

# Electrocardiogram Signal Compression Using Multiwavelet Transform

Morteza Moazami-Goudarzi, and Mohammad. H. Moradi

**Abstract**—In this paper we are to find the optimum multiwavelet for compression of electrocardiogram (ECG) signals. At present, it is not well known which multiwavelet is the best choice for optimum compression of ECG. In this work, we examine different multiwavelets on 24 sets of ECG data with entirely different characteristics, selected from MIT-BIH database. For assessing the functionality of the different multiwavelets in compressing ECG signals, in addition to known factors such as Compression Ratio (CR), Percent Root Difference (PRD), Distortion (D), Root Mean Square Error (RMSE) in compression literature, we also employed the Cross Correlation (CC) criterion for studying the morphological relations between the reconstructed and the original ECG signal and Signal to reconstruction Noise Ratio (SNR). The simulation results show that the *cardb2* by the means of identity (Id) prefiltering method to be the best effective transformation.

**Keywords**—ECG compression, Multiwavelet, Prefiltering.

## I. INTRODUCTION

Compressing biological signals, especially ECG has an important role in diagnosis, taking care of patients and signal transfer through communication lines. Therefore, ECG data compression has been one of the most active research areas in biomedical engineering.

Techniques for ECG compression which have been reported in the literature fall mainly into two categories: (1) direct compression such as Amplitude-Zone-Time Epoch Coding (AZTEC) method, the coordinate reduction time coding system (CORTES), turning point (TP) technique, Scan-Along Polygonal Approximation (SAPA), and the long-term prediction (LTP). (2) transformational methods such as Fourier transform, Walsh Transform, Karhunen-Loeve Transform (KLT), and Wavelet Transform (WT). In most cases, direct methods are superior to transform methods with respect to system simplicity and error. However, transform methods usually achieve higher compression rates and are insensitive to noise contained in original ECG signal [1].

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Among the methods mentioned above, wavelet transformation is an efficient tool in signal processing aimed at compressing ECG signals, detection of QRS complex, analysis of ventricular late potential, etc. The purpose of this paper is to employ the multiwavelet as an extension of wavelet and the primary results of applying multiwavelets in signal processing [2,3,4,5], compression [4,6,7] and noise elimination [4,8,9] indicate the superiority of multiwavelet to wavelet.

## A. A Brief History of Multiwavelet

Multiwavelets constitute a new chapter which has been added to wavelet theory in recent years. Recently, much interest has been generated in the study of the multiwavelets where more than one scaling functions and mother wavelet are used to represent a given signal.

The first construction for polynomial multiwavelet was given by Alpert, who used them as a basis for the representation of certain operators. Later, Geronimo, Hardin and Massopust constructed a multi-scaling function with 2 components using fractal interpolation.

In [10], multiwavelets based on Cardinal Hermite splines were constructed. In spite of the many theoretical result on multiwavelet, their successful applications to various problem in signal processing are still limited.

Unlike scalar wavelets in which Mallat's pyramid algorithm have provided a solution for good signal decomposition and reconstruction, a good framework for the application of the multiwavelet is still not available. Nevertheless, several researchers have proposed method of how to apply a given multiwavelet filter to signal and image decomposition. For example, Xia *et al* [10,11] have proposed new algorithm to compute multiwavelet transform coefficients by using appropriate pre- and post-filtering filters, and have indicated that the energy compaction for discrete multiwavelet transform may be better than that obtained using conventional discrete scalar wavelet transforms.

So, based on problems mentioned above, finding a multiwavelet that has the most energy compaction is an important subject in signal compression. Our motivation in this work is to find multiwavelet that has the best energy compaction for a different ECG signals.

## B. Multi-scaling Functions and Multiwavelets

The concept of multi-resolution analysis can be extended from the scalar case to general dimension  $r \in \mathbb{N}$ . A vector valued

function  $\Phi = [\phi_1 \ \phi_2 \ \dots \ \phi_r]^T$  belonging to  $L^2(\mathbb{R})^r$ ,  $r \in \mathbb{N}$  is called a multi-scaling function if the sequence of closed spaces:

$$V_j = \overline{\text{span}\left\{2^{j/2}\phi_i(2^j \cdot -k): 1 \leq i \leq r, \ k \in \mathbb{Z}\right\}}, \quad j \in \mathbb{Z}$$

constitute a multi-resolution analysis of multiplicity  $r$  (MRA) for  $L^2(\mathbb{R})$ .

Now let  $W_j$  denote a complementary space of  $V_j$  in  $V_{j+1}$ . The vector valued function  $\Psi = [\psi_1 \ \psi_2 \ \dots \ \psi_r]^T$  such that:

$$W_j = \overline{\text{span}\left\{2^{j/2}\psi_i(2^j \cdot -k): 1 \leq i \leq r, \ k \in \mathbb{Z}\right\}}$$

is called a multiwavelet. Multi-scaling and multiwavelet functions must satisfy the two-scale equation:

$$\Psi(t) = \sqrt{2} \sum_k H_k \Phi(2t - k)$$

Where  $H_{k \in \mathbb{Z}} \in L^2(\mathbb{Z})^{r \times r}$  is an  $r \times r$  matrix of coefficients [4, 9].

### C. Multiwavelet in Comparison with Wavelet

The multiwavelet idea originates from the generalization of scalar wavelets. Instead of one scaling function and one wavelet, multiple scaling functions and wavelets are used. This leads to more degree of freedom in constructing wavelets. Therefore opposed to scalar wavelets, properties such as compact support, orthogonality, symmetry, vanishing moments, short support can be gathered simultaneously in multiwavelets, which are fundamental in signal processing [4,5].

The increase in degree of freedom in multiwavelets is obtained at the expense of replacing scalars with matrices, scalar functions with vector functions and single matrices with block of matrices. However, prefiltering is an essential task which should be performed for any use of multiwavelet in the signal processing.[4,12]

### D. Prefiltering of the Data

One of the challenges in realizing multiwavelets is the efficient prefiltering. In the case of scalar wavelets, the given signal data are usually assumed to be the scaling coefficients that are sampled at a certain resolution, and hence, we directly apply multiresolution decomposition on the given signal. But the same technique can not be employed directly in the multiwavelet setting and same prefiltering has to be performed on the input signal prior to multiwavelet decomposition. The type of the prefiltering employed is critical for the success of the results obtained in application.

There could be infinitely many ways to do such prefiltering. There exist well known prefilters in literature [11,13,14]. The most obvious way to get second input row is

just to repeat the first on eand use two identical rows of length  $n$ .

A different way to get the input rows for the multiwavelet filter bank is to preprocess the given scalar signal  $f(n)$ . In our implementation, first we refer to repeated row (rr) and second we refer to approximation prefilter (app).

For the balanced multiwavelet, the identity (ID) prefilter is used. This prefilter just separates the input data in two streams: one consisting of even numbered samples, the other consisting of odd numbered samples.

### E. Multiwavelet Decomposition

The goal of this sub-section is to apply nine multiwavelets with prefiltering mentioned above on ECG signals. We retain the same number of largest coefficients for each multiwavelet and then invert the algorithm to reconstruct the signal. All of our tests are applied on the first 2048 samples of the Lead I from MIT-BIH records 100, 101, 102, 103, 104, 105, 106, 107, 118, 119, 200, 201, 202, 203, 205, 207, 208, 209, 210, 212, 214, 215, 217, 219. A simple threshold compression method has been applied based on the following steps:

- 1) Prefiltering and decomposition up to 6 levels.
- 2) Retention of the first  $N$  largest coefficients of the decomposition.
- 3) Reconstruction from  $N$  coefficients.

For simplicity, we have considered  $N = 125$ , corresponding to compression ratios of 16.384 for all signals.

## II. ASSESSMENT CRITERIA

An ECG compression algorithm is judged by its ability to minimize the distortion while retaining all significant features of the signal. The distortion in reconstruction has been computed by means of the following formula:

$$D = \frac{\|x_{or} - x_{re}\|^2}{\|x_{or}\|^2}$$

where  $x_{or}$  is the original signal and  $x_{re}$  is the reconstructed signal.

Other methods that can be used to measure distortion are PRD. So  $x_{or}$  and  $x_{re}$  are signals of length  $N$ , PRD can be defined as:

$$PRD = \sqrt{\frac{\sum (x_{or} - x_{re})^2}{\sum (x_{or})^2}} \times 100\%$$

Another criterion we use for measuring distortion is RMSE. In data compression, we are interested in finding an optimal approximation for minimizing this criterion as defined by the following formula:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{or} - x_{re})^2}$$

However, since the similarity of the reconstructed and original signal are very important from the clinical point of view, the

CC is employed to evaluate the similarity between the original signal and its reproduced version, defined as:

$$CC = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2} \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \bar{y})^2}}$$

where  $x_i$  and  $y_i$  are the samples of the original signal and its reproduced version,  $\bar{x}$  and  $\bar{y}$  are their average values, respectively.

Another criterion that is used here is SNR and is given by:

$$SNR = 20 \log \left( \frac{\hat{\sigma}_{x_{or}}}{\hat{\sigma}_{x_{or}-x_{re}}} \right) \quad (\text{dB})$$

Where  $\hat{\sigma}$  denotes the standard deviation estimator. In signal processing we are interested in finding an optimal approximation for maximizing this criterion.

The results are displayed in Table I, presenting the average RPD, D and CC calculated for each multiwavelet for the 24 different records. As observed from these results, cardbal2 by the means of Id prefiltering method exhibits the best results comparing the others.

Finally, in order to investigate the effect of compressing ECG signals using the cardbal2 by the means of Id prefilter from the clinical point of view, three waveforms including original, reconstructed waveforms and difference between original and reconstructed signal (error) of records 107, 119 and 219 are shown in Figs. 1, 2 and 3. Note that reconstructed ECG signals are smoothed versions of the original signals, but error increases when the original signal changes abruptly.

security clearances.

#### IV. CONCLUSION

In this paper, we studied the optimum multiwavelet for compressing the ECG signal. It should be noted that a further improvement in results may be achieved with more sophisticated quantization (for example the SPIHT coder suggested in [15]), and also with other multiwavelet bases and new prefiltering approaches.

#### III. RESULTS AND DISCUSSION

TABLE I  
SUMMARY OF COMPRESSION RESULTS. PRD AND CC ARE IN PERCENT

|                             |                  |                  |                       |                    |             |
|-----------------------------|------------------|------------------|-----------------------|--------------------|-------------|
| GHM <sup>1</sup>            | 10.199464        | 98.988928        | 1.2055274e-002        | 0.043585007        | 17.6        |
| GHM <sup>2</sup>            | 10.442661        | 98.922792        | 1.2616885e-002        | 0.044773102        | 17.9        |
| CL <sup>1</sup>             | 9.3516668        | 99.145806        | 1.0522501e-002        | 0.040056692        | 18.9        |
| CL <sup>2</sup>             | 9.1507510        | 99.192476        | 9.9911897e-003        | 0.039211513        | 18.7        |
| SA4 <sup>1</sup>            | 9.6601638        | 99.102202        | 1.0957484e-002        | 0.041052841        | 18.9        |
| SA4 <sup>2</sup>            | 9.2169610        | 99.166599        | 1.0220815e-002        | 0.038997280        | 18.4        |
| bih52s <sup>1</sup>         | 11.212887        | 98.710719        | 1.6118575e-002        | 0.047393910        | 17.0        |
| bih52s <sup>2</sup>         | 11.682872        | 98.589613        | 1.7440181e-002        | 0.049827301        | 17.3        |
| bih54n <sup>1</sup>         | 97.075842        | 99.053456        | 1.2457205e-002        | 0.041163568        | 18.8        |
| bih54n <sup>2</sup>         | 10.555499        | 98.884445        | 1.4600110e-002        | 0.044330958        | 18.7        |
| bighm2 <sup>1</sup>         | 34.906194        | 91.819611        | 1.4646049e-001        | 0.14521119         | 6.1         |
| bighm2 <sup>2</sup>         | 48.399680        | 85.512920        | 2.7072476e-001        | 0.19750880         | 8.4         |
| Cardbal4 <sup>3</sup>       | 8.9235645        | 99.224869        | 9.5386841e-003        | 0.038196125        | 19.1        |
| Cardbal3 <sup>3</sup>       | 9.0019501        | 99.208416        | 9.7816903e-003        | 0.038472232        | 19.0        |
| <b>Cardbal2<sup>3</sup></b> | <b>8.8986281</b> | <b>99.230226</b> | <b>9.4111670e-003</b> | <b>0.038030973</b> | <b>19.1</b> |

1 prefiltered with rr      2 prefiltered with app      3 prefiltered with Id

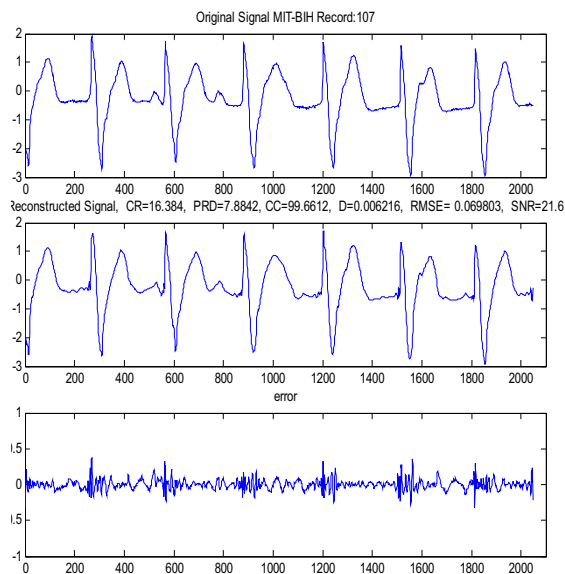


Fig. 1 – Compressing ECG using the cardbal2 with Id prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 107 are used. CR=16.384, PRD=7.8842%, CC=99.6612%, D=0.006216, RMSE=0.069803, SNR=21.6 (dB)

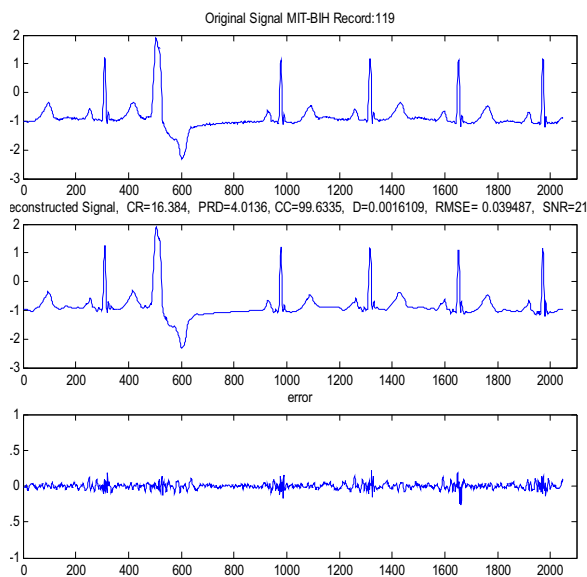


Fig. 2 – Compressing ECG using the cardbal2 with Id prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 119 are used. CR=16.384, PRD=4.0136%, CC=99.6335%, D=0.0016109, RMSE=0.039487, SNR=21.3 (dB)

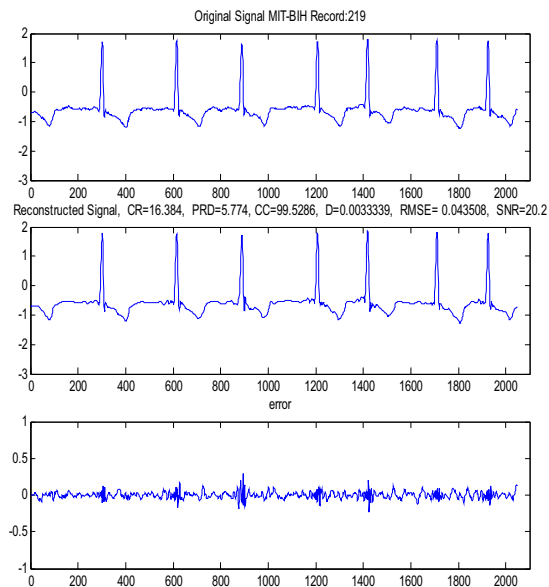


Fig. 3 – Compressing ECG using the cardbal2 with Id prefiltering method. The above figure shows the original signal, the middle shows reconstructed signal after compression and the bottom shows error between them. The first 2048 samples of MIT-BIH record 219 are used. CR=16.384, PRD=5.774%, CC=99.5286%, D=0.0033339, RMSE=0.043508, SNR=20.2 (dB)

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