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A Search Algorithm for Solving the Economic Lot Scheduling Problem with Reworks under the Basic Period Approach

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***Abstract*—**In this study, we are interested in the economic lot scheduling problem (ELSP) that considers manufacturing of the serviceable products and remanufacturing of the reworked products. In this paper, we formulate a mathematical model for the ELSP with reworks using the basic period approach. In order to solve this problem, we propose a search algorithm to find the cyclic multiplier k_i of each product that can be cyclically produced for every k_i basic periods. This research also uses two heuristics to search for the optimal production sequence of all lots and the optimal time length of the basic period so as to minimize the average total cost. This research uses a numerical example to show the effectiveness of our approach.

***Keywords*—**Economic lot, reworks, inventory, basic period.

I. Introduction

THIS study is an extension of the economic lot scheduling problem (ELSP). We are interested in the ELSP with reworks in which a production system deals with two sources of products: manufacturing of the serviceable products and remanufacturing of the reworked products. The reworked products that are defective items produced during some manufacturing processes inside a factory must be repair or remanufacturing in order to resale.

Similar to the (conventional) ELSP, the focused variant with reworks is concerned with the scheduling of cyclical production of more ($n \geq 2$) than two products on a single facility in equal lots over an infinite planning horizon, assuming stationary and known demands for each product. The objective of the ELSP with reworks, which is abbreviated as the ELSPR, is to determine the lot size and the schedule of production of each product so as to minimize the average total cost incurred per unit time.

In past decades, most researchers discussed ELSPR under the common cycle approach. However, it is obvious that the basic period based approaches can obtain better solutions than the common cycle approach. The cost of the common cycle approach always is viewed as the upper bound of the ELSP. We did not find any article solving the ELSPR in the literature under the basic period based approaches though a lot of researchers pay more attention to the lot sizing and scheduling problems with reworks recently.

In this paper, we focus on the ELSPR under the basic period approach in which only one manufacturing lot and only one reworked lot for each product exist during a production cycle, and all the products have different replenishment cycles. For the conventional ELSP, it is straightforward to obtain a production schedule under the CC approach since the objective function value remains the same for different sequences of the manufacturing and reworked lots for different products. On the other hand, for the ELSP with reworks, different sequences (i.e., different starting times) of the manufacturing/reworked lots may lead to significant change in additional holding costs. Therefore, we have to pay attention to the sequencing of the manufacturing/reworked lots in the production schedule when searching for the optimal solution for the ELSPR.

In this paper, we present a mathematical model for the ELSPR under the basic period (BP) approach. Under the basic period approach, all products can have different cycles which are integer k_i multiples of a time period termed as a ‘basic period’. For product i, the integer k_i is called as the cyclic multiplier. It means a production cycle of product i is $T_i = k_i * B$.

In this study, we suggest a search algorithm for solving the ELSPR under the basic period approach, and it can find the optimal or near optimal cyclic multipliers of all products. We also propose two heuristics that not only determine the optimal cycle time and the optimal production sequence, but also utilize a simple scheduling heuristic to schedule the starting times of all the manufacturing and reworked lots so as to minimize the average total cost.

II. Literature Review

The ELSP has been applied for production planning and inventory control in industries such as plastics extrusion, metal stamping, textile manufacturing, bottling, printing and packing (see Boctor [1]). The solution methodologies for the ELSP may be divided into two major categories, namely, analytical approaches and heuristics. The analytical approaches include the independent solution (IS) approach, the Common Cycle (CC) approach, the basic period (BP) approach and the extended basic period (EBP) approach. The solution of the IS approach can be viewed as the lower bound of the cost for the ELSP. The solution of the CC approach can be considered as the upper bound of the cost for the ELSP. The BP and EBP approaches always obtain better solutions than the CC

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approach. For the BP or EBP approaches, it is difficult to search for cyclic multipliers $\{k_i\}$ of all products in order to generate a feasible schedule.

The reworked products after remanufacturing are usually sold at a lower price than newly products, e.g., retreated tires and reconditioned copiers. In other cases, co-assembly and remanufactured products supplied the same market as manufactured products; e.g., single used cameras, pallets and containers, and service parts of cars and computers. One may refer to Tang and Teunter [5] for further details. Schrady [3] pioneered the studies on the lot sizing with the remanufacturing of returns. Teunter [6] discussed inventory systems with recovery, and he derived simple formulae that determine the optimal lot sizes for the production/procurement of new items and for the recovery of returned items. These formulae are valid for the finite and infinite production rates as well as the finite and infinite recovery rates. Recently, Tang and Teunter [5] proposed a mathematical model for the ELSP with returns. For solving the ELSP with shelf life considerations, Soman *et al*. [4] propose an efficient deep search procedure to search for the cyclic multipliers $\{k_i\}$ of all products.

To the best our knowledge, Chang and Yao [2] first studied the ELSP with reworks. They use a heuristic to solve the ELSP with reworks under the CC approach.

However, the cost of the CC approach can be viewed as the upper bound for solving the ELSP. The basic period based approaches that include the BP and EBP approaches can obtain better solutions. To the best of the authors' knowledge, no researchers paid attention to the ELSPR under the BP or EBP approaches in the literature. It is difficult to check the feasibility of a production schedule under the EBP approach. Therefore, we are motivated to study the ELSPR under the BP approach in this paper.

III. The Assumptions and the Mathematical Model of the ELSPR

We first introduce the assumptions and notations in our mathematical model of the ELSPR as follows.

A. Assumptions and Notations

In this paper, the assumptions about the ELSPR are listed as follows.

1. Each facility can produce only one product at any time point.
2. A facility has enough capacity to produce the demand of the produced (serviceable and reworked) items during a production cycle.
3. The setup costs and setup times of the products are independent of their production sequence on a facility.
4. No shortage is allowed.
5. The parameters for each product are known and fixed at any time point.
6. The facility will not generate any more defective item when producing the reworked lot for any product.
7. Only one manufacturing lot and only one reworked lot for each product exit during a production cycle T_i.

We categorize the notations into two groups, namely, parameters and decision variables as follows.

Parameters:

a_m, a_r : The setup costs of the manufacturing lot and the reworked lot of product i respectively.

h_m, h_r: The holding costs of the manufacturing lot and the reworked lot of product i respectively.

p_m, p_r: The production rates of the manufacturing lot and the reworked lot of product i respectively.

s_m, s_r : The setup times of the manufacturing lot and the reworked lot of product i respectively.

d_i : The demand rate of product i.

n: The number of the products.

β_i : The defective rate of product i.

Decision Variables:

k_i: The cyclic multiplier of product i.

B: The time length of a basic period.

x_m , x_r :The starting times of the manufacturing lot and the reworked lot of product i respectively during a cycle T_i.

We note that the actual starting time of the setup for manufacturing lot i_m is $x_i^m - s_i^m$ and that of reworked lot i_r is $x_i^r - s_i^r$.

B. The Mathematical Model of the ELSPR

Here, we present a mathematical model for the ELSPR. The objective function can be divided into two parts as follows (see Fig. 1).

1. IC_i, which named as the "ideal cost" expressed as Eq. (2), includes the setup cost and inventory holding cost for the ideal situation where the timing of the manufacturing and the remanufacturing (reworked) lots is such that the inventory is always 0 when the production of a lot starts.
2. AC_i, which named as the "additional cost" expressed as Eq. (3), is the holding serviceable inventory caused by the non-ideal timing of manufacturing and reworked lots.

Eq. (4) is a capacity constraint that the c^{th} basic period must have enough capacity to produce all lots that should be produced at this period. If the manufacturing (reworked) lot of product i is produced at the c^{th} period, $z_{i,c}^m$ ($z_{i,c}^r$)=1; otherwise, $z_{i,c}^m$ ($z_{i,c}^r$)= 0.

In this study, we propose a search algorithm for solving ELSPR under the basic period approach. The search algorithm includes two heuristics, namely, a simple scheduling heuristic and a bisection search, to search for the optimal cyclical multipliers $\{k_i\}$, the optimal production sequence and the optimal basic period, respectively. Here, we present the whole procedure of the search algorithm. Please refer to Chang and Yao [2] for the details about the two heuristics.

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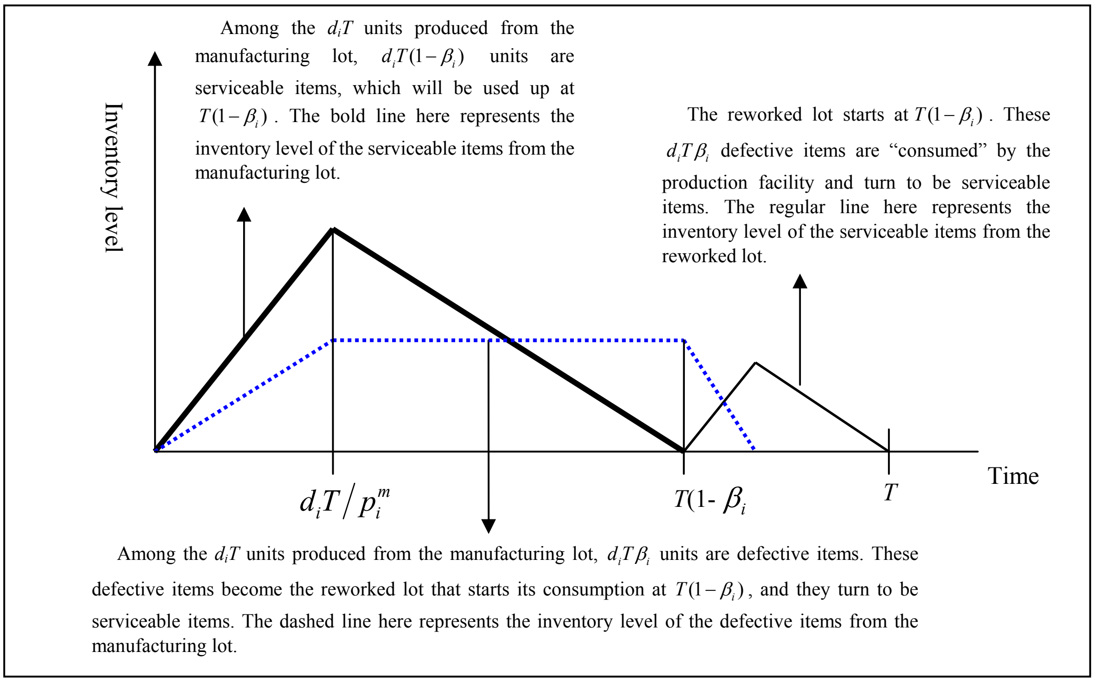

Fig. 1 The inventory level of product i in the ideal situation

$$\min TC = TC_{IC}+TC_{AC}=\sum_i IC_i + \sum_i AC_i \tag{1}$$

$$IC_i=\frac{a_i^m + a_i^r}{k_i B} + h_i^r k_i B d_i \beta_i \left[1+\beta_i\left(\frac{d_i}{2p_i^r}-1\right)-\frac{d_i}{2p_i^m}\right] + \frac{h_i^m k_i B d_i}{2}\left[(1-\beta_i)\left((1-\beta_i)-\frac{d_i}{p_i^m}\right)+\beta_i^2\left(1-\frac{d_i}{p_i^r}\right)\right] \tag{2}$$

$$AC_i = h_i^m d_i \left\{(1-\beta_i)\left[f\left(x_i^r - x_i^m\right)-k_i B(1-\beta_i)\right]^+ + \beta_i\left[f\left(x_i^r - x_i^m\right)-k_i B(1-\beta_i)\right]^-\right\} \tag{3}$$

$$\sum_{i,c}\left[z_{i,c}^m\left(s_i^m + \frac{d_i(1-\beta_i)k_i B}{p_i^m}\right)+z_{i,c}^r\left(s_i^r + \frac{d_i\beta_i k_i B}{p_i^r}\right)\right] \le B \tag{4}$$

$$[Z]^+ = \max(Z, 0) \text{ and } [Z]^- = \max(-Z, 0) \tag{5}$$

$$f(x_i^r - x_i^m) \begin{cases} (x_i^r - x_i^m) & \text{if } x_i^r - x_i^m \ge 0 \\ k_i B + (x_i^r - x_i^m) & \text{if } x_i^r - x_i^m < 0 \end{cases} \tag{6}$$

$$\sqrt{\frac{2\sum_{i=1}^n \{(a_i^m + a_i^r)/k_i\}}{\sum_{i=1}^n d_i k_i \{h_i^r \beta_i + h_i^m[\beta_i^2(1-d_i/p_i^r)+(1-\beta_i)^2(1-d_i/p_i^m)]\}}} \tag{7}$$

$$\sqrt{\frac{2(a_i^m + a_i^r)}{d_i\{h_i^r\beta_i + h_i^m[\beta_i^2(1-d_i/p_i^r) + (1-\beta_i)^2(1-d_i/p_i^m)]\}}} \quad (8)$$

We first show an iterative procedure to simultaneously determine the maximum value $k_i^{\max}$ of each product's multiplier k_i and the time length of basic period B. under the power-of-two policy as follows.

Step 1

Use Eq. (8) to determine T_i independently for each product.

Step 2

Select the smallest T_i as the initial estimate of the basic period B.

$$B = \min(T_i)$$

Step 3

Determine the integer multiple $k_i^{\max}$ for each product defined by $k_i^{\max} \geq T_i/B$ where $k_i^{\max} = \{1, 2, 4, 8, 16, \ldots\}$ is the next higher power of two integer multiplier.

Step 4

Use Eq. (7) to re-compute the basic period B using the new estimates of $k_i^{\max}$.

Step 5

Return to Step 3 to determine new $k_i^{\max}$ by using B from Step 4. The procedure terminates when consecutive iterations produce identical values of $k_i^{\max}$ at Step 4.

The whole search algorithm for solving the ELSPR under the BP approach can be described as follows.

Step 1

Use the above procedure to obtain the maximum value $k_i^{\max}$ of the cyclic multiplier k_i of product i.

Step 2

Let $\{k_i\} = 1$ for all product and use Eq. (7) to compute the upper bound B_{UB} of the basic period B. A solution TC_{CC} can be obtained by using $\{k_i\} = 1$ and B_{UB}. If constraints Eq. (4) can be satisfied, this solution TC_{CC} is feasible and can be viewed as the upper bound of the cost. Save TC_{CC} as the 'current best solution' TC_{OPT}. Put $(\{k_i\}, TC_{CC})$ into a list L.

Step 3

(1) Choose a solution with the lowest cost form list L. Remove this solution from list L.
(2) Let $i = 1$, Choose product i.
(3) If $k_i < k_i^{\max}$, double the value of k_i. Then use (7) to calculate the basic period B.
(4) Use a simple scheduling heuristic and a bisection search to search for an optimal basic period and production sequence according to $\{k_i\}$ and B. Please refer to Chang and Yao [2] for these two heuristics.
(5) Use Eq. (2) and (3) compute the ideal cost and the additional cost in order to obtain the average total cost TC.
(6) Use the capacity constraint Eq. (4) to check whether the solution $(\{k_i\}, TC(\{k_i\}))$ is feasible or not.
(7) If the solution $(\{k_i\}, TC(\{k_i\}))$ is feasible and $TC(\{k_i\}) < TC_{OPT}$, let $TC_{OPT} = TC(\{k_i\})$ and $\boldsymbol{k}_{OPT} = \{k_i\}$.
(8) If $TC(\{k_i\})$ is less than TC_{OPT}, insert the solution $(\{k_i\}, TC(\{k_i\}))$ into list L.
(9) Let $k_i = k_i/2$ and $i = i+1$. If $i > n$, go to Step 4; otherwise, choose product i and go to Step 3 (3).

Step 4

Repeat to run Step 3 until no solution can be chosen form list L.

Step 5

Output TC_{OPT} and $\boldsymbol{k}_{OPT}$.

Using the above algorithm to solve the ELSPR, we can find an optimal or near optimal solution with the lowest cost.

IV. A Numerical Example

Here, we show the effectiveness and quality of our search algorithm using a 5-product example. Table I presents the parameters of all the products in this example. We use a heuristic from Chang and Yao [2] to solve this example under the CC approach. The average total cost of the example is \$7.294 with $B = 55.766$, IC_i = \$7.173 and AC_i = \$0.121. This research uses a search algorithm to solve this example and the optimal solution (called as Solution A) listed in Table II is \$6.903 with $B = 40.901$, IC_i = \$6.846 and AC_i = \$0.058. The run time of our search algorithm is less than 3 seconds. Solution B is utilized to show how a bisection search improves the solution quality of the ELSPR. It is obvious that our optimal solution is better than the lower bound of the cost using the CC approach.

V. Conclusion

In this study, we are interested in the ELSPR that deals with two sources of products: manufacturing of the serviceable products and remanufacturing of the reworked products. To the best of the authors' knowledge, no researchers studied the ELSPR under the basic period based approaches in past years. We formulate a mathematical model for the ELSPR using the basic period approach. In order to solve this problem, we propose a search algorithm to search for the optimal cyclic multipliers of all products, the optimal production sequence of all lots and the optimal length of the basic period. Our numerical example demonstrates the effectiveness and quality of our search algorithm.

TABLE I
THE PARAMETERS OF ALL PRODUCTS IN THE EXAMPLE

Product No.	d_i	a_i^m	s_i^m	p_i^m	h_i^m	a_i^r	s_i^r	p_i^r	h_i^r	β_i
1	9	20	0.25	80	0.00175	20	0.25	80	0.00088	0.2
2	9	20	0.25	80	0.00263	20	0.25	80	0.00132	0.3
3	9	20	0.25	80	0.0035	20	0.25	80	0.00175	0.3
4	30	20	0.25	80	0.00438	20	0.25	80	0.00219	0.2
5	3	20	0.25	80	0.00525	20	0.25	80	0.00263	0.2

TABLE II
THE SOLUTIONS OF THE EXAMPLE

		the length of the basic period	ideal cost	additional cost	average total cost	Note
Solution A $\{k_i\}=\{2, 2, 1, 1, 2\}$	B is set as $B_{IC}(\{k_i\})$	40.901	6.846	0.058	6.903	No improvement using a bisection search
Solution B $\{k_i\}=\{1, 2, 2, 1, 2\}$	B is set as $B_{IC}(\{k_i\})$	39.784	7.044	0.269	7.313	
	Use a bisection search to find an optimal *B*	38.498	7.048	0.262	7.310	

TABLE III
THE PRODUCTION LOAD OF THE EXAMPLE UNDER THE BASIC PERIOD APPROACH

lot style	Product No.	$k_i^{\max}$	k_i	1th period	2th period	3th period	4th period
mfg. lot	1	4	2	7.612	0.000	7.612	0.000
	2	4	2	6.692	0.000	6.692	0.000
	3	2	1	3.471	3.471	3.471	3.471
	4	2	1	12.520	12.520	12.520	12.520
	5	4	2	2.704	0.000	2.704	0.000
reworked lot	1	4	2	0.000	2.091	0.000	2.091
	2	4	2	0.000	3.011	0.000	3.011
	3	2	1	1.630	1.630	1.630	1.630
	4	2	1	3.318	3.318	3.318	3.318
	5	4	2	0.000	0.864	0.000	0.864
period load				37.948	26.904	37.948	26.904

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