

# Determining Optimal Production Plan by Revised Surrogate Worth Trade-off Method

Tunjo Perić, Zoran Babić

**Abstract**—The authors of this work indicate by means of a concrete example that it is possible to apply efficaciously the method of multiple criteria programming in dealing with the problem of determining the optimal production plan for a certain period of time. The work presents: (1) the selection of optimization criteria, (2) the setting of the problem of determining an optimal production plan, (3) the setting of the model of multiple criteria programming in finding a solution to a given problem, (4) the revised surrogate trade-off method, (5) generalized multicriteria model for solving production planning problem and problem of choosing technological variants in the metal manufacturing industry.

In the final part of this work the authors reflect on the application of the method of multiple criteria programming while determining the optimal production plan in manufacturing enterprises.

**Keywords**—multi-criteria programming, production planning, technological variant, Surrogate Worth Trade-off Method.

## I. INTRODUCTION

**P**RODUCTION planning is one of the most important activities in manufacturing enterprises. The production plan involves a set of particular kinds of products to be produced in a particular period of time and the structure of output. The optimal production plan is a dynamic phenomenon and it can only be realised by continuous effort to respond to the market by using available capital and human resources. The process of determining the optimal production plan in a manufacturing company is complex and dynamic therefore it has to be the object of continuous consideration at the company level. Determining the optimal production plan for a particular period is even more significant if it involves the choice of optimal technological variants.

The result of production planning is the production program of the company which includes the production variants for each product. The choice of the optimal production program and optimal technological variants is significantly reflected in the company market position and in the quality of company operation.

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The basic objectives of this work are:

- (1) To point to the shortcomings of one-criteria linear programming (LP) and the necessity to apply multicriteria linear programming (MLP),
- (2) To select production plan optimization criteria,
- (3) To formulate a general model for production plan optimization,
- (4) To demonstrate the optimal production plan determining by use of MLP methods on the concrete example,
- (5) To apply the revised interactive Surrogate Worth Trade-off Method on the concrete problem
- (6) To analyse the obtained solutions and point to the direction of further research in this field.

The papers presenting the use multicriteria programming methods in production problems are numerous (see Agrell et al. [1], Kangas et al. [8], Kwak et al. [10]). In this work the authors try to simultaneously determine the optimal production plan and the optimal production variant by using a modification of the SWT method that should be more acceptable to the decision-makers.

## II. OPTIMIZATION OF PRODUCTION PLAN

By determination of the optimal production plan in this paper we assume simultaneous determining of the optimal production program and optimal technological variants.

Production program is a set of particular kinds of products to be produced in a particular period of time and the structure of output. The optimal production program is a dynamic phenomenon and as an aspect of business policy it plays the decisive role for company survival and progress. Determining the optimal production program is a complex and dynamic process which has to be planned both in the short run and in the long run and has to be subject to continuous consideration. Production program of a manufacturing company is significantly reflected in its market position and business performance. Company production program is measurable by economic parameters. The best production program is the one which ensures the highest level of performance expressed in economic parameters. Such production program is the optimal one for a particular period of time.

Technological variant is the production procedure for a particular product. Numerous products can be produced by different production procedures. Different technological variants require different expenditure of material, energy, labour, machine operation, etc. Consequently, a company may achieve different results by using different technological procedures for the same product. Therefore, if there is a possibility to use different technological procedures, to operate rationally a company has to choose the most suitable variant.

The best technological variant for production of a particular product is the variant which allows achievement of the best result expressed by economic parameters. Such technological variant represents the optimal variant for production of a particular product.

The selection of optimal technological variants for particular products and determination of an optimal production program is a special aspect of the company operational policy reflected in company performance and development. Determination of the optimal technological variant is a complex and dynamic process that is subject to both short-term and long-term planning and subject to continuous consideration.

#### *A. Production Plan Optimization Criteria*

Production plan optimization criteria depend on the company operation objectives and on the specific operation goals in that company.

Determination of objectives is one of the most significant tasks of company operation, because the company as a business system depends on the definition of its objectives. The degree of company freedom to determine its objectives depends on the social system and on the conditions posed by the company environment. In the present business conditions companies are perfectly free to determine their objectives.

The company objectives are significant because they:

- Determine and direct the entire company activity in a particular period
- Eliminate the effect of contingency and thus allow an even and stable development of the company.

Each company has its own system of objectives which differs from other systems in size, contents, etc. What is common for all such systems is that they comprise two groups of objectives:

- Economic objectives and
- Social objectives.

Economic objectives comprise survival, efficient operation and progress, and they are realised by achieving the determined performance. Social objectives comprise relationships within the company, workers' motivation, living conditions, etc. Economic and social objectives are related and interdependent which makes defining of objectives even more complex (Humble [7]).

Consequently, economic objectives of company operation comprise maximization of: output, capacity utilization, total revenues, profit, labour productivity, operational profitability, operational economy, growth rate, etc.

In addition to the company objectives, there are also specific production objectives that contribute to the better realisation of objectives and higher goals.

Specific production objectives can be for example:

- Maximization of product quality
- Minimisation of scrap
- Minimisation of pollution, etc.

In addition to the above mentioned ones, optimization of production by use of multicriteria programming methods requires determination of some specific goals such as maximization of exports, etc.

All the above mentioned objectives are not equally important, which has to be taken into account in production optimization by multicriteria programming methods.

From the stated objectives production optimization criteria are derived, which can be:

- Output (in pieces)
- Capacity utilization (in hours of machine operation)
- Total revenues (in monetary units)
- Profit (in monetary units)
- Working hours (in hours)
- Working capital (in monetary units)
- Costs (in monetary units)
- Growth rate (in percentages)
- Quality (in particular units depending on the character of production)
- Scrap (in pieces)
- Pollution (measured dependently of its character)
- Value of exports (in monetary units), etc.

Whether all these criteria will be considered in production optimization depends on the character of the production and on the development level of company information system. Namely, if the information system is not highly developed, some of these criteria cannot be objectively included either due to lack of the appropriate data or due to lack of information on the behaviour regularity in criteria functions.

From the above list it is obvious that there are a large number of criteria for production plan optimization. All the listed criteria are interdependent to a higher or lower degree, which has to be taken into account in optimization. Namely, criteria analysis in each concrete case of optimization has shown that their number can be reduced, which reduces the cost of searching for the optimal solution.

It has to be pointed out however, that optimization criteria selection depends on the character of the problem which is being solved. In optimization of the production program and technological variants all the mentioned criteria can be taken into account, if the problem in question requires that, and also some additional criteria may be included that are not

mentioned in this section.

### B. MP Model for Selection of Optimal Production Program and Technological Variants

Here we will present an MCP model for optimization of production plan by simultaneous determination of optimal technological variants and optimal production program. In order to solve this problem for a particular company we will start from the following assumptions:

- Criteria for selection of the optimal production program and technological variants are given.

- Limited capacity of machines, materials, labour and market are available.

The company intends to produce  $n$  products by  $m$  technological variants available.

Let us introduce the following marks:

- $f_j$  = criteria for the selection of the optimal production program and technological variants ( $j = 1, \dots, k$ );

- $x_{ig}$  = the quantity of  $i$ -product produced by  $g$ -technological variant ( $i = 1, \dots, n; g = 1, \dots, m$ );

- $c_{igj}$  =  $i$ -coefficient of the  $g$ -technological variant,  $j$ -criterion function;

- $a_{ig}$  = production time per unit of  $i$ -product by  $g$ -technological variant ( $i = 1, \dots, n; g = 1, \dots, m$ );

- $q_{itg}$  = normative expenditure of  $t$ -raw material to produce  $i$ -product by  $g$ -technological variant ( $i = 1, \dots, n; t = 1, \dots, l; g = 1, \dots, m$ );

- $r_{ig}$  = normative labour necessary to produce  $i$ -product by  $g$ -technological variant ( $i = 1, \dots, n; g = 1, \dots, m$ );

- $b_g$  = capacity of  $g$ -technological variant ( $g = 1, \dots, m$ );

- $q_t$  = available quantity of  $t$ -raw material ( $t = 1, \dots, l$ );

- $r$  = labour capacity;

- $d_i$  = market capacity of  $i$ -product – minimal quantity needed

- $u_i$  = market capacity of  $i$ -product – maximal sale possibility.<sup>1</sup>

Based on the above marks we can formulate the mathematical model in the following way:

$$(\max) \underline{f} = [f_1(\underline{x}), \dots, f_k(\underline{x})]$$

$$\text{s.t.} \quad \sum_{i=1}^n a_{ig} x_{ig} \leq b_g \quad g = 1, \dots, m$$

$$\sum_{i=1}^n \sum_{g=1}^m q_{itg} x_{ig} \leq q_t \quad (t = 1, \dots, l)$$

$$\sum_{i=1}^n \sum_{g=1}^m r_{ig} x_{ig} \leq r$$

$$d_i \leq x_i \leq u_i \quad (i = 1, \dots, n)$$

$$x_{ig} \geq 0 \quad (i = 1, \dots, n; g = 1, \dots, m).$$

If all the criteria functions of the above model are linear, then it is an MLP model which can be solved by all the available MLP methods. Criteria functions of the model have the following form:

$$(\max) \underline{f} = \left[ \sum_{i=1}^n \sum_{g=1}^m c_{ig1} x_{ig}, \dots, \sum_{i=1}^n \sum_{g=1}^m c_{igk} x_{ig} \right].$$

If, however, some of the criteria functions are nonlinear, then the model is a multiple criteria nonlinear programming (MNP) model. Depending on the characteristics of nonlinear criteria functions, the MNP model will be solved by the appropriate MNP methods.

Based on the above model it is possible to solve the corresponding problems of selection of optimal production program and optimal technological variants for a particular period.

### III. SETTING THE PROBLEM OF THE OPTIMAL PRODUCTION PROGRAM AND OPTIMAL TECHNOLOGICAL VARIANTS SELECTION

In the following sections we will present the application of the above model on the selection of the production program and technological variants in a metal processing company.

#### A. Company Production Characteristics needed for Establishment of the MCP Model Criteria Functions

For the current year January – December period the company is to produce, in addition to single products made to order, 11 different products marked by numbers from 1 to 11.

Net sale price and net-profit per product made by a particular technological variant are shown in the following table:

<sup>1</sup> To take into account market limitations we have to observe that:

$$x_i = \sum_{g=1}^m x_{ig}, \quad (i = 1, \dots, n)$$

TABLE I NET SALE PRICE AND NET-PROFIT PER PRODUCT

Product $x_i$	Net sale Price $c_{ig3}$	Net-profit - $c_{ig1}$			
		Var. U-7	Var. U-11	Var. NC-P	Var. NC-A
1	12.80	0.35	-	0.77	0.95
2	78.00	3.5	-	4.68	-
3	14.50	0.45	0.57	0.87	1.11
4	10.80	0.50	0.65	-	-
5	9.81	0.33	0.41	0.59	0.71
6	13.60	0.51	0.62	0.82	0.99
7	15.80	0.88	0.95	-	-
8	20.30	1.19	1.22	-	-
9	19.80	0.88	0.95	1.19	-
10	13.45	0.57	0.68	0.81	0.99
11	218.50	9.12	10.5	13.11	-

Technological variants represent different ways to produce the same product. In our example the variant U-7 presents production by a special machine requiring high participation of labour, while by the variant NC-A the production is completely automated with the minimal participation of workforce.

For production of different products the needed direct labour will be different in different technological variants. The direct labour required for particular products and variants, and equivalency coefficients obtained by division of the necessary working time for production of each product by different variants with the needed time to produce product 3 (freely chosen by the authors) by different variants are shown in the following table:

TABLE II DIRECT LABOUR REQUIRED AND EQUIVALENCY COEFFICIENTS

Product $x_i$	Direct labour required in minutes and equivalency coefficients - $c_{ig2}$							
	Var. U-7	Equiv. coeff.	Var. U-11	Equiv. coeff.	Var. NC-P	Equiv. coeff.	Var. NC-A	Equiv. coeff.
1	18	1,28	-	0,00	9	1,80	7	1,40
2	132	9,43	-	0,00	66	13,2	-	0,0
3	14	1,00	10	1,00	5	1,00	5	1,0
4	15	1,07	9	0,90	-	0,00	-	0,0
5	9	0,64	8	0,80	5	1,00	3	0,6

6	13	0,93	12	1,20	6	1,20	5	1,0
7	22	1,57	20	2,00	-	0,00	-	0,0
8	30	2,14	25	2,50	-	0,00	-	0,0
9	18	1,28	14	1,40	11	2,20	-	0,0
10	14	1,00	11	1,10	7	1,40	5	1,0
11	250	17,8	200	20	145	29	-	0,0

### B. Company Production Characteristics needed for Establishment of the MCP Model Constraints

#### B1. Constraints in terms of available capacity

The turning plant consists of seven U-7 lathes, one U-11 lathe, two NC-P machines and one NC-A machine.

The time needed to manufacture one product on these machines is shown in the Table 3.

TABLE III LATHES OPERATION TIME

Product $x_i$	Lathes operation time in minutes <sup>2</sup> - $a_{ig}$			
	Var. U-7	Var. U-11	Var. NC-P	Var. NC-A
1	10	-	6	5
2	80	-	40	-
3	8	8	4	4
4	5	5	-	-
5	4	5	3	2
6	6,5	6,5	3	3
7	6	5,5	-	-
8	25	21	-	-
9	6,5	5,8	5	-
10	12	10	7	5
11	47	39	30	-
Available capacity - $b_g$	405000	101000	130460	99490

The available capacity of particular groups of machines is calculated according to the form (in minutes):  $b_i = p_i \cdot m_i \cdot n_i \cdot s_i \cdot \eta_i - K_i$ , where  $b_i$  = available capacity of the  $i$ -lathe group,  $p_i$  = number of lathes of the  $i$ -group,

<sup>2</sup> The time required to manufacture a product besides turning operation includes some other operations (cutting, milling, polishing, heat treatment, etc.) Some of these operations are carried out on the lathe depending on the type of product and type of lathe. Turning operation on different lathe types will be of different grade, which results in additional operations and consequently in additional time needed to finish the product. As in this process turning is the most important operation the selection of technological variants will be carried out with this operation in mind.

$m_i$  = number of working days in the given period,  $n_i$  = number of working hours per shift  $s_i$  = number of shifts per day,  $\eta_i$  = utilization level of the  $i$ -lathe group (taking into account the waste of time resulting from technological, organisational, market and other factors). Based on the analysis of data from the previous period it is assumed that the utilization level of the U-7 lathe group will be 0.85, while U-11, NC-P i NC-A will be 0.95.  $K_i$  is the capacity of the  $i$ -lathe group intended for production to order (single units and small series). Thus:

$$b_1 = 6 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.85 \cdot 60 - 150390 = 405000$$

$$b_2 = 1 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.95 \cdot 60 = 101000,$$

$$b_3 = 2 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.95 \cdot 60 - 76450 = 130460,$$

$$b_4 = 1 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.95 \cdot 60 - 3965 = 99490.$$

**B2. Constraints in terms of labour**

In the given period labour is not restricted because in the labour market there are a large number of workers with the required skills.

**B3. Constraints in terms of available materials**

Consumption of the basic material in kilograms needed to manufacture single products on particular lathes is shown in the Table 4.

TABLE IV CONSUMPTION OF BASIC MATERIAL IN KILOGRAMS

Product	Consumption of basic material in kilograms – $q_{ig}$			
	Var. U-7	Var. U-11	Var. NC-P	Var. NC-A
1	0,41	-	0,40	0,40
2	1,75	-	1,70	-
3	0,35	0,35	0,34	0,34
4	0,40	0,40	-	-
5	0,49	0,47	0,45	0,44
6	0,36	0,35	0,35	0,33
7	0,71	0,69	-	-
8	0,66	0,65	-	-
9	0,42	0,41	0,41	-
10	0,33	0,31	0,30	0,30
11	3,15	3,10	3,05	-

In the given period the possibility to purchase the basic material is restricted to  $q_1 = 46000$  kg.

There are no restrictions for the purchase of other materials.

**B4. Constraints in terms of market**

As the products in question are specific and intended for limited market segments the company has restricted possibility of sale. Consequently, in the subsequent sales plan period the maximal sale will be 7500 units of the product 1 ( $u_1$ ); 4500 units of the product 2 ( $u_2$ ); 14500 units of the product 3 ( $u_3$ ); 8000 units of the product 4 ( $u_4$ ); 25500 units of the product 5 ( $u_5$ ); 15500 units of the product 6 ( $u_6$ ); 9500 units of the product 7 ( $u_7$ ); 4500 units of the product 8 ( $u_8$ ); 8500 units of the product 9 ( $u_9$ ); 8500 units of the product 10 ( $u_{10}$ ) i 4500 units of the product 11 ( $u_{11}$ ).

Based on the above data we will form the model containing three criteria functions: net-profit, output, and revenues from exports. The stated criteria functions have to be maximized.

**IV. MULTICRITERIA LINEAR PROGRAMMING MODEL**

Technological variants are different ways of producing the same product. In our example the variant U-7 represents production on a special machine which requires high participation of labour in the manufacturing process, while in the variant NC-A the manufacturing process is wholly automated with minimal participation of labour.

Let  $x_{ig}$  = quantity of  $i$ -product produced by  $g$ -technological variant ( $i = 1, \dots, 11$ ;  $j = 1, \dots, 4$ ).

**a) Criteria functions**

Net-profit:

$$(\max) f_1 = \sum_{i=1}^{11} \sum_{g=1}^4 c_{ig1} x_{ig}, \text{ where } c_{ig1} \text{ is net-profit from the}$$

Table 1.

Output:

$$(\max) f_2 = \sum_{i=1}^{11} \sum_{g=1}^4 c_{ig2} x_{ig}, \text{ where } c_{ig2} \text{ are equivalency}$$

coefficients from the Table 2.

Revenues from exports:

$$(\max) f_3 = \sum_{i \in I} \sum_{g=1}^4 c_{ig3} x_{ig}$$

where  $c_{ig3}$  are net sale prices from the Table 1, where they are the same at any variant. As only some of the products are exported  $I$  represents a set of indices of exported products or  $I = \{1, 3, 4, 5, 10\}$ .

It has to be pointed out that some products cannot be manufactured by all variants. Thus by the second variant U-11 the first and the second product cannot be produced, i.e.  $x_{12} = x_{22} = 0$ , which can be easily perceived from the Table 1.

**b) Constraints**

Lathes U-7, U-11, NC-P i NC-A

$$(1) \sum_{i=1}^{11} a_{i1}x_{i1} \leq 405000, \quad (2) \sum_{i=1}^{11} a_{i2}x_{i2} \leq 101000,$$

$$(3) \sum_{i=1}^{11} a_{i3}x_{i3} \leq 130460, \quad (4) \sum_{i=1}^{11} a_{i4}x_{i4} \leq 99490,$$

where  $a_{ig}$ , ( $g = 1, \dots, 4$ ) is the lathe operation time in a particular variant from the Table 3.

Material capacity:

As only one material has a limited capacity ( $t = 1$ ) this constraint is obtained from the Table 4., where  $q_{ilg}$  are consumption indicators of the basic material from that table.

$$(5) \sum_{i=1}^{11} \sum_{g=1}^4 q_{ilg}x_{ig} \leq 46000.$$

Additional constraints result from market constraints depending on the possibility of sale, as explained above, and naturally from the non-negativity constraints.

$$d_i \leq x_i \leq u_i, \quad (i = 1, \dots, 11)$$

$$x_{ig} \geq 0 \quad (i = 1, \dots, 11; g = 1, \dots, 4).$$

*A. Solving the Production Program and Technological Variants Optimization Model by MLP Methods*

The presented model is first solved by application of linear programming maximizing separately each of the three criteria functions on the set of allowable solutions. The following solutions are obtained:

TABLE V OPTIMAL (MARGINAL) SOLUTIONS

Solution	Variable values	Criteria functions values		
		$f_1$	$f_2$	$f_3$
$x^1$	$x_{21}=2253,$ $x_{33}=1638,$ $x_{3,4}=12862,$ $x_{42}=8000,$ $x_{54}=772,$ $x_{64}=15500,$ $x_{71}=9500,$ $x_{81}=4500,$ $x_{91}=8500,$ $x_{10,2}=4658;$ $x_{11,2}=370;$ $x_{11,3}=4130$	127073 (100%)	226165 (93.7% of $f_2^*$ )	366870 (48.5% of $f_3^*$ )

$x^2$	$x_{14}=7500,$ $x_{21}=4254,$ $x_{34}=3872,$ $x_{41}=8000,$ $x_{51}=4369,$ $x_{64}=15500,$ $x_{72}=9500,$ $x_{91}=1112,$ $x_{92}=7388,$ $x_{11,2}=151,$ $x_{11,3}=4349$	122720 (96.6% of $f_1^*$ )	241245 (100%)	281410 (37,2% of $f_3^*$ )
$x^3$	$x_{11}=7500,$ $x_{31}=14500,$ $x_{41}=8000,$ $x_{51}=25500,$ $x_{10,1}=6000,$ $x_{10,2}=2500$	26685 (21% of $f_1^*$ )	57480 (23.8% of $f_2^*$ )	757130 (100%)

From the Table 5 it is obvious that by maximizing function  $f_1$  the obtained maximal value for that function is significantly different from the value when functions  $f_2$  and  $f_3$  are maximized respectively. A significant difference in the value of particular functions also appears at maximization of the other two criteria functions.

Consequently, it is obvious that the application of linear programming is inadequate for determination of the optimal production program and selection of optimal technological variants, and that it is necessary to apply multicriteria linear programming. Namely, the company that has to choose one solution for realisation is restricted only to optimal (marginal) solutions of one of the criteria functions, which differ significantly (one criterion assuming the maximal value while the other two criteria are insufficiently satisfied), unless the MLP methods are used. Solving the model by the MLP methods will result in a compromising solution that will provide acceptable values for criteria functions.

A1. Solving the model by interactive approach by the redesigned surrogate worth trade-off method (SWT)

#### A1.1. Surrogate worth trade-off method (SWT)

Surrogate Worth Trade-off Method was developed by Haimes, Hall and Friedman [6]. This method starts from the idea that for the decision maker it is much easier to estimate the relative trade-off value of marginal increases and reductions between the two criteria functions than their absolute values.

The method contains two stages: stage 1 – identification and creation of nondominated solutions formed by trade-off functions, and stage 2 – searching for the preferred solution within nondominated solutions. The position of the preferred solution is determined in interaction with the decision maker who evaluates the nondominated links by using newly introduced surrogate worth trade-off function. Solving a number of MP models by the SWT method we have found out that for the decision makers it is hard to understand trade-off functions due to different measure units by which criteria functions are expressed. We started from the idea that it is easier for the decision maker to evaluate trade-off functions if all criteria functions were expressed by the same measure unit. The most suitable way of reducing the criteria functions of the model to one measure unit seemed to be reduction of criteria functions to nondimensional space by dividing the criteria function by the maximal value on the given set of allowable solutions. This enabled us to reduce the SWT method to solving of the following model:

Trade-off functions are determined from the dual variables linked with the constraints of the following model:

$$(\max) f = \frac{f_1(\underline{x})}{f_1^*} \cdot 100$$

s.t.

$$\frac{f_j(\underline{x})}{f_j^*} \cdot 100 \geq \varepsilon_j, \quad j = 2, \dots, k \quad (1)$$

$$\underline{x} \in X$$

where  $X$  is a set of possible solutions defined by the constraints of the model,  $\varepsilon_j$  are deviations from the optimal values which are parameter varied in the process of construction of nondominated solutions and trade-off functions. Creation of nondominated solutions by application of the model (1) is identical to the method of  $\varepsilon$ -constraints.

Generalised Lagrangian,  $L$ , of the model (1) is

$$L = \hat{f}_1(\underline{x}) + \sum_{i=1}^m \mu_i g_i(\underline{x}) + \sum_{j=2}^k \lambda_{1j} (\hat{f}_j(\underline{x}) - \varepsilon_j) \quad (2)$$

where  $\hat{f}_1(\underline{x}) = \frac{f_1(\underline{x})}{f_1^*} \cdot 100$ ,  $\hat{f}_j(\underline{x}) = \frac{f_j(\underline{x})}{f_j^*} \cdot 100$ , and

$\mu_i$ ,  $i = 1, \dots, m$ , and  $\lambda_{1j}$ ,  $j = 2, \dots, k$  are generalised

Lagrange multipliers. The mark  $1j$  at  $\lambda$  denotes that  $\lambda$  is a Lagrange multiplier linked to the first criteria function,  $\hat{f}_1(\underline{x})$  and  $j$ -constraint. Lagrange multiplier  $\lambda_{1j}$  will be generalised as  $\lambda_{lj}$  linked to the  $l$ -criteria function and  $j$ -constraint. Let  $\hat{X}$  be the set of all  $\underline{x}$ , and  $\Omega$  the set of all  $\lambda_{lj}$  satisfying Kuhn – Tucker's conditions (Kuhn and Tucker [9]) for the model (2). The following conditions are of interest for us:

$$\lambda_{1j} (\hat{f}_j(\underline{x}) - \varepsilon_j) = 0, \quad j = 2, \dots, k$$

$$\lambda_{1j} \geq 0.$$

Consequently, if  $\hat{f}_j(\underline{x}) > \varepsilon_j$ , for some  $C$  (condition is not obligatory), then the corresponding Lagrange multiplier  $\lambda_{1j} = 0$ . The value  $\lambda_{1j}$ ,  $j = 2, \dots, k$ , which corresponds to obligatory constraints ( $\hat{f}_j(\underline{x}) - \varepsilon_j = 0$ ) is interesting as it indicates the marginal benefit (cost) of the criteria function  $\hat{f}_j(\underline{x})$  caused by reduction of one unit  $\varepsilon_j$ .

By deriving the model (2) we get:

$$\lambda_{1j}(\varepsilon_j) = -\partial L / \partial \varepsilon_j, \quad j = 2, \dots, k.$$

However, for  $\underline{x} \in \hat{X}$ ,  $\lambda_{1j} \in \Omega$  for all  $j$ ,  $\hat{f}_1(\underline{x}) = L$ , then

$$\lambda_{1j}(\varepsilon_j) = -\partial \hat{f}_1(\cdot) / \partial \varepsilon_j, \quad j = 2, \dots, k. \quad (3)$$

In performing trade-off functions in the SWT method of interest are only those values  $\lambda_{1j} > 0$  that correspond to  $\hat{f}_j(\underline{x}) - \varepsilon_j = 0$ , as they correspond to nondominated solutions. Consequently,  $\hat{f}_j(\underline{x}) = \varepsilon_j$ , (3) can be written as:

$$\lambda_{1j}(\varepsilon_j) = -\partial \hat{f}_1(\cdot) / \partial \hat{f}_j(\cdot), \quad j = 2, \dots, k \quad (4)$$

(3) can be generalised, where achievement index is the  $l$ -function of MP model instead of the first criteria function. Thus:

$$\lambda_{lj}(\varepsilon_j) = -\partial \hat{f}_l(\cdot) / \partial \hat{f}_j(\cdot), \quad l \neq j, \quad l, j = 1, \dots, k.$$

Haimes, Hall and Friedman [6] consider several approaches to determine  $\lambda_{lj}(\cdot)$  when criteria functions are nonlinear.

Surrogate worth function  $w_{lj}$  ensures interaction between the decision-maker and the model.  $w_{lj}$  represents the worth of decision-maker's estimation of how much (on a scale of, say, from -10 to +10, with zero denoting equal preference) he/she prefers trading-off  $\lambda_{lj}$  percentages of marginal units of the  $l$ -

criteria function  $f_l$  for one percentage of the marginal unit of  $j$ -criteria function  $f_j$ , whereby the worth of other criteria functions is not changed.  $w_{ij}$  is defined as:

$w_{ij} > 0$ , when  $\lambda_{ij}$  marginal percentages of  $f_l(x)$  are preferred to one marginal percentage of  $f_j(x)$ , whereby all criteria are satisfied on the level  $\varepsilon_j$ ,  $j = 1, \dots, k$ .

$w_{ij} = 0$ , when  $\lambda_{ij}$  marginal percentages of  $f_l(x)$  are equivalent to one marginal percentage of  $f_j(x)$ , whereby all criteria are satisfied on the level of  $\varepsilon_j$ ,  $j = 1, \dots, k$ .

$w_{ij} < 0$ , when  $\lambda_{ij}$  marginal percentages of  $f_l(x)$  are not preferred to one marginal percentage of  $f_j(x)$ , whereby all criteria are satisfied on the level of  $\varepsilon_j$ ,  $j = 1, \dots, k$ .

In order to find a set of indifferent nondominated solutions<sup>3</sup> the decision-maker is asked whether  $\lambda_{ij}$  percentages of criteria function  $f_l(x)$  are more, less, or equally preferred to one percentage of the criteria function  $f_j(x)$ . The worth of  $\lambda_{ij}^*$  is selected so that  $w_{ij}(\lambda_{ij}^*) = 0$ .

The interaction with the decision-maker goes on until a single solution  $\underline{f}^*$  is found for which all  $w_{ij}(\lambda_{ij}^*)$  are equal to zero. This may not be realised in the first attempt. Then it is necessary to approximate the process by developing the approximate link functions for  $\hat{w}_{ij}(\underline{f})$  (by regression or interpolation) determining the solution  $\underline{f}^*$  ( $\underline{f}^*$  does not contain  $f_l$ ) from the set of equations:

$$\hat{w}_{ij}(\underline{f}^*) = 0, \quad j = 1, \dots, k, \quad j \neq l.$$

Then the following model is solved:

$$\begin{aligned} &(\max) \hat{f}_l(x) \\ &\text{s.t.} \\ &\hat{f}_j(x) \geq f_j^{**} \quad j = 1, \dots, k; \quad j \neq l \\ &g_i(x) \leq 0 \quad i = 1, \dots, m. \end{aligned} \quad (5)$$

If there is an optimal solution to the model (5) then it should be an indifferent solution. However, if the solution of the

given model is not indifferent, it is necessary to repeat the process creating more nondominated solutions around  $\hat{w}_{ij} = 0$  and continue interaction with the decision maker.

Another way to obtain an indifferent solution, if it has not been obtained in the previous step, involves solving the model (5), whereby  $f_j^{**}$  represents  $f_j(x)$ ,  $j = 1, \dots, k$ ,  $j \neq l$ , for each  $\lambda_{ij}^*$ . The solution of the given model has to be an indifferent solution.

#### A1.2. Model solving

The decision-maker first determines the most important criteria function. That is function  $f_1$ .

By the SWT method the following LP model is solved:

$$(\max) f = \frac{f_1(x)}{f_1^*} \cdot 100$$

s.t.

$$\begin{aligned} &\underline{x} \in X \\ &\frac{f_2(x)}{f_2^*} \cdot 100 \geq \varepsilon_2 \\ &\frac{f_3(x)}{f_3^*} \cdot 100 \geq \varepsilon_3 \end{aligned}$$

where  $X$  is the set of allowable solutions,  $f_1(x)$ ,  $f_2(x)$  i  $f_3(x)$  are the model criteria functions,  $f_1^*$ ,  $f_2^*$  and  $f_3^*$  are the optimal values of criteria functions on the given set of allowable solutions, and  $\varepsilon_2$  and  $\varepsilon_3$  are values in percentages given by analysts in order to obtain allowable solutions that completely satisfy these two constraints.

By solving the LP model with different values for " $\varepsilon$ " the following nondominated solutions are obtained:

<sup>3</sup> Indifferent nondominated solution is the one for which  $w_{ij}(\lambda_{ij}^*) = 0$ .

TABLE VI NONDOMINATED SOLUTIONS

Solution	Criteria functions values			$\lambda_{12}$	$w_{12}$	$\lambda_{13}$	$w_{13}$
	$f_1$	$f_2$	$f_3$				
$\underline{x}^1$	95.7273	92	90	-0.1549	-2	-0.1535	+1
$\underline{x}^2$	94.9530	92	95	-0.0636	-9	-0.3519	+4
$\underline{x}^3$	94.1841	93	95	-2.9813	+8	-0.6893	+6
$\underline{x}^4$	98.6920	98	59	-1.0071	+5	-0.1831	+2
$\underline{x}^5$	99.2479	96	59	-0.1695	-2	-0.1291	-2
$\underline{x}^6$	99.4151	94	59	-0.0161	-7	-0.1093	-3
$\underline{x}^7$	97.1047	98	66	-1.8709	+6	-0.3037	+4
$\underline{x}^8$	98.0365	97	66	-0.2960	0	-0.1516	0
$\underline{x}^9$	98.5133	95	66	-0.1663	-2	-0.1287	-2
$\underline{x}^{10}$	98.6663	93	66	-0.0161	-7	-0.1093	-3
$\underline{x}^{11}$	98.6824	92	66	-0.0161	-7	-0.1093	-3
$\underline{x}^{12}$	93.7269	98	73	-3.7093	+10	-0.5698	+6
$\underline{x}^{13}$	96.3978	97	73	-0.2282	-2	-0.3031	+4
$\underline{x}^{14}$	97.5674	95	73	-0.2960	0	-0.1516	0
$\underline{x}^{15}$	97.7786	94	73	-0.1663	-2	-0.1287	-2
$\underline{x}^{16}$	97.9175	92	73	-0.0161	-2	-0.1093	-3
$\underline{x}^{17}$	96.6578	95	79	-0.2960	0	-0.1516	0
$\underline{x}^{18}$	97.2767	93	78	-0.1612	-2	-0.1397	-1
$\underline{x}^{19}$	97.1317	93	79	-0.1612	-2	-0.1397	-1
$\underline{x}^{20}$	97.2320	90	79	-0.0086	-8	-0.1212	-2
$\underline{x}^{21}$	95.1142	95	85	-2.9813	+8	-0.6893	+6
$\underline{x}^{22}$	94.4249	95	86	-2.9813	+8	-0.6893	+6
$\underline{x}^{23}$	96.2722	93	85	-0.2537	-1	-0.1509	-1
$\underline{x}^{24}$	96.1206	93	86	-0.2581	-1	-0.1521	+1
$\underline{x}^{25}$	96.3758	91	86	-0.0086	-8	-0.1212	-2
$\underline{x}^{26}$	95.4204	92	92	-0.1549	-2	-0.1535	+1
$\underline{x}^{27}$	95.2668	92	93	-0.1549	-2	-0.1535	+1
$\underline{x}^{28}$	93.4887	92	99	-0.4128	+2	-0.4114	+5
$\underline{x}^{29}$	93.6942	90	99	-0.0656	-9	-0.3475	+4

Solving the given models results in obtaining the corresponding values  $\lambda_{12}$  and  $\lambda_{13}$  for each non-dominated solution represented by values of dual variables linked to constraints  $\frac{f_2(x)}{f_2^*} \cdot 100 \geq \varepsilon_2$  and  $\frac{f_3(x)}{f_3^*} \cdot 100 \geq \varepsilon_3$ , respectively.

Based on the values  $\lambda_{12}$  and  $\lambda_{13}$  the decision maker estimates the  $w_{12}$  and  $w_{13}$  (surrogate worth functions) for

each nondominated solution. From the above table it is obvious that the solution  $\underline{x}^8$  with criteria function values:

$$f_1 = 98.0365\% = 0.980365 \cdot f_1^* = 124578,$$

$$f_2 = 97\% = 0.97 \cdot f_2^* = 234008,$$

$$f_3 = 66\% = 0.66 \cdot f_3^* = 499706,$$

has  $w_{12}$  and  $w_{13}$  equal to zero, and also the highest value of the function  $f_1$  among these solutions. Consequently, from

the viewpoint of decision-maker this solution is indifferent and can be taken as the preferred one.

The values of variables in this solution are:

$x_{11} = 6932$ ,  $x_{21} = 2316$ ,  $x_{34} = 14500$ ,  $x_{41} = 2869$ ,  $x_{42} = 5131$ ,  
 $x_{64} = 13830$ ,  $x_{72} = 9182$ ,  $x_{91} = 5234$ ,  $x_{92} = 3266$ ,  $x_{10,1} = 8500$ ,  
 $x_{11,2} = 151$ ,  $x_{11,3} = 4349$ .

Hereby all the machines work at full capacity and the complete quantity of basic material (46000 kg) is consumed. All products with the exception of  $x_5$  are produced. It has to be pointed out that some products are manufactured partly by one and partly by some other variant. The choice of variants, or the possibility of such a solution by which a particular product is produced by only one of the four possible variants, requires introduction of 0-1 variables and will not be considered in this work.

## V. CONCLUSION

The research presented in this paper points to the possibility of an efficient application of MLP methods in solving the problem of determining an optimal production plan in the metal industry enterprise, whereby by an optimal production plan we mean an optimal production program and optimal technological variants. The research also shows that the application of the MLP method in solving this problem is a "necessity" resulting from the fact that within the enterprise there are multiple conflicting objectives, so that optimization in terms of criteria suitable for one objective leads to the failure, or a partial failure in achieving other objectives. Application of an appropriate MLP method results in a compromise solution (a nondominated one) providing values of the criteria functions that are acceptable to the decision-maker, which eliminates this failure.

It has to be stressed that we have used a concrete example to point to the fact that optimization of production program and technological variants is by its nature a multicriteria problem. Namely, given certain assumptions, this problem can be viewed as a deterministic MLP problem. The obtained results reveal the multicriteria nature of the problem and the need for cooperation between the decision-maker and the analyst (an expert in multicriteria programming and decision making). The presented optimal production plan has to be used by the decision maker as the sound basis for the final decision on the structure of the production program and technological variants of production for a particular period. Based on a thus obtained production plan the company can organise in a more efficient way its other functions such as purchase, sales, finance, development, etc.

When determining an optimal production program and technological variants by application of the MLP methods, it is necessary to involve experts from all the company functions in order to include all the constraints into the model in the best possible way, and to utilize all the available capabilities of the

enterprise. The optimal production program and the optimal technological variants must not be seen as a static category but as a dynamic ongoing process. It is necessary to continually follow all the relevant internal and external changes, to include them into the model, and to present the obtained results to the decision maker. The optimal production program and optimal technological variants have to be related to a particular period of time: a year, a half-year, a quarter, a month, a week, a day. A well developed information system, available experts in multicriteria programming and decision making, and a certain degree of trust in information obtained from the analyst, are the preconditions for an efficient application of the MLP methods in this field. In our opinion, the company which bases determining of its production program and technological variants on the results obtained by application of the MLP methods will achieve results that are better in comparison to those resulting merely from reliance on experience and intuition of the decision-maker.

In order to improve the application of the MLP methods in solving the problems of this kind we propose the building-up of a decision making support system involving the proposed MLP model for optimization of production plan and the MLP methods to be applied for obtaining nondominated solutions from which the decision-maker selects the preferred solution. That is a project that should be further developed.

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