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Big Bang – Big Crunch Optimization Method in Optimum Design of Complex Composite Laminates

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Abstract—An accurate optimal design of laminated composite structures may present considerable difficulties due to the complexity and multi-modality of the functional design space. The Big Bang – Big Crunch (BB-BC) optimization method is a relatively new technique and has already proved to be a valuable tool for structural optimization. In the present study the exceptional efficiency of the method is demonstrated by an example of the lay-up optimization of multilayered anisotropic cylinders based on a three-dimensional elasticity solution. It is shown that, due to its simplicity and speed, the BB-BC is much more efficient for this class of problems when compared to the genetic algorithms.

Keywords—Big Bang – Big Crunch method, optimization, composite laminates, pressure vessel.

I. Introduction

A cylindrical pressure vessels is one of the most commonly used structures made of composite materials. Such shells are used as reservoirs, chemical containers, pipes, aircraft and ship elements. Along with the stress-strain analysis, the design optimization of filament-wound pressure vessels is of considerable industrial interest in view of the resulting material and weight savings. The multidimensional design of anisotropic pressure vessels presents considerable difficulties due to the complexity, multimodality, as well as very high sensitivity to the change in parameters, of the functional design space.

Stress analysis and design optimization of composite pressure vessels have been studied by many authors. The present exact three-dimensional solution was developed in [11] and is based on theoretical works by Lekhnitskii [8], [9] and Mitinskii [10], who considered the case of single-layered anisotropic cylinder under internal and external pressures. In the present paper, the design of laminated composite pressure vessels is based on the use of the Big Bang – Big Crunch (BB-BC) optimization algorithm. The BB-BC optimization method was proposed by Erol and Eksin [2] in 2006 as a new evolutionary algorithm. According to the authors, the BB-BC algorithm relies on one of the evolution theories of the universe, namely the Big Bang – Big Crunch theory. In the Big Bang phase the population of feature vectors is randomly fills the space, while in the Big Crunch phase these points are drawn into a dense cluster with the center of gravity being the optimum solution of the optimization problem. Another version of this approach, called “Big Crunch” optimization method, was given by Kripka *et al.* [6], where the Universal Gravitation Law (derived by Newton) was incorporated into the algorithm.

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The BB-BC method has quickly demonstrated its superiority over other heuristic population-based search techniques when employed to perform structural optimization tasks. For example, for the optimal design of space trusses [1], [4], skeletal structures [5], for parameter estimation in structural systems [12]. Other reported successful applications of the BB-BC algorithm is fuzzy system control [7] and automatic target tracking [3].

II. Big Bang – Big Crunch algorithm

The Big Bang – Big Crunch algorithm is a heuristic population-based evolutionary optimization method. Among the merits of this method are computational simplicity, ability to handle multidimensional problems and very fast convergence. However, it seems that the implementation of it can be problematic when a noisy multimodal functional space is encountered, where there are a few local minima or maxima of a similar magnitude. Fortunately, such problems are rare in structural design problems.

The optimization problem can be stated as extreme-value problem where the main objective is to find such a set of parameters $(x_1, x_2, \ldots, x_n)$ which maximize or minimize a quantity dependent upon them. In the present paper, by finding the maximum possible burst pressure in the pressure vessel, we maximize the objective function.

The BB–BC optimization procedure can be briefly outlined as follows:

1) The initial population of feature vectors is randomly generated and spread over the entire search space, allowing also some individuals (within the range of 10%) be generated outside the search space. Then all the points which fall outside the prescribed limits are placed at the boundaries. This will guarantee that the optimum solution point will not fall outside the domain filled in by the candidate points. The number of individuals in the population must be big enough in order not to miss the optimum point. However, the population size can be significantly reduced as the search domain shrinks.
2) The fitness values are computed for every individual and, in the case of maximization, the center of mass is calculated as follows

$$x_c^{(k)} = \frac{\sum\limits_{i=1}^{N_{pop}} f_i x_i^{(k)}}{\sum\limits_{i=1}^{N_{pop}} f_i}, \qquad k = 1, 2, \ldots, n \qquad (1)$$

where n is the number of parameters and N_{pop} is the population size.

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3) Determine the boundaries of new contracted space as

$$\sigma_k = \frac{|x_{max}^{(k)} - x_{min}^{(k)}|}{N_{gen} + 1}, \qquad k = 1, 2, \ldots, n \tag{2}$$

where N_{gen} is the generation (iteration) number. Then the limits of the parameters are calculated:

$$\begin{aligned} x_{min}^{(k)} &= \beta x_c^{(k)} + (1-\beta) x_{best}^{(k)} - \sigma_k \\ x_{max}^{(k)} &= \beta x_c^{(k)} + (1-\beta) x_{best}^{(k)} + \sigma_k \end{aligned} \tag{3}$$

Here the empirical parameter β $(0 \leqslant \beta \leqslant 1)$ controls the influence of the global best solution on the boundaries of new search space.

4) The new search space is now randomly filled with points and thus a new population is created.
Hence the algorithm is repeated until the stop criteria are met. As the search space is contracted with each new iteration the algorithm arrives at the optimum point very fast.

III. Failure criteria and optimization method

The structure under consideration is a cylindrical shell of finite length made from an anisotropic material (see Fig. 1). The axis of anisotropy coincides with the axis of symmetry of the cylinder and the stresses act on the planes normal to the generator and do not vary along the generator. The analysis used is an exact three-dimensional elasticity solution which takes also into account the effect of closed ends. Unfortunately, the mathematical foundations of the analysis are quite cumbersome and thus cannot be accommodated here. The interested reader can find the detailed solution in [11]. The strength of filamentary composites is determined by the tensile and compressive strengths in the fibre directions and by the shear strength of the composite material. Failure in tension usually occurs when the fibres break, whereas failure in compression involves debonding of the fibres and the matrix material as a result of micro-buckling. Failure in shear is usually characterized by crack propagation through the composite material. In composite structures, tensile, compressive and shear stresses may result even from simple loading conditions, and therefore the failure mode of composite structures is rather complicated.

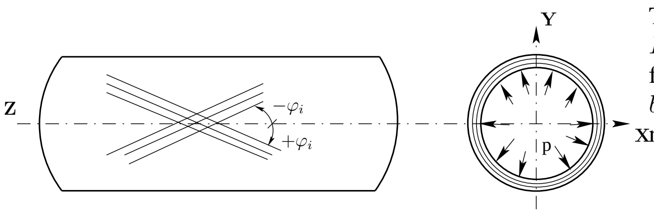

Fig. 1. Geometry of an anisotropic cylindrical pressure vessel.

The assumption of the Tsai–Wu three-dimensional failure criterion [13] is that there exists a failure surface in the stress space expressed in the following scalar form

$$f(\sigma_k) = F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1 \tag{4}$$

where $k,\ i,\ j = 1, 2, \ldots, 6$; F_i and F_{ij} are strength tensors of the second and forth rank, respectively. It is noted that this equation is applied to each layer to check for failure or otherwise. In case of laminated pressure vessels possessing cylindrical anisotropy, equation (4) for the m-th layer can be written in the following expanded form:

$$\begin{aligned} & F_{11}^{(m)} \sigma_1^{(m)2} + F_{33}^{(m)} (\sigma_3^{(m)2} + \sigma_2^{(m)2}) + F_{44}^{(m)} \tau_{12}^{(m)2} \\ + \quad & 2F_{31}^{(m)} (\sigma_3^{(m)} + \sigma_2^{(m)}) \sigma_1^{(m)} + 2F_{32}^{(m)} \sigma_3^{(m)} \sigma_2^{(m)} \\ + \quad & F_3^{(m)} (\sigma_3^{(m)} + \sigma_2^{(m)}) + F_1^{(m)} \sigma_1^{(m)} - 1 = 0 \end{aligned} \tag{5}$$

where

$$\begin{gathered} F_{11}^{(m)} = \frac{1}{X_t^{(m)} X_c^{(m)}},\ F_{33}^{(m)} = \frac{1}{Y_t^{(m)} Y_c^{(m)}},\ F_{44}^{(m)} = \frac{1}{S^{(m)2}} \\ F_3^{(m)} = \frac{1}{Y_t^{(m)}} - \frac{1}{Y_c^{(m)}}, \qquad F_1^{(m)} = \frac{1}{X_t^{(m)}} - \frac{1}{X_c^{(m)}} \\ F_{31}^{(m)} = -\frac{1}{2} \sqrt{F_{33}^{(m)} F_{11}^{(m)}}, \qquad F_{32}^{(m)} = -\frac{1}{2} F_{33}^{(m)} \end{gathered} \tag{6}$$

and $X_t,\ X_c$ are, respectively, longitudinal tensile and compressive strengths, $Y_t,\ Y_c$ are those for the transverse direction and S is the shear strength. It should be noted that the normal stresses $\sigma_i^{(m)},\ i = 1, 2, 3$ and shear stress $\tau_{12}^{(m)}$ are stresses in the material coordinates and can be computed as

$$\begin{aligned} \sigma_1^{(m)} &= \sigma_z^{(m)} \cos^2 \varphi_m + \sigma_\theta^{(m)} \sin^2 \varphi_m - \tau_{\theta z}^{(m)} \sin 2\varphi_m \\ \sigma_2^{(m)} &= \sigma_z^{(m)} \sin^2 \varphi_m + \sigma_\theta^{(m)} \cos^2 \varphi_m + \tau_{\theta z}^{(m)} \sin 2\varphi_m \\ \sigma_3^{(m)} &= \sigma_r^{(m)} \\ \tau_{12}^{(m)} &= (\sigma_\theta^{(m)} - \sigma_z^{(m)}) \sin \varphi_m \cos \varphi_m - \tau_{\theta z}^{(m)} \cos 2\varphi_m \end{aligned} \tag{7}$$

The design objective is the maximization of the burst pressure P_{cr} subject to the failure criterion (4). The design problem for a multilayered pressure vessel of a given thickness ratio b/a (the ratio of the external radius b to the internal radius a), number of layers nl can be stated as

$$P_{max} \stackrel{\text{def}}{=} \max_{\bar{\varphi}} P_{cr}(\bar{\varphi}, r) = \max_{\bar{\varphi}} \min_{r} P_{cr} \tag{8}$$

where

$$\bar{\varphi} = \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \vdots \\ \varphi_{nl} \end{Bmatrix} \tag{9}$$

and $P_{cr}(\bar{\varphi}, r)$ can be easily calculated from the quadratic equation

$$\left(F_{ij} \sigma_i^{(m)} \sigma_j^{(m)} \right) P_{cr}^{2(m)} + \left(F_i \sigma_i^{(m)} \right) P_{cr}^{(m)} - 1 = 0 \tag{10}$$

where the stresses are calculated for an applied unit pressure $P_{cr}(\bar{\varphi}, r) = \min_m P_{cr}^{(m)}$. Solution of the equation (10) gives

$$P_{cr}^{(m)} = -\left(\frac{\xi^{(m)}}{2\delta^{(m)}} \right) + \sqrt{\left(\frac{\xi^{(m)}}{2\delta^{(m)}} \right)^2 + \frac{1}{\delta^{(m)}}} \tag{11}$$

where

$$\begin{aligned} \delta^{(m)} &= F_{33}^{(m)}(\sigma_3^{(m)2}+\sigma_2^{(m)2})+2F_{32}^{(m)}\sigma_3^{(m)}\sigma_2^{(m)} \\ &+ 2F_{31}^{(m)}(\sigma_3^{(m)}+\sigma_2^{(m)})\sigma_1^{(m)} \qquad (12) \\ &+ F_{11}^{(m)}\sigma_1^{(m)2}+F_{44}^{(m)}\tau_{12}^{(m)2} \\ \xi^{(m)} &= F_3^{(m)}(\sigma_3^{(m)}+\sigma_2^{(m)})+F_1^{(m)}\sigma_1^{(m)} \end{aligned}$$

The negative root for $P_{cr}^{(m)}$ does not have any physical meaning and the positive value only must be taken into consideration.

The optimization procedure involves the stages of iteratively improving $\varphi_{opt}^{(m)}$, $m = 1, 2, \ldots, nl$ in order to maximize P_{cr} for a given radius, thickness ratio and axial force.

The problem considered is highly complicated and requires the use of a reliable multi–dimensional optimization method. With an increasing number of layers, the terrain of the functional multi–dimensional space becomes very complex and the use of calculus-based methods proves to be ineffective. Such methods suffer from the lack of robustness, and therefore they are hampered by inauspicious features in the multi-dimensional space like “ridges”, “canyons”, “flat spots” and multiple extrema. In addition to these limitations, they are local in scope; the optima they seek are the best in a neighborhood of the current point.

Originally (see [11]) the optimization procedure was carried out using the genetic algorithm techniques. Genetic algorithms are unconstrained optimizing algorithms, which means that the method does not converge in the normal sense as calculus-based methods do. After some number of generations, the improvement of a GA search slows down and eventually stops. In the best scenario, this indicates that the optimum set of parameters has been found. Unfortunately this is not always true. It might indicate premature “convergence”; though due to the mutation operator, the algorithm may recover after some number of iterations. It is useful to check the GA improvement regularly after some number (depending on the particular problem) of generations. The behaviour of the GA can be controlled to some extent by the mutation operator. In the case of a binary string, a local search can be carried out, where bit values are swapped, one after another (0 to 1 and 1 to 0). This is a time consuming procedure, and therefore, only the best individuals should be used. At every step the fitness is checked and the best combination is found. The number of generations can be also rather high. The use of the Big Bang - Big Crunch optimization method in the present study has demonstrated an incomparably greater efficiency than the genetic algorithms.

IV. Numerical Results

The design optimization of closed-ended cylinders (pressure vessels) of different thickness ratios and number of layers was carried out using the BB - BC optimization procedure. The material used is T300/5208 graphite-epoxy, and the material data used is taken from [14] and are as follows

$E_r = E_\theta = 1.03{\cdot}10^4$ MPa; $E_z = 1.81{\cdot}10^5$ MPa; $\nu_{\theta z} = 0.28$ $G_{\theta z} = 7170$ MPa; $X_t = X_c = 1500$ MPa; $Y_t = 40$ MPa; $Y_c = 246$ MPa; $S = 68$ MPa.

Table 1. Optimum ply angles and the maximum burst pressure

b/a	No of Layers (nl)	Optimum Angle Combination (inside to outside)	Burst Pressure (MPa)
1.01	1	54.2	4.49
	2	51/57	4.51
	3	49/53/59	4.53
	4	49/51/55/60	4.54
	5	50/50/53/56/61	4.55
	10	50/50/50/51/52/53/55/57/59/61	4.57
	20	50/51/51/50/51/50/50/51/51/52/ 53/54/54/55/56/57/58/59/60/61	4.57
1.1	1	54.5	47.90
	2	50/58	48.14
	3	48/47/64	48.93
	4	46/49/49/67	49.95
	5	46/46/46/50/76	51.46
	10	47/46/46/46/46/46/50/55/64/88	53.34
	20	47/47/46/46/47/47/47/47/47/46/ 46/48/50/52/54/57/61/66/73/89	54.04
1.2	1	54.7	104.27
	2	49/59	105.00
	3	49/48/64	107.30
	4	51/48/49/67	109.18
	5	53/50/49/49/69	110.35
	10	60/54/52/51/51/50/49/49/48/74	112.80
	20	85/53/53/52/52/51/51/51/50/49/ 50/49/49/48/48/48/48/52/60/77	114.67
1.3	1	56.3	150.10
	2	60/51	150.43
	3	59/58/50	150.46
	4	42/80/62/48	151.97
	5	38/76/66/59/46	154.69
	10	33/47/66/74/70/64/59/57/51/43	159.70
	20	30/37/44/53/62/74/70/73/59/60/ 72/63/63/62/61/58/53/50/46/42	162.40
1.5	1	57.3	181.00
	2	64/46	183.60
	3	69/50/44	184.08
	4	72/55/46/43	184.14
	5	22/85/67/55/50	186.16
	10	15/40/88/85/70/61/56/52/51/48	196.42
	20	19/32/46/68/89/88/87/82/68/66/ 63/62/54/51/50/48/47/47/47/46	200.95

Optimization study shows that for the single-layered cylinder, thickness has marginal effect on the optimal φ, and φ_{opt} is in the range $54°$–$57°$ for $1.01 \le b/a \le 1.5$.

For a multilayered CFRP T300/5208 cylinder, the results were The population size used is 100 individuals and the control coefficient $\beta = 0.7$. obtained and are given in the Table 1, for layers of equal thickness and a layer thickness

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of $(b/a - 1)/nl$ where nl is the number of layers. However, the equal thickness vessels do not necessarily give the highest burst pressure. All values of the burst pressure were calculated at the weakest point within the cylinder thickness. In the case of T300/5208 the failure point shifts from outside surface to the inside as the wall thickness increases. At about $b/a = 1.15$ this transition occurs. However, the thicker the cylinder, the smoother the transition of the burst pressure from one layer to another. It is also observed that the increase in number of layers does not significantly improve the performance of the pressure vessel, and a complex system for the reinforcement of the layer package is not justifiable. At the same time, the technique developed might be used for the optimization of other composite structures and produce interesting results.

Some results are given above for thin, medium and thick closed-ended cylinders. Such optimization problems can be highly complicated because of the huge number of fibre combinations as well as the very complex functional spaces. Figs. 2 and 3 show a graphical representation (functional space) of the three–dimensional optimizing problem for a thin ($b/a = 1.1$) and thick ($b/a = 1.5$) two–layered cylinder. Even in this case the functional surface is rather complicated and it is hardly possible to imagine how complex a hyper-surface in a multi-dimensional optimizing problem will be. It is also observed that the change in burst pressure is smoother in the thick cylinder.

In the case of 20 layers we have $90^{20} \approx 1.216{\cdot}10^{39}$ different combinations of the reinforcement. Besides that, the problem is very sensitive to any changes in the fibre orientations. Such calculations have only become possible due to sophisticated present evolutionary algorithms. Fig. 4 illustrates an incredible efficiency of the algorithm. As is seen, the optimum result for twenty-dimensional problem is achieved after about twenty five generations, or to put it differently, in less than one minute of computing time on an ordinary computer. The population size used is 100 individuals and the control coefficient $\beta = 0.7$.

A distinguishing feature of this algorithm is its much the same number of iterations for small and large number of parameters. This is because the space is contracted at the same rate for all these problems. The dynamics of this contraction can be seen in Figs. 5–8 by the example of two-dimensional optimization problem of a pressure vessel with radii ratio $b/a = 1.1$. These figures make it clear that after only twenty generations the points are already densely concentrated around the optimum solution point.

V. Conclusion

The multidimensional design optimization of composite structures presents considerable difficulties due to complex "terrain" of the functional space. The first reliable optimization method applied to this class of problems was the genetic algorithms. With the advent of the Big Bang - Big Crunch method such optimizations became even more efficient. As this paper shows, the convergence speed of the BB - BC is unmatched with the GAs. Moreover, the number of iterations is the same for any number of the design parameters. The above technique can be used for optimizing various structural problems with a large number of optimizing variables of different nature.

As any other technique the BB -BC method is not perfect and requires a certain caution when used, particularly a relatively large population size is required at the start in order not to miss the optimum point. However, the population size can be drastically reduced later.

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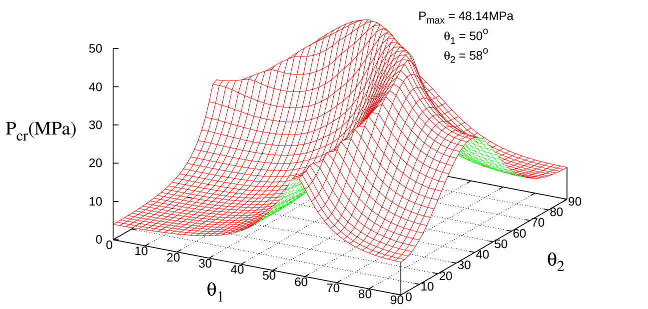

Fig. 2. Burst pressure plotted against ply angles for a two–layered pressure vessel with the radii ratios: $b/a = 1.1$.

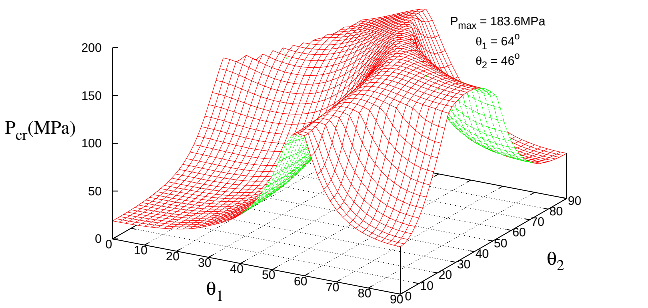

Fig. 3. Burst pressure plotted against ply angles for a two–layered pressure vessel with the radii ratios: $b/a = 1.5$.

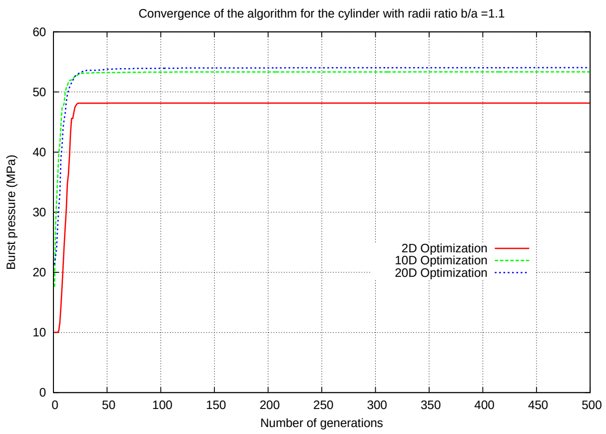

Fig. 4. Convergence of the BB–BC algorithm for the cylinder with radii ratio $b/a = 1.1$

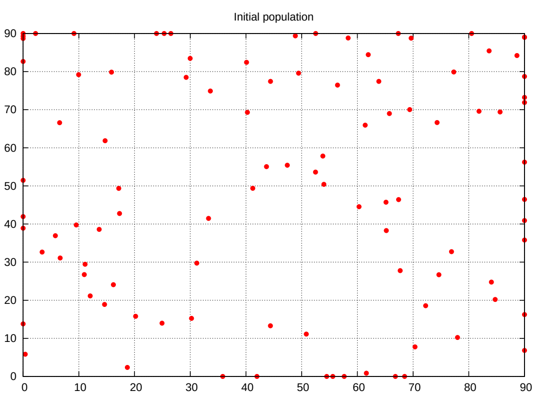

Fig. 5. The initial population of 100 individuals.

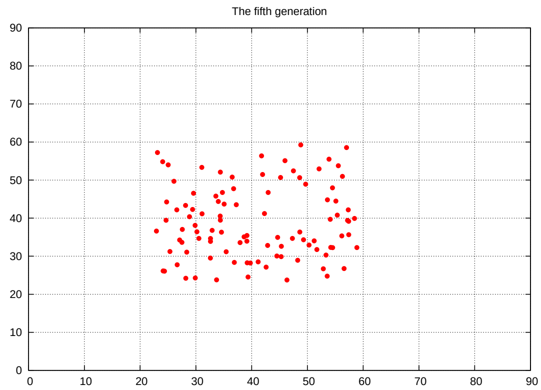

Fig. 6. The fifth generation.

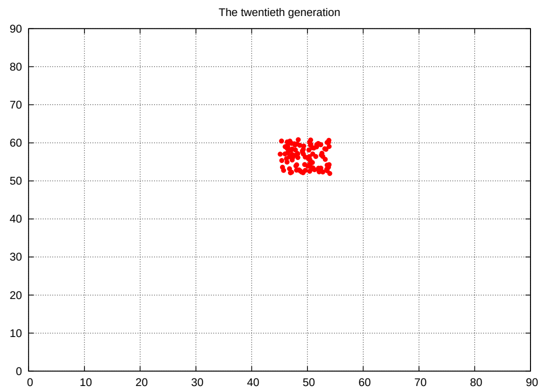

Fig. 7. The twentieth generation.

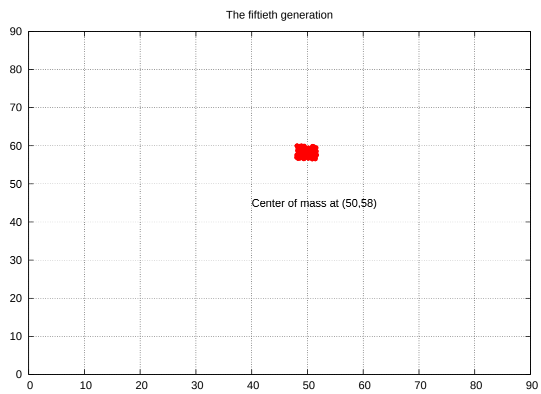

Fig. 8. The fiftieth generation.