n}(s_i[\Delta f_i - \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i)])$$

$$\le -\mathbf{s^T As} + \sum_{i=1}^{n}(|s_i|\left|\Delta f_i - \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i)\right|) . \tag{27}$$

Following (20), ω_i can be chosen as small as possible such that

$$\left|\Delta f_i - \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i)\right| \le \omega_i \le \gamma_i |s_i| , \tag{28}$$

where $0 < \gamma_i < 1$. Further, multiplying (28) with s_i yields

$$|s_i|\left|\Delta f_i - \Theta_{k_{id}}^T \mathbf{\psi}_{k_i}(s_i)\right| \le \gamma_i |s_i|^2 = \gamma_i s_i^2 . \tag{29}$$

Hence, one can obtain

$$\dot{V} \le -\mathbf{s^T As} + \sum_{i=1}^{n} \gamma_i s_i^2 . \tag{30}$$

The right hand side of (30) is

$$\sum_{i=1}^{n}(-a_i+\gamma_i)s_i^{\,2}=-\mathbf{s}^{\mathbf{T}}(\mathbf{A}-\mathbf{\Gamma})\mathbf{s}\,, \tag{31}$$

where $\mathbf{\Gamma}=diag[\gamma_1,\gamma_2,\cdots,\gamma_n]$. Since $a_i>\gamma_i$, $i=1,2,\cdots,n$, and $(\mathbf{A}-\mathbf{\Gamma})$ is a positive matrix, it is clear that

$$\dot{V}\le\sum_{i=1}^{n}-a_is_i^{\,2}+\gamma_is_i^{\,2}=-\mathbf{s}^{\mathbf{T}}(\mathbf{A}-\mathbf{\Gamma})\mathbf{s}\le 0\,. \tag{32}$$

In (32), $(\mathbf{A}-\mathbf{\Gamma})$ is a positive matrix. The situation of $\dot{V}=0$ implies $\mathbf{s}=0$. Therefore, the adaptive control law guarantees the existence of sliding mode. In order words,

$$\lim_{t\to\infty}\mathbf{s}=\lim_{t\to\infty}(\dot{\mathbf{e}}+\mathbf{\Lambda e})=0\,. \tag{33}$$

This implies

$$\lim_{t\to\infty}\mathbf{q}=\mathbf{q_d}\,. \tag{34}$$

Therefore, the proposed adaptive fuzzy sliding mode control law indeed achieves the tracking control effectively.

III. Simulation Results

A three-degree-of-freedom SCARA robotic manipulator control system is applied to verify the advantage of the proposed control method. The dynamic equation of the considered robotic manipulator is acquirable by using the Euler-Lagrange method as presented in [21]. The desired trajectories of the three-degree-of-freedom SCARA robotic manipulator are

$$q_{d1}=3+0.1(\sin(t)+\sin(2t))\,, \tag{35}$$

$$q_{d2}=3+0.1(\sin(t)+\sin(3t))\,, \tag{36}$$

$$q_{d3}=3+0.1(\sin(t)+\sin(4t))\,. \tag{37}$$

The proposed AFSMC is applied. The results are shown in Figs. 3 to 6. In the simulation, $\mathbf{A}=diag[1,1,1]$ and $\mathbf{\Lambda}=diag[50,100,400]$ are chosen. As seen in Fig. 3, all the joint angles track the desired trajectories. The tracking performance is very good and maintained very well as shown in Fig. 4. The sliding function responses are shown in Fig. 5. All the sliding modes are reached and maintained in less than 0.5 sec. Since the adaptive tuning algorithm with fuzzy logic is applied, the sliding surface functions are very smooth. It is worth noting that smooth sliding surface function implies smooth control input. Referring to Fig. 6, the control input chattering is effectively eliminated by using the proposed AFSMC.

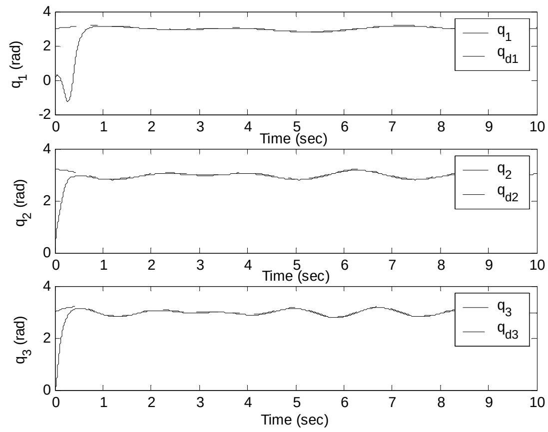

Fig. 3 Joint angle tracking response with AFSMC

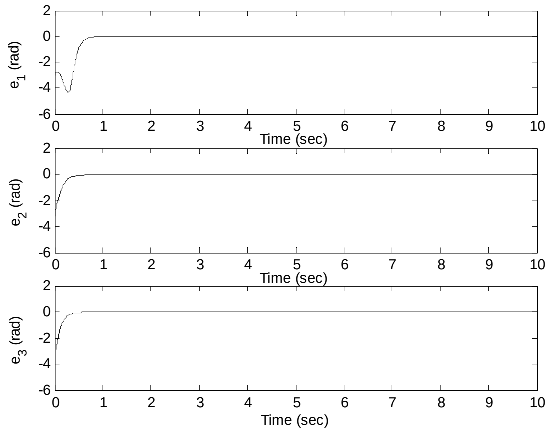

Fig. 4 Tracking error performance with AFSMC

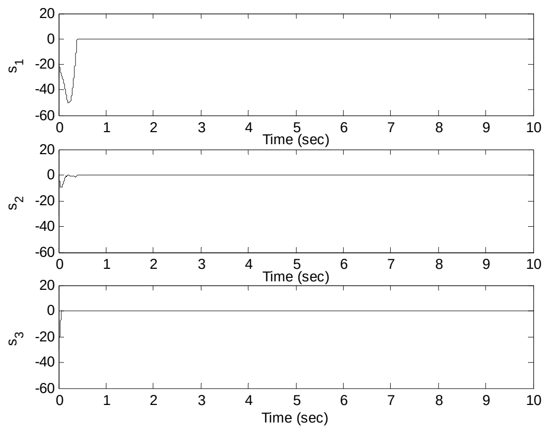

Fig. 5 Sliding surface function response with AFSMC

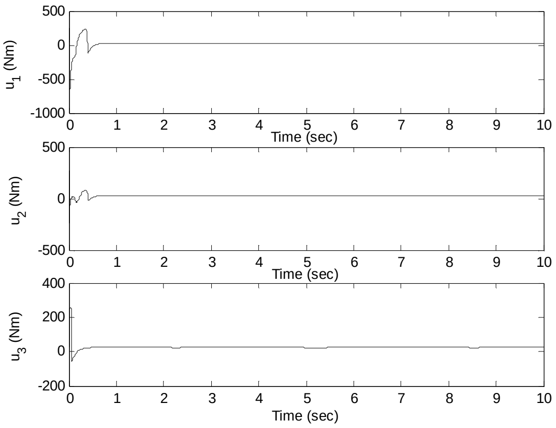

Fig. 6 Control input performance with AFSMC

IV. Conclusion

This paper proposes an adaptive fuzzy sliding mode control method for the robust trajectory control of robotic manipulators. Consider that a large control input switching gain usually causes the chattering problem when the sliding mode controller works alone. In this study, an adaptive tuning algorithm with fuzzy logic is developed to online adjust the switching input gain. Chattering problem in the sliding mode is thus overcome. The convergence and stability of the proposed control system are proved by using the Lyapunov method. Computer simulation of a three-degree-of-freedom robotic manipulator is carried out. Simulation results show that the control input chattering can be eliminated by using the proposed adaptive fuzzy sliding mode control method. Robust tracking performance is achieved effectively.

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References

[1] P. R. W. Ouyang, J. Zhang, and M. M. Gupta, "An adaptive switching learning control method for trajectory tracking of robot manipulators," *Mechatronics*, vol. 16, no. 1, pp. 51-61, 2006.
[2] Z. Qi, J. E. McIrony, and F. Jafari, "Trajectory tracking with parallel robots using low chattering, fuzzy sliding mode controller," *J. Intell. Robot. Syst.*, vol. 48, no. 3, pp. 333-356, 2007.
[3] V. Parra-Vega, S. Arimoto, Y. H. Liu, G. Hirzinger, and P. Akella, "Dynamic sliding PID control for tracking of robot manipulators: theory and experiments," *IEEE Trans. Robot. Automat.*, vol. 19, no. 6, pp. 967-976, 2003.
[4] H. Hu and P. Y. Woo, "Fuzzy supervisory sliding-mode and neural-network control for robotic manipulators," *IEEE Trans. Ind. Electron.*, vol. 53, no. 3, pp. 929-940, 2006.
[5] H. F. Ho, Y. K. Wong, and A. B. Rad, "Robust fuzzy tracking control for robotic manipulators," *Sim. Mod. Pract. Theory*, vol. 15, no. 7, pp. 801-816, 2007.
[6] A. Ferrara and L. Magnani, "Motion control of rigid robot manipulators via first and second order sliding mode," *J. Intell. Robot. Syst.*, vol. 48, no. 1, pp. 23-36, 2007.
[7] V. I. Utkin, *Sliding Mode in Control and Optimization*. Springer-Verlag, New York, 1992.
[8] J. Y. Hung, W. Gao, and J. C. Hung, "Variable structure control: a survey," *IEEE Trans. Ind. Electron.*, vol. 40, no. 1, pp. 2-22, 1993.
[9] K. D. Young, V. I. Utkin, and Ü. Özgüner, "A control engineer's guide to sliding mode control," *IEEE Trans. Contr. Syst. Tech.*, vol. 7, no. 3, pp. 328-342, 1999.
[10] J. J. E. Slotine and W. Li, *Applied Nonlinear Control. Prentice-Hall*, Englewood Cliffs, NJ, 1991.
[11] P. Guan, X. J., Liu, and J. Z. Liu, "Adaptive fuzzy sliding mode control for flexible satellite," *Eng. Appl. Arti. Intell.*, vol. 18, no. 4, pp. 451-459, 2005.
[12] L. X. Wang, *A Course in Fuzzy Systems and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1997.
[13] L. Astudillo, O. Castillo, P. Melin, A. Alanis, J. Soria, and L. T. Aguilar, "Intelligent control of an autonomous mobile robot using type-2 fuzzy logic," *Engineering Letters*, vol. 13, no. 2, pp. 93-97, 2006.
[14] H. N. Wu and M. Z. Bai, "Active fault-tolerant fuzzy control design of nonlinear model tracking with application to chaotic systems," *IET Control Theory & Applications*, vol. 3, no. 6, pp. 642-653, 2009.
[15] R. E. Precup, S. Preitl, I. J. Rudas, M. L. Tomescu, and J. K. Tar, "Design and experiments for a class of fuzzy controlled servo systems," *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 1, pp. 22-35, 2008.
[16] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, pp. 338–353, 1965.
[17] O. Kaynak, K. Erbatur, and M. Ertugrul, "The fusion of computationally intelligent methodologies and sliding mode control, a survey," *IEEE Trans. Ind. Electron.*, vol. 48, no. 1, pp. 4–17, Feb. 2001.
[18] C. Hwang and C. Kuo, "A stable fuzzy sliding mode control for affine nonlinear system with application to four-bar linkage system," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 238–252, Apr. 2001.
[19] B. K. Yoo and W. C. Ham, "Adaptive control of robot manipulator using fuzzy compensator," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 2, pp. 186–199, 2000.
[20] F. L. Lewis, C. T. Abdallah, and D. M. Dawson, *Control of Robot Manipulators.* Macmillan New York, 1993.
[21] A. G. AK and G. Cansever, "Three link robot control with fuzzy sliding mode controller based on RBF neural network," *Proc. 2006 IEEE Int. Symp. Intell. Contr.*, pp. 2719-2724, 2006,
[22] Y. Guo and P. Y. Woo, "An adaptive fuzzy sliding mode controller for robotic manipulators," *IEEE Trans. Syst. Man Cybern.*, vol. 33, no. 2, 149-159, 2003.