International Journal of Business, Human and Social Sciences
ISSN: 2517-9411
Vol:6, No:5, 2012

Uncertainty Propagation and Sensitivity Analysis During Calibration of an Integrated Land Use and Transport Model

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Abstract—In this work, propagation of uncertainty during calibration process of TRANUS, an integrated land use and transport model (ILUTM), has been investigated. It has also been examined, through a sensitivity analysis, which input parameters affect the variation of the outputs the most. Moreover, a probabilistic verification methodology of calibration process, which equates the observed and calculated production, has been proposed. The model chosen as an application is the model of the city of Grenoble, France. For sensitivity analysis and uncertainty propagation, Monte Carlo method was employed, and a statistical hypothesis test was used for verification. The parameters of the induced demand function in TRANUS, were assumed as uncertain in the present case. It was found that, if during calibration, TRANUS converges, then with a high probability the calibration process is verified. Moreover, a weak correlation was found between the inputs and the outputs of the calibration process. The *total effect* of the inputs on outputs was investigated, and the output variation was found to be dictated by only a few input parameters.

Keywords—Uncertainty propagation, sensitivity analysis, calibration under uncertainty, hypothesis testing, integrated land use and transport models, TRANUS, Grenoble.

I. Introduction

Integrated land use and transport modeling has attracted the attention of researchers, recently. In this regard, the main thrust has been to develop models which are generic, user friendly and robust. Over the years, a large number of such models have come into existence [1], [2]. It is well known that integration of land use and transport system creates a complex nonlinear system, which evolves in different scales [2]. Analyzing these complex systems is typically a hard problem, especially in the presence of uncertainty, whose effects may be difficult to assess [3]. In such cases, calibration plays a central role, as it helps us determine optimal parameters, creating a robust model [4], [5]. Propagation of uncertainty and assessing sensitivity of the input parameters on the outputs during the calibration process, is significant for proper tuning of the model and ensuring better predicting capabilities. Moreover, if a large number of parameters are involved, such an analysis can help to significantly reduce the dimension of the whole calibration problem, which in turn helps parameter estimation [6], [7]. On the other hand, it is necessary that the calibrated model converges and the outputs produced by the calibrated model match actual observed data [8]. The goal here is to create a model that is robust to uncertainties, while verifying its consistency with observations.

Uncertainty can come from various sources in an integrated land use and transportation model (ILUTM), and is well documented [9], [3]. Moreover, while developing such a model, little can be done regarding lack of data and mis-measurement thereof, biased sampling and mis-specification [10]. Hence, in practice, quantifying uncertainty for these models is challenging [11]. Regarding evolution of uncertainty, scientists have investigated the impact of uncertainty in land use part of UrbanSim [12], [13], and several other ILUTMs like combination of DRAM-EMPAL and UTPP [3]. Researchers have also investigated the role of uncertainty in decision making and use parameter estimation in this case [14]. Sevcíková *et al.* have used Bayesian inference for estimation of optimal parameters in UrbanSim [15], [16]. Regarding model calibration, Clay *et al.* investigates multivariate uncertainty analysis along with validation exercises in MEPLAN, another ILUTM [17], [18]. But an intensive analysis, focusing on uncertainty propagation and sensitivity analysis of the calibration process of ILUTMs has largely been ignored. In this work, we have investigated effect of uncertainty during model calibration process of TRANUS, an ILUTM [19], when applied to model the region of Grenoble in France. In particular, we analyze the propagation of the uncertainty and we perform a sensitivity analysis of the calibration process developed in TRANUS. Moreover, a probabilistic verification methodology for TRANUS calibration is proposed to check if the calibration task has been achieved.

TRANUS has been used by city planners and modelers to simulate land use and transport structures in several scenarios [20], [21]. It predicts the land use and travel demand of a model and can help to evaluate some optimal parameter values for sustainable development [22]. Calibration in TRANUS is important as it estimates price adjustment factors which are then used for future predictions [19]. Uncertainty and sensitivity analysis during calibration of TRANUS is significant as it helps ranking input parameters according to their impact on variability of the adjustment factors.

The rest of the paper is organized as follows. First, TRANUS and its modeling philosophy is described in brief. Next, TRANUS calibration methodology is presented, along with method used to perform sensitivity analysis and uncertainty propagation, and then the proposed probabilistic verification methodology is formulated. Afterward, we describe the Grenoble model and the parameters assumed as uncertain during calibration process. Finally, the results achieved from calibration of Grenoble model are presented, and an analysis of the results obtained from uncertainty propagation and sensitivity analysis are carried out.

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II. TRANUS DESCRIPTION

TRANUS provides a generic framework to model land use and transportation in an integrated manner, both in urban and regional levels. The region of interest is divided into *economic sectors* and *spatial zones*. Then TRANUS combines two modules: the *land use and activity* module which simulates a spatial economic system by assessing the activity locations and economic sector interactions; and a *transportation* module, which estimates the use of the transport network and the associated disutility.

The land use and activity module estimates the productions and the consumptions for a zone at a given period, and the demands of flows that this activity generates. These demands of flows are then fed to the transportation module. In this way the movements of people or freight are explained as the results of the economic and spatial interaction between activities, the transport system and the real estate market. Then from the transportation network, once the corresponding trips are generated, travel flows are allocated to the network according to travel demand. In turn, the accessibility that results from the transport system influences the location and interaction between activities through transport disutilities, also affecting land rent.

The two modules in the system use discrete choice logit models, linked together in a consistent way. This includes activity-location, land-choice, and multi-modal path choice and assignment. The modules are then run iteratively, such that production and consumption demands for each area are met and equilibrium is achieved.

For the sake of brevity, an in-depth analysis using mathematical equations has not been presented to describe TRANUS land use and transportation modules. Interested readers may consult [23] for detailed mathematical treatment of the subject.

III. CALIBRATION OF THE LAND USE MODULE

The current work focuses primarily on the land use module. TRANUS land use algorithm consists in the resolution of a system of around 20 deterministic nonlinear equations and inequalities. The solution of such a system represents an economic equilibrium between supply and demand. This system of equations contains a number of economic parameters (e.g. demand elasticity parameters, location dispersion parameters, etc.) which can be *a priori* roughly estimated with a more or less important degree of uncertainty. In practice, it is crucial to precisely calibrate these parameters in order to get satisfactory simulations. In TRANUS, this calibration is not automatic. It is done via a series of trials and errors and requires lots of expertise. In addition to these economic parameters, TRANUS contains a set of intrinsic parameters called “adjustment factors” which allow the adjustment of the prices in order to correct the modeling errors. By adding these parameters, the system of equations is then relaxed. This increases the set of solutions and thus, in a sense, the system then becomes better-posed. Contrary to the economic parameters, TRANUS contains a procedure enabling the automatic assessment of these adjustment factors. Hence, the goal of this paper is to specifically work on this adjustment factor estimation procedure. We analyze its relevance and its sensibility with respect to economic parameters.

TRANUS calibration is generally is done with respect to a “base year”, for which the production values, real estate prices and transport flows are available as observed data for the region of interest. Let us consider a region, divided into N sectors and M zones. Productions and prices are available as observed data for a given base year. We denote by $X^{obs} \in \mathbb{R}^{N\times M}$ and $P^{obs} \in \mathbb{R}^{N\times M}$ the set of observed production data and observed data prices respectively. The data and economic parameters serve as inputs to the calibration process, and adjustment factors serve as outputs. The calibration process can be mathematically described with the following input-output model,

$$[X, H]^\top = f(X^{obs}, P^{obs}, \rho), \tag{1}$$

where $\rho = \{\rho_i\} \in \mathbb{R}^{\mathcal{P}}$ is the vector of economic parameters, which are uncertain. $X = \{X_{nm}\} \in \mathbb{R}^{N\times M}$ is the matrix of computed productions with X_{nm} being the production for sector n in zone m. The adjustment factors $H \in \mathbb{R}^{N\times M}$ allow to correct the primary given prices by assigning proper values of unit production prices $P = \{P_{nm}\}$,

$$P = H + P^{obs}, \tag{2}$$

in such a way that the computed productions X match the observed productions X^{obs}. In other words, H are introduced to correct the model in order to get a solution which verifies

$$X \cong X^{obs}. \tag{3}$$

Prices are in effect often quite volatile and, generally, the price data is the least accurate among all data. The adjustment factors H allow thus to compensate for this and also for the various other imperfections of the model.

The approach used to estimate the adjustment factors H and the productions X is iterative. The process loops until convergence is reached for the prices P and productions X, or until a maximum given number of iterations if not the case. The process is explained through a flow diagram in fig. 1.

A. Uncertainty and Sensitivity Analysis

Given ρ admits a PDF $\mathbb{P}(\rho)$, the PDF of H, $\mathbb{P}(H)$, and the sensitivity of $\mathbb{P}(H)$ to ρ, are to be determined. In this work, Monte Carlo approach has been used, both for uncertainty propagation and sensitivity analysis [24].

Let $\rho^{(k)}, k = 1, 2, \ldots, K$ be an unbiased sample of $\mathbb{P}(\rho)$, then, $\forall \rho^{(k)}$ we can compute $H^{(k)}$ from eqn. (1). The moments of H then can be approximated using Monte Carlo (MC) approximations, i.e.,

$$\mathbb{E}[H] = \frac{1}{K}\sum_{i=1}^{K} H^{(k)}, \quad \mathbb{E}[H^2] = \frac{1}{K}\sum_{i=1}^{K} {H^{(k)}}^2, \quad \ldots \tag{4}$$

$\mathbb{P}(H)$ can be then estimated using histogram approximation [25]. As for sensitivity analysis, we are interested in studying the effect of each parameter on the outputs. We are primarily focused on finding the total effects of inputs [6], i.e. examining the effect of variability of just a parameter or combination

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ISSN: 2517-9411
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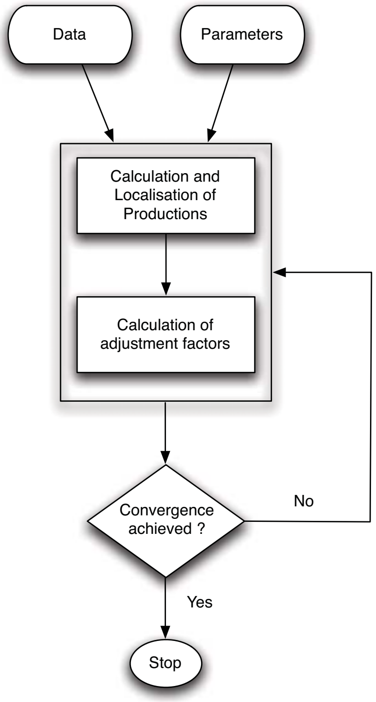


Fig. 1. Calibration process of TRANUS

of them on H. In mathematical terms, we are interested in calculating,

$$S_{T^i} = \frac{\mathbb{E}[\mathbf{V}[H|\rho_{\setminus i}]]}{\mathbf{V}[H]}, \quad S_{T^{ij}} = \frac{\mathbb{E}[\mathbf{V}[H|\rho_{\setminus(i,j)}]]}{\mathbf{V}[H]}, \quad \ldots \quad (5)$$

where, $\quad \mathbf{V}[\cdot] = \mathbb{E}\left[\cdot^2\right] - \mathbb{E}\left[\cdot\right]^2$

where $i, j \in \{1, 2, \ldots, \mathcal{P}\}$ represent the parameter's indexes, $\setminus i = (1, \ldots, i-1, i+1, \ldots, \mathcal{P})$ and $\setminus(i,j) = (1, \ldots, i-1, i+1, \ldots; 1, \ldots, j-1, j+1, \ldots, \mathcal{P})$. S_{T^i} and $S_{T^{ij}}$ represent total effects of ρ_i alone, and combination of ρ_i and ρ_j, respectively. For example, S_{T^i} is calculated varying only ρ_i over its domain and calculating its effect.

Here, we are interested to know the value of total effects of each input parameter on the variability of the adjustment factors.

B. Methodology for Verification of Calibration

The inbuilt calibration in TRANUS stops when the following criteria are met,

$$\max_{n,m} \left| \frac{P^{\tau}_{nm} - P^{\tau-1}_{nm}}{P^{\tau-1}_{nm}} \right| \leq \epsilon_1 \quad \max_{n,m} \left| \frac{X^{\tau}_{nm} - X^{\tau-1}_{nm}}{X^{\tau-1}_{nm}} \right| \leq \epsilon_2, \quad (6)$$

where τ is the iteration index (in the loop represented in fig. 1). However, as previously mentioned, if the model is sufficiently well calibrated, the production values calculated by the model, and the observed production should match. Hence, the calibration step must be verified for consistency in production values.

In this work, it has been tested that given eqn. (6) holds then the L_2 norm of the difference between the productions is less than a pre-specified value ϵ, i.e. :

$$\left|\left|X^{obs} - X\right|\right|_2 < \epsilon. \quad (7)$$

A statistical method has been employed here, where inference is drawn based on whether the sample passes a hypothesis test. The verification methodology, in the context of model calibration of TRANUS is presented next.

Given the value of ϵ we define the following random variable,

$$\mathcal{I}_X = \begin{cases} 1, & ||X^{obs} - X||_2 < \epsilon, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Let us further define another random variable Y, which is a function of $\mathcal{I}_X$. For a sample of size K of X, received from sampling the parameter space and subsequently applying the calibration methodology,

$$Y = \frac{1}{K} \sum_{k=1}^{K} \mathcal{I}_{X^{(k)}}. \quad (9)$$

Clearly, the random variable Y follows a binomial distribution: it can be proved as the random variable $\mathcal{I}_X$ is a Bernoulli random variable, hence a finite sum of them will result in a binomial random variable. Hence, $Y \sim \mathrm{B}(p, K)$, where p is the probability that eqn. (7) holds.

For the discrete random variable Y, a z-test for proportion is then carried out [26], to verify TRANUS calibration. The null (H_0) and the alternate (H_1) hypothesis are set as follows:

- $H_0 : Y > Y_0$,
- $H_1 : Y \leq Y_0$,

where Y_0 is a user specified proportion, quantifying the fraction of sample, that the user believes should pass the $z-$test. In other words, Y_0 is the user's perception on what fraction of an ensemble of input parameter vector ρ, will satisfy eqn. (7) given the ensemble satisfies eqn. (6), after outputs X and H have been obtained applying eqn. (1) and eqn. (2).

Typically, for a model that has been well designed and calibrated, Y_0 is set at a high value. The null hypothesis is accepted at a significance level α for a sample. If the null hypothesis is accepted we infer that the calibration task has been verified.

IV. TRANUS Grenoble Model

In this work, TRANUS has been applied to model the urban area of Grenoble, France [27]. The region is divided into 225 zones, and the transport network is composed of 2413 nodes. The meshes are of variable size, so that the transport network and the geographic distribution are coherent. Public transport modes (buses, tramways, trains) as well as private modes (walk, bike, and 4 categories of cars) have been implemented. The data of the transport network were obtained from the SMTC[1]. The transport data on peak hours were mainly gathered from SMTC, AURG[2], INRETS[3], and the survey EMD[4] made on Grenoble in 2010.

[1]Syndicat Mixte des Transport en Commun
[2]Agence d'Urbanisme de la Région Grenobloise
[3]Institut National de REcherche sur les Transports et leur Sécurité
[4]Enquète Ménage Déplacement

The Grenoble model considers 22 economical sectors. The population has been divided into 7 sectors: the households are classified into 4 categories depending on their income, as well as the retired households that are classified into 2 categories. The students are considered as a separate sector. The retired households and the students are made exogenous, as their location choice is more explained by attractivity criterion than by accessibility criterion. As for the 8 employment sectors, the industries and immaterial industries are made basic. Service employments are classified into 3 sectors, all induced by the model: community-based services for daily use, community-based services for occasional use and supermarkets. Schools, university and other public employments corresponds to 3 induced sectors. Finally, the real estate is divided into 7 categories: individual / collective / social housing, and 4 categories of net gross floor areas depending in their location, and use. The data for calibrating the land use model were obtained from INSEE[5], AURG, UNEDIC[6] and EMD. These data have been processed to obtain population and employment by type and by zone, as well as the prices of the real estate.

The economic sectors can also be classified as transportable and non-transportable sectors. Non-transportable sectors are the ones that must be consumed where they are produced. Typically they correspond to the real estate, i.e. housings and net gross floor areas. All sectors for employment and population are transportable. In the case of transportable sectors, because the model is distributing a total production amount to zones, the magnitude of the adjustment factors are of little significance, as long as their standard deviation over all zones is small. Otherwise it means that the adjustment factors are dominating the zonal distribution of production. In the case of non-transportable sectors, there is no zonal distribution, so the values of the adjustment factors for all zones, must be as small as possible. Detailed explanation can be found in [28]. For the Grenoble model, 7 of the 22 sectors are non-transportable and rest are transportable [27].

A. *Uncertain Parameters in the Model*

As said previously, TRANUS contains a large number of economic parameters (e.g. demand elasticity parameters, location dispersion parameters, etc.). Among these parameters, some are more difficult to estimate, but has a major impact on the calibration result. This is the case of the parameters involved in the calculation of the induced demand, that are taken here as uncertain parameters of interest [28]. The demand function is given by an exponential model,

$$a^i_{nn'} = \min_{nn'} + (\max_{nn'} - \min_{nn'})\, e^{-\delta_{nn'} U^i_{n'}} \tag{10}$$

where $a^i_{nn'}$ is the amount of production of sector n' demanded by a unit of sector n in zone i, $\max_{nn'}$ and $\min_{nn'}$ are minimum and maximum amount of n' required by a unit production of n respectively, $\delta_{nn'}$ is the elasticity parameter of n with respect to the cost of input n', and $U^i_{n'}$ is the consumption disutility of n' in i. Details on how consumption disutility $U^i_{n'}$ is calculated can be found in [23].

In this work, $\max_{nn'}$, $\min_{nn'}$ and $\delta_{nn'}$ are assumed to be uncertain. Amongst 231 possible inter-sector combination for n and n', 16 were selected. We focused on the parameters describing the demand of the housing sectors required by the various population sectors. Figure 2 describes these sector's interaction. Hence there were 48 input parameters which were uncertain. The resulting uncertainty in adjustment factors were studied, consisting of $225 \times 22 = 4950$ outputs for all zones and sectors combined. However, a comprehensive analysis, assuming all the parameters as uncertain, leading to calculation of production has not been done here, and is a topic of our future research.

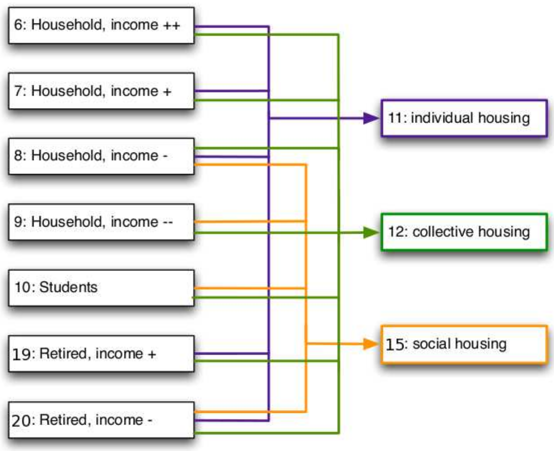


Fig. 2. Sector interactions for which we consider uncertain the parameters of the demand function. The numbers correspond to the sector id in the model. The direction of arrows correspond to directions of consumption, e.g. students consume collective or social housing.

V. Simulation Results

Monte Carlo simulations were performed with 9 samples, each of sample size 200. Hence total number of input parameter sets were 1800. As stated earlier, 16 inter-sector interactions were considered and a multivariate Gaussian distribution was assumed about a pre-specified mean with 5% standard deviation about the mean value. The parameters considered uncertain, and their mean values are given in table I, where the symbols have the same meaning as in eqn. (10). The calibration mean values correspond to approximate values obtained from opinions of practitioners and experts after analyzing the parameters for each sector-sector combination individually. Also, in the present case we assume that the random variables considered are independent of each other.

As stated in section III, the calibration methodology of TRANUS is computationally intensive. The computational time taken to run TRANUS for one set of input parameters, on an average is $350.8s$. Hence, in this work large number

[5]Institut National de la Statistique et des Etudes Economiques
[6]Union Nationale Interprofessionnelle pour l'Emploi dans l'Industrie et le Commerce

TABLE I
MEAN OF THE INITIAL GAUSSIAN PDFS AND THE CORRESPONDING SECTOR-SECTOR COMBINATION

	Elasticities ($\delta_{nn'}$)		Minimum ($\min_{nn'}$)		Maximum ($\max_{nn'}$)	
No.	Sect-Sect	Mean	Sect-Sect	Mean	Sect-Sect	Mean
1	6-11	0.075	6-11	63	6-11	172
2	6-12	0.08	6-12	51	6-12	142
3	7-11	0.0786	7-11	59	7-11	168
4	7-12	0.082	7-12	45	7-12	134
5	8-11	0.0886	8-11	50	8-11	163
6	8-12	0.095	8-12	46	8-12	132
7	8-15	0.07495	8-15	45	8-15	125
8	9-12	0.11	9-12	36	9-12	122
9	9-15	0.1	9-15	35	9-15	114
10	10-12	0.1345	10-12	14	10-12	37
11	10-15	0.1035	10-15	14	10-15	37
12	19-11	0.0886	19-11	64	19-11	148
13	19-12	0.102	19-12	48	19-12	86
14	20-11	0.098	20-11	50	20-11	145
15	20-12	0.137	20-12	45	20-12	88
16	20-15	0.0745	20-15	40	20-15	85

of samples were not considered. However, in future, we plan to work with larger number of samples. First we apply the proposed verification methodology on each sample. Then, we will present the results of uncertainty propagation and sensitivity analysis.

A. Verification Results

At first we check whether for each input set in a sample, if TRANUS converges, or if eqn. (6) holds. Then we perform the hypothesis test to verify if indeed convergence has been achieved in the sense of eqn. (7). Amongst the 1800 input sets 1509 of them satisfied eqn. (6). For the hypothesis test, the significance level α was fixed to 0.95 and ϵ and Y_0 were accordingly varied. The results received for some ϵ and Y_0 combination are given in table II.

TABLE II
RESULTS FOR THE z-TEST BY VARYING ϵ AND Y_0.

Sample	$\epsilon = 50$,$Y_0 = 0.85$		$\epsilon = 70$,$Y_0 = 0.85$		$\epsilon = 70$,$Y_0 = 0.95$	
	p value	Result	p value	Result	p value	Result
1	0.957	Pass	1	Pass	0.97045	Pass
2	0.1943	Fail	1	Pass	0.82237	Fail
3	0.9996	Pass	1	Pass	0.97495	Pass
4	0.7543	Fail	1	Pass	0.95027	Pass
5	0.5706	Fail	1	Pass	0.99143	Pass
6	0.999	Pass	1	Pass	0.98859	Pass
7	0.9993	Pass	1	Pass	0.98296	Pass
8	0.9966	Pass	1	Pass	0.9303	Fail
9	0.9998	Pass	1	Pass	0.99411	Pass

It can be observed that given $Y_0 = 0.85$ for a low ϵ value, 3 amongst 9 fail the test and for $\epsilon = 70$ and $Y_0 = 0.95$ two of them fail. It is seen, most of the samples pass the test if we increase the value of ϵ. Figure 3, shows number of samples passing the $z-$test when ϵ and Y_0 are varied. The colored lines have the same Y_0 value. It is observed that, all the samples pass the test if $\epsilon > 90$ with $Y_0 = 0.95$.

From the data, it is observed that, the L_2 norm of X^{obs} for Grenoble region is at least $||X^{obs}||_2 > 1.2 \times 10^8$. Hence for $\epsilon = 90$, $\epsilon/||X^{obs}||_2 < 7.5 \times 10^{-7}$. Thus, clearly $\epsilon = 90$ represents a very small fraction of the total production of the

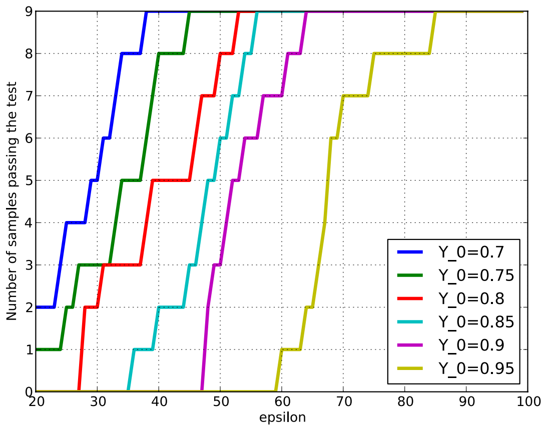


Fig. 3. Plot showing variation number of samples passing hypothesis test with ϵ for different values of Y_0

Grenoble region. It can be said that for given set of input parameters, if the TRANUS calibration process converges in the sense of eqn. (6), then for an acceptable value of ϵ, eqn. (7) is true with a significance level of $\alpha = 0.95$.

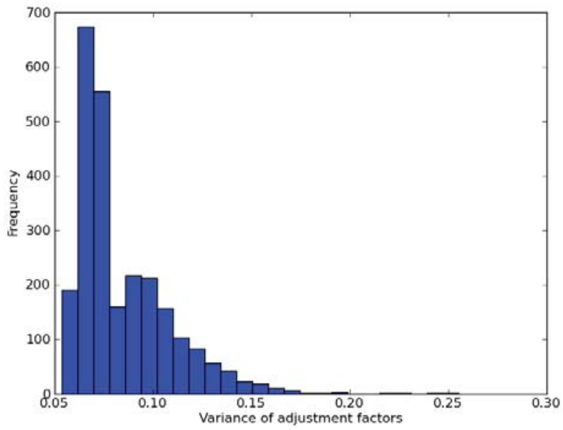
B. Uncertainty Propagation

This section assesses, how the PDF of the adjustment factors are affected when the input is uncertain. To do this, we employ the Monte Carlo approach explained in section III-A. From the verification methodology only those input sets which satisfy eqn. (6) are chosen. The price adjustment factors are normalized and presented as increase over the observed price P^{obs}. In other words, Hs in eqn. (1) and eqn. (2) are normalized to get H/P^{obs}. The adjustment factors are expected to lie between $[-1, 1]$ ideally [19]. The results are presented in an aggregate level, for the sake of ease in representation.

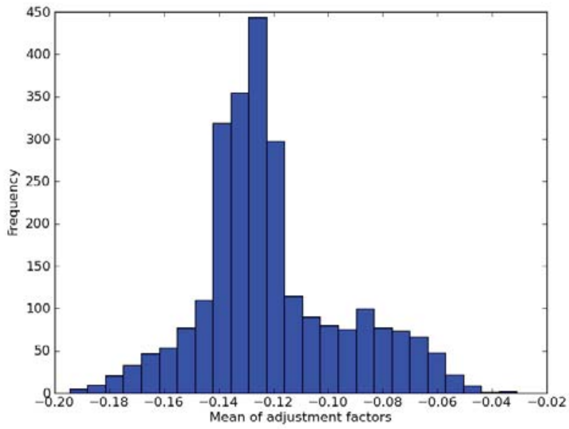
Figure 4 shows the frequency distribution for mean and variances of the adjustment factors, calculated for every sector-zone combination. It can be observed that for most cases the variances are low with a maximum frequency for $0.05 - 0.06$. However for some cases the adjustment factors have high variance. Amongst 4950 combination 5 of them were observed to have variance over 0.2. The maximum variance in adjustment factors was observed for zone 45 and and sector 2 which was 0.2417, fig. 5 shows the histogram plot.

Next the correlation structure between the adjustment factors was studied. All the sector-zone combination were considered here giving rise to 3184026 possible pairings. Amongst them 3621 sector-zone combination were highly correlated having correlation coefficient greater than 0.9. However, 11 pairings had correlation coefficient less than -0.4. The frequency distribution of the correlation coefficient has been plotted in fig. 6. It can be observed that most of the adjustment factors do not exhibit a strong dependence with other factors, as the correlations are crowded around zero.

The input-output correlation structure has been plotted in fig. 7. It can be seen that they are clustered around zero.



(a) Variance



(b) Mean

Fig. 4. Histogram of a) variance and b) mean of the adjustment factors.

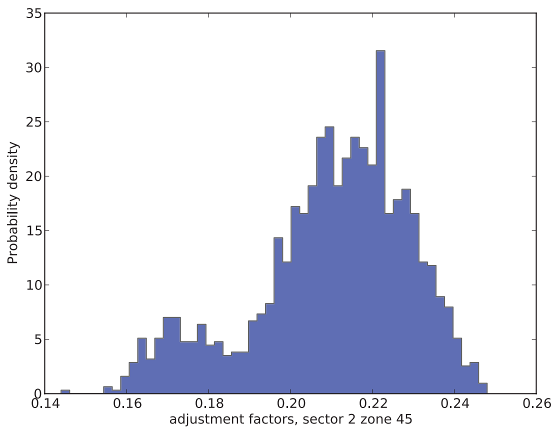


Fig. 5. PDF of the adjustment factor of zone 45 and sector 2.

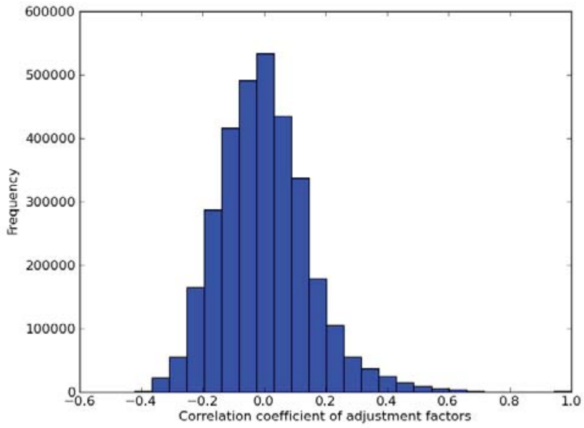


Fig. 6. Frequency distribution plot for correlation coefficient between outputs.

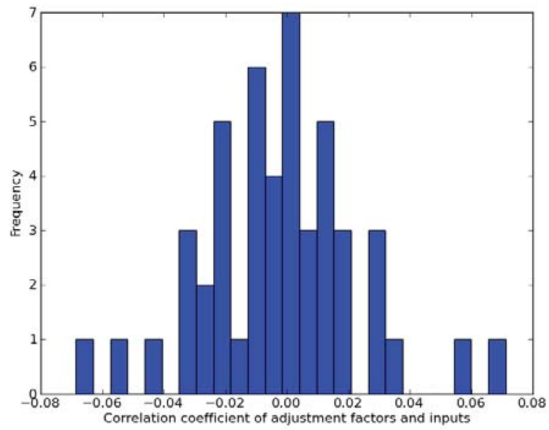


Fig. 7. Frequency distribution plot for correlation coefficient between inputs and outputs.

This indicates the absence of a linear relationship between outputs and inputs. However, though strong dependence is not exhibited, nothing can be intuitively concluded regarding actual relation between inputs and outputs.

C. *Adjustment Factors for Transportable and Non-Transportable Sectors*

In this section we investigate the variation of the standard deviation (σ_n) and maximum value H_n^{max} of the adjustment factors, for a given sector over all the zones. If the adjustment factors are defined as H_{nm}, for sector n and zone m (refer section III), then the two random variables can be defined as,

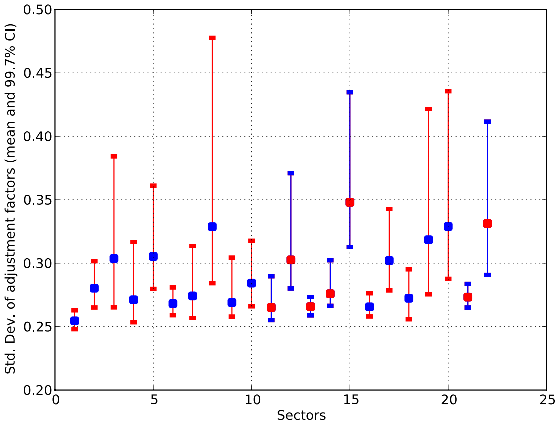


Fig. 8. Plot for mean and 99.7% confidence intervals (CI) for standard deviation of adjustment factors over all zones. Transportable sectors- blue dots and red lines. Non-transportable sectors- red dots and blue line.

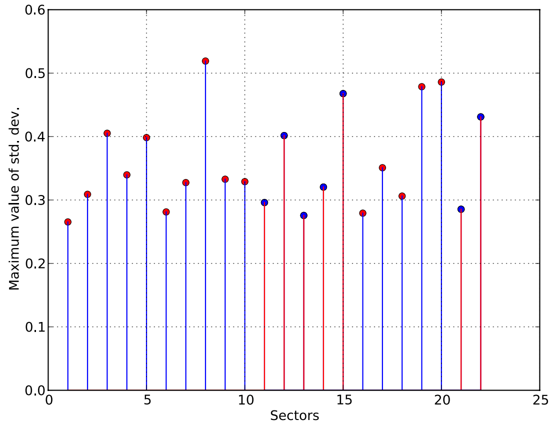


Fig. 9. Maximum of the standard deviation of adjustment factors for each sector over all zones. Transportable sectors- red dots. Non-transportable sectors- blue dots.

$$\sigma_n = \frac{1}{M-1}\sum_{m=1}^{M}\left[H_{nm} - \mathbb{E}\left[H_n\right]\right]\left[H_{nm} - \mathbb{E}\left[H_n\right]\right]^{\top} \quad (11)$$

$$H_n^{max} = \max_{m\in[1,M]} H_{nm} \quad (12)$$

where, $$\mathbb{E}\left[H_n\right] = \frac{1}{M}\sum_{m} H_{nm}$$

As stated in section IV, the transportable sectors should have a low σ_n, which is the standard deviation of H_{nm} for the sector n over all zones. Hence, we are interested in σ_n for transportable sectors. However, for non-transportable sectors, the adjustment factors itself should be low. Thus, here we investigate the maximum adjustment factor obtained in a given sector for all zones, i.e. H_n^{max}. The non-transportable sectors in the Grenoble model are, sector number 11, 12, 13, 14, 15, 21 and 22. Rest of the sectors are transportable. We consider the Monte Carlo input sets obtained from uncertainty propagation in the previous section for the present analysis.

Figure 8 shows the mean and the 99.7% confidence intervals of σ_n for all sectors, and fig. 9 shows their maximum values obtained from the samples. It can be observed that amongst transportable sectors, sector number 3, 8, 19 and 20 have maximum values for σ_n greater than 0.4. Moreover the variability of σ_n for these sectors are higher than any other transportable sector. For instance, the standard deviation of the adjustment factor for sector number 8 can go as high as 0.47 with a non zero probability.

Figure 10 shows, plots for mean and 99.7% confidence intervals for H_n^{max} for all sectors and fig. 11 shows their maximum values. It can be observed that, amongst non-transportable sectors, sector number 12, 15 and 22 have value of H_n^{max}, which is greater than 3. Moreover, the variability of H_n^{max} for sector 22 is higher than any other sector. It can be inferred from fig. 10 that +99.7% confidence interval of H_n^{max}, for sectors 11, 12, 14 and 15 extend significantly beyond their mean values. So, on an average the maximum

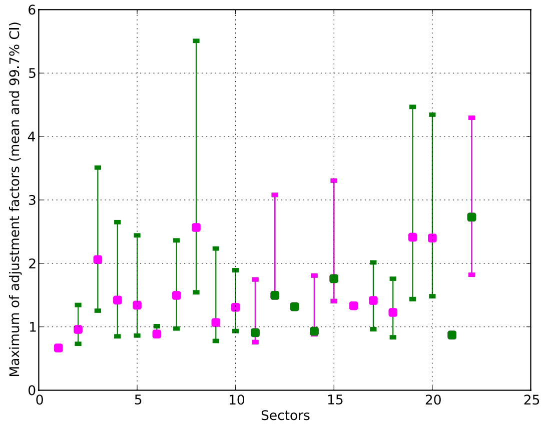


Fig. 10. Mean and 99.7% confidence intervals for H_n^{max} for each sector over all zones. Transportable sectors- magenta dots and green lines. Non-transportable sectors- green dots and magenta lines.

value of the adjustment factors for these sectors are low, but large variations may occur.

D. Sensitivity Analysis

The total effect of the uncertain input variables on the output is investigated here, using methods explained in section III-A. Here, effect of each input parameter, individually has been considered, hence, for subsequent input parameter sets just one parameter value is varied where as others are kept fixed. We work with the samples that were drawn initially for uncertainty propagation. The number of inputs were 48, corresponding to 16 elasticity, minimum and maximum consumption value parameters. For the sake of convenience in representation, the uncertain parameters have been replaced by numbers corresponding to column 1 of table I. The number of total effects observed were $4950 \times 48 = 237600$. As it is not

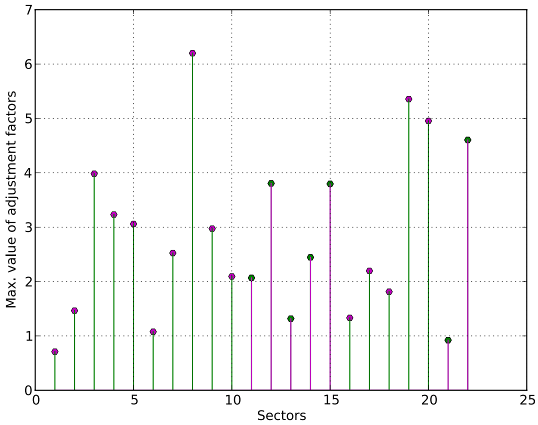


Fig. 11. Maximum H_n^{max} for each sector over all zones. Transportable sectors- magenta dots. Non-transportable sectors- green dots.

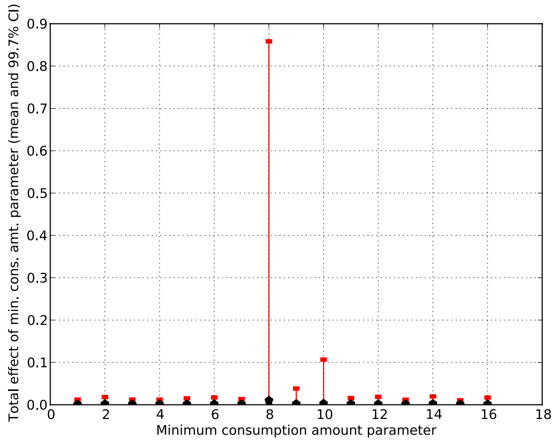


Fig. 13. Confidence intervals of the total effect of minimum consumption amount parameters ($\min_{nn'}$), over all sectors and zones. Mean- black star. CI- red lines.

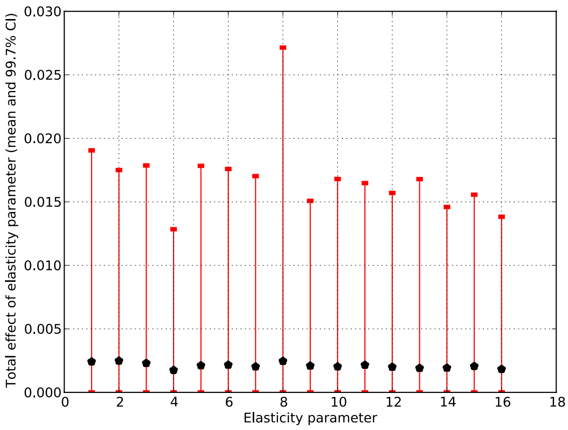


Fig. 12. Confidence intervals of the total effect of elasticity parameters ($\delta_{nn'}$), over all sectors and zones. Mean- black star. CI- red lines.

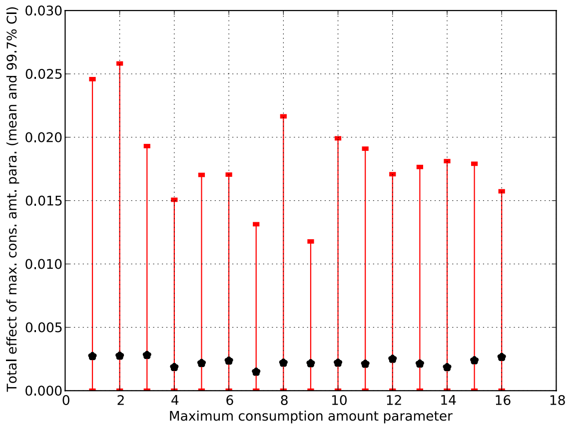


Fig. 14. Confidence intervals of the total effect of maximum consumption amount parameters ($\max_{nn'}$), over all sectors and zones. Mean- black star. CI- red lines.

possible to present all the results individually, the results are presented in an aggregate level, covering all zones and sectors, with notes on important observations.

Figure 12 shows the plot of means and 99.7% CIs for total effect of each elasticity parameter $\delta_{nn'}$, over all sectors and zones. It can be seen that elasticity parameter 8 i.e. δ_{9-12} has the $+99.7\%$ CI spanning above 0.025, which is maximum amongst any other parameters. Amongst elasticity parameters, the total effect of δ_{9-12} most significant. The effect of minimum consumption amount parameters in eqn. (10) are observed to follow a similar trend. It is seen that the parameter for sector-sector combination of 9 and 12 have the maximum total effect. From fig. 13 it can be observed that $\min_{9-12}$ can potentially be as high as 0.85, which is 8.5 times more than the next largest value any other parameter can take.

However, the total effects of maximum consumption amount parameters, behave differently. Figure 14 shows the 99.7% CI and mean for total effect of $\max_{nn'}$ over all sectors and zones. It is observed that, unlike the previous two cases, the positive end of the confidence intervals do not show large variation. The spread of the total effect of $\max_{6-12}$, is larger than any other maximum consumption amount parameter, with 0.026 being the $+99.7\%$ CI, but the positive end for $\max_{9-12}$ is just 0.003 less.

VI. Conclusion

In the current work, we have performed, propagation of uncertainty and sensitivity analysis during calibration process of TRANUS. Moreover a probabilistic verification methodology, has been proposed. The calibration process was presented as an input output model with uncertain inputs as the parameters of the induced demand function and outputs being the adjustment factors computed during calibration of TRANUS. The calibration verification methodology, sought to equate the observed production data with the calculated production during calibration. The land use and transportation model of Grenoble, France was used as an application here.

A $z-$test for hypothesis was employed for verification. It was found that with a significance level of $\alpha = 0.95$ the calibration process is verified if the TRANUS calibration algorithm converges.

The weak correlation was observed between the adjustment factors and between adjustment factors and inputs, with correlation coefficients clustered around zero. This serves as an indicator that no strong linear relationship exists between the adjustment factors and the inputs.

The variation of the standard deviation and maximum value of the adjustment factors, for a given sector over all the zones was investigated next. Amongst transportable sectors sector number 8 was observed to have the highest $+99.7\%$ confidence interval for standard deviation. As for the non-transportable sectors, sector number 22 had the largest $+99.7\%$ confidence interval for maximum value of adjustment factors.

The total effect of the input parameters on outputs was investigated next. It was found that the $\min_{9-12}$ parameter has the maximum effect amongst all other parameters. Amongst elasticity parameters, elasticity for sector combination $9-12$ has the maximum total effect, and maximum consumption amount for sector combination $6-12$ had higher total effect than any other maximum consumption parameter.

In future, we plan to use the results obtained from this analysis, to estimate parameters, and to employ techniques for Bayesian inference to get the PDF of input parameters. It is our future aim to calibrate the Grenoble model such that the land use and transportation scenarios, can be predicted and validated.

ACKNOWLEDGMENT

The authors would like to thanks Tomás de la Barra, the designer of TRANUS, for his immense help in developing the Grenoble model and expert opinion on calibration. The also authors would like to thank Mathieu Vadon and Hugo-Luis Manterola for their help with the codes of TRANUS optimization, and Alejandro Deymonnaz for help in scientific computing. We also thank the CNRS "Programme Interdisciplinaire de Recherche Ville et Environnement" (PIRVE) for the support provided to this work through the SOCLE3 project.

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