

Improvement over DV-Hop Localization Algorithm for Wireless Sensor Networks

Shrawan Kumar, and D. K. Lobiyal

Abstract—In this paper, we propose improved versions of DV-Hop algorithm as QDV-Hop algorithm and UDV-Hop algorithm for better localization without the need for additional range measurement hardware. The proposed algorithm focuses on third step of DV-Hop, first error terms from estimated distances between unknown node and anchor nodes is separated and then minimized. In the QDV-Hop algorithm, quadratic programming is used to minimize the error to obtain better localization. However, quadratic programming requires a special optimization tool box that increases computational complexity. On the other hand, UDV-Hop algorithm achieves localization accuracy similar to that of QDV-Hop by solving unconstrained optimization problem that results in solving a system of linear equations without much increase in computational complexity. Simulation results show that the performance of our proposed schemes (QDV-Hop and UDV-Hop) is superior to DV-Hop and DV-Hop based algorithms in all considered scenarios.

Keywords—Wireless sensor networks, Error term, DV-Hop algorithm, Localization.

I. INTRODUCTION

A wireless sensor network (WSN) is an interconnected set of sensor nodes. Each sensor node consists of three components: data sensing, data processing, and wireless communication [1]. These nodes have limited resources in terms of memory, battery power, and processing. WSNs have been used in different settings such as target tracking [2], environmental monitoring, smart home applications, disaster management, and intelligent transportation [3]. All these applications require location of events because sensed data are meaningful in the applications only if labeled with geographic location information. This makes location estimation of nodes (localization) a key issue in WSNs [4]. Global positioning system (GPS) is the simplest way of localizing nodes [5]. Many applications use hundreds or thousands of sensor nodes. The use of GPS in each node makes the network expensive, and overall consumption of energy increases. These issues make GPS infeasible for localization in WSN [6]. Designing a realistic localization algorithm for WSNs with less hardware requirement, limited power consumption, and lower computational cost is a challenging assignment [7].

Various localization algorithms have been proposed for WSNs, which have been categorized as range-based and range-free algorithms [8]. The range-based algorithms use

absolute point-to-point distance estimates or orientation information between neighbor nodes for localization [9]. These algorithms provide higher localization accuracy but require additional hardware for measurement of distance or angle information and are thus expensive for large-scale sensor networks [10]. On the other hand, range-free algorithms do not need the distance or orientation information between nodes. They need only network connectivity information for localization of nodes. Although range-free algorithms provide more cost-effective localization, their results are less precise than range-based algorithms [11]. The cost-effectiveness and simplicity in application of range-free algorithms motivate researchers to improve their localization accuracy [12]. Some typical range-free algorithms are Centroid [13], Amorphous [14], approximate point-in triangle test (APIT) [8], and distance vector-hop (DV-Hop) [15], of which DV-Hop is the most popular because of its facility, feasibility, and good coverage quality.

In this paper, we focus only on the range-free algorithm DV-Hop and propose improvement over the DV-Hop localization algorithm for WSNs. The proposed methods improve localization accuracy without increasing hardware cost and communication messages in the network. In these approaches, first error terms are separated from estimated distances between unknown node and anchor nodes, which then minimizes these error terms by using quadratic programming to improve the localization accuracy (which will be referred to as QDV-Hop algorithm in this paper). Furthermore, computationally complex quadratic programming problem solver is replaced by unconstrained formulation, which results in solving a system of linear equations; we call it UDV-Hop algorithm. Simulation results show that the performance of our proposed methods (QDV-Hop and UDV-Hop) are superior to typical DV-Hop algorithm, improved DV-Hop localization algorithm for WSNs (IDV-Hop) [16] and improved DV-Hop localization algorithm with reduced node localization error for WSNs (RNLEDV-Hop) [17].

The rest of the paper is organized as follows: section II presents related work. The proposed works are described in section III. In section IV, simulation results are shown and localization performances are discussed. Section V illustrates the communication cost and computational efficiency of the algorithms. Finally in section VI, we conclude the paper.

II. RELATED WORK

In the DV-Hop algorithm, location of unknown nodes (sensor nodes that do not know their location) is calculated by

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using some anchor nodes (nodes that know their location by GPS). The main drawback of the DV-Hop algorithm is that the estimated distances between nodes are prone to error. The error in the estimated distance between nodes is the cause of poor localization accuracy.

Numerous improvements of DV-Hop algorithm such as [16], [17]–[19] etc. have been proposed to improve localization accuracy. Furthermore, some improvements to the DV-Hop algorithm are also based on range-based techniques such as received signal strength indicator (RSSI). In [20], distance between the nodes is estimated by using RSSI to reduce the ranging error, but [21] shows that RSSI is not an appropriate distance measurement in a noisy environment. The accuracy of using RSSI is not only highly sensitive to multipath, fading, and other sources of interference but also requires additional hardware. To make discussion easy, here we discuss DV-Hop and its improvements [16] and [17] in brief.

A. DV-Hop Algorithm

The DV-Hop algorithm consists of three steps. In the first step, each anchor node broadcasts beacon packets with its location information and hop count value is initialized by 1. Each node in the network, which receives beacon packets, maintains a table (x_i, y_i, hop_i) for every anchor node, where (x_i, y_i) is the coordinate of anchor node i and hop_i is the minimum number of hops from anchor node i . If a received packet contains the lesser hop count value to a particular anchor node, the hop count value of the table is replaced with hop count value of the received packet, and this packet is forwarded in the network with increasing hop count value by one. Otherwise this packet is discarded. By this mechanism, all nodes in the network obtain minimum hop count value from every anchor node.

In the second step, the anchor node calculates the average size for one hop from other anchor nodes by (1)

$$HopSize_i = \left(\sum_{j \neq i} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \right) / \left(\sum_{j \neq i} h_{ij} \right) \quad (1)$$

where (x_i, y_i) and (x_j, y_j) are the coordinate of anchor node i and j , and h_{ij} is the minimum number of hops between anchor nodes i and j . After calculating hop-size, each anchor node broadcasts its hop-size in the network by using controlled flooding. When an unknown node receives this hop-size information, it saves the first arrived message (hop-size) and then transmits to neighbors. By this, most nodes receive hop-size of the nearest anchor node. When an unknown node p receives hop-size information from an anchor node, it calculates the distance between itself and anchor node by (2).

$$d_{pk} = HopSize_i \times hop_{pk} \quad (2)$$

where $HopSize_i$ is the hop-size that the unknown node p obtains from the nearest anchor node i and hop_{pk} is the minimum number of hops between unknown node p and anchor node k .

In the final step, each unknown node estimates its location by polygon method. Let location (coordinate) of unknown node p is (x, y) , location of i^{th} anchor node is (x_i, y_i) , and the distance between the anchor node i and unknown node p is d_i . Therefore, distance of unknown node p from n anchor nodes is given by (3)

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 = d_1^2 \\ (x - x_2)^2 + (y - y_2)^2 = d_2^2 \\ \vdots \\ (x - x_n)^2 + (y - y_n)^2 = d_n^2 \end{cases} \quad (3)$$

Subtracting the last equation from previous $n-1$ equations, simplifying and writing in matrix form, we obtain;

$$AX = B, \quad (4)$$

where A , B , and X are given as:

$$A = \begin{bmatrix} 2(x_1 - x_n) & 2(y_1 - y_n) \\ 2(x_2 - x_n) & 2(y_2 - y_n) \\ \vdots & \vdots \\ 2(x_{n-1} - x_n) & 2(y_{n-1} - y_n) \end{bmatrix}, B = \begin{bmatrix} x_1^2 + y_1^2 - x_n^2 - y_n^2 + d_n^2 - d_1^2 \\ x_2^2 + y_2^2 - x_n^2 - y_n^2 + d_n^2 - d_2^2 \\ \vdots \\ x_{n-1}^2 + y_{n-1}^2 - x_n^2 - y_n^2 + d_n^2 - d_{n-1}^2 \end{bmatrix}$$

$$\text{and } X = \begin{bmatrix} x & y \end{bmatrix}$$

Applying least square method in (4), location of unknown node p is obtained by (5).

$$X = (A' A)^{-1} A' B \quad (5)$$

where A' represents the transpose of matrix A .

1. Error Analysis of DV-Hop Algorithm

In the DV-Hop algorithm, it is assumed that the minimum hop path between nodes is similar to a straight line, but in practical applications, it is not so. The communication range of each node in the network is not a standard circle ideally because it is anomalous polygon. This is associated with the influence of network topology that makes every hop distance much different from others. If the average distance of per hop is used to estimate the distance between an anchor node and an unknown node, the estimated distance is different from the true distance. The difference between the estimated and true distance is called ranging error, which is the cause of localization error [22]. Example 1 explains the generating

process of ranging error [18], [23].

Example 1:

In Fig.1, $A1$, $A2$, and $A3$ are anchor nodes. U is the unknown node that needs to be localized. These anchor nodes know the distance from each other; in Fig.1, the distance is 40, 40, and 30. The number of hops between U and $A1$ is 1, between U and $A2$ is 3, and between U and $A3$ is 2. The distance between $A1$ and $A2$ is 5. Let the length of other each hop is 10. In DV-Hop algorithm, anchor nodes $A1$, $A2$, and $A3$ calculate the average distance of per hop (hop-size) as follows:

$$A1 : (40+30)/(4+3) = 10;$$

$$A2 : (40+40)/(4+5) = 8.88;$$

$$A3 : (30+40)/(3+5) = 8.75;$$

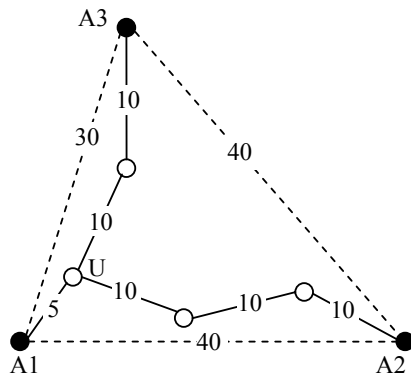


Fig. 1 Error analysis of DV-Hop algorithm

After calculating the hop-size, anchor nodes broadcast it in the network. The unknown node saves the first received value of hop-size. In Example 1, anchor nodes $A1$, $A2$, and $A3$ will broadcast their hop-size 10, 8.88, and 8.75, respectively. The unknown node U is only one hop away from the anchor node $A1$; therefore, it will receive the first message from $A1$. The hop-size of the unknown node U will be 10. The unknown node U estimates the distance between itself and anchor nodes. The distance between unknown node U and $A1$ is 10, between U and $A2$ is 30, and between U and $A3$ is 20. The true distance between U and $A1$ is 5, whereas the estimated distance is 10. Then ranging error for $A1$ is $10 - 5 = 5$, which is 50%. Due to this error, estimated location of unknown node U will be erroneous.

On the basis of the above analysis, we conclude that the localization accuracy of the DV-Hop algorithm is affected by ranging errors.

B. An Improved DV-Hop Algorithm (IDV-Hop) [16]

IDV-Hop improves the localization accuracy of DV-Hop algorithm by modifying the second and third steps of DV-Hop algorithm.

In the second step, each anchor node broadcasts a packet that contains its hop-size with its identity in the network. If a node receives this packet, it adds this information in its table and broadcasts it to neighbor nodes. When an unknown node obtains hop-size of every anchor node, which is estimated by anchor nodes in the second step of DV-Hop algorithm, the average hop-size is calculated by (6)

$$HopSize_{avg} = (\sum HopSize_i) / n \quad (6)$$

where n is the number of anchor nodes.

Now, the unknown node estimates its distance from the i^{th} anchor node by (7)

$$d_i = hops_i \times HopSize_{avg} \quad (7)$$

where $hops_i$ is the number of hops between unknown node and the i^{th} anchor node.

In the third step, the 2-D Hyperbolic location algorithm [24] is used in place of traditional triangulation algorithm to estimate the location of unknown nodes.

In the second step of this algorithm, unknown node receives the hop-size from all anchor nodes and forwards it to neighbor nodes to estimate the average hop-size. In receiving and forwarding hop-size messages from all anchor nodes, communication cost of the algorithm increases. Therefore, the improvement in localization accuracy compared with DV-Hop costs increased communication.

C. An Improved DV-Hop Algorithm with Reduced Node Location Error (RNLEDV-Hop) [17]

In RNLEDV-Hop, the second and third steps of DV-Hop are modified to improve the localization accuracy. In the second step, after calculating the hop-size by the second step of the DV-Hop, each anchor node i estimates the distance from every other anchor node j by (8).

$$d_{est}^{i,j} = HopSize_i \times h_{i,j} \quad \forall i \neq j \quad (8)$$

where $h_{i,j}$ is number of hops between anchor nodes i and j .

On the other hand, the true distance between anchor nodes i and j is

$$d_{true}^{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (9)$$

where (x_i, y_i) and (x_j, y_j) are coordinate of anchor node i and j .

The difference between the estimated distance and the true distance is corresponding to estimation error denoted as $e^{i,j}$ in (10).

$$e^{i,j} = d_{est}^{i,j} - d_{true}^{i,j} \quad (10)$$

By using this estimated error, the anchor node estimates its effective average hop-size by (11)

$$\text{HopSize}_{\text{eff}}^{i,j} = \text{HopSize}_i - \frac{e^{i,j} + e^{i,k}}{h_{i,j} + h_{i,k}} \quad (11)$$

where k is the nearest anchor node from the anchor node i .

Each anchor node broadcasts its effective hop-size in the network. When unknown node p receives effective hop-size from any anchor node, it estimates the effective distance from the anchor node j by (12)

$$d_{\text{eff}}^{p,j} = \text{HopSize}_{\text{eff}}^{i,j} \times h_{p,j} \quad (12)$$

where i is the nearest anchor node from unknown node p .

In the third step, by using the effective distance, unknown node estimates its location by 2-D Hyperbolic location algorithm. The covariance matrix of range estimation error is also used as weighted matrix for localization improvement.

Localization accuracy of this algorithm is better than DV-Hop. But each anchor node involves extra computation of effective hop-size to refine the hop-size. Again, the computation of the covariance matrix of range estimation error increases computational work.

III. PROPOSED ALGORITHMS

In this paper, we mainly focus on the third step of DV-Hop algorithm. Communication ranging error introduces error in estimating the distance between the unknown node and the anchor node, calculated by hop-size and number of hops that affects the localization accuracy of DV-Hop algorithm. In the proposed approaches, we try to enhance the localization accuracy by two methods, viz., QDV-Hop and UDV-Hop. In QDV-Hop algorithm, we first separate error terms from estimated distances between the unknown node and anchor nodes and then minimize the effect of these error terms by using quadratic programming to obtain better localization accuracy. Quadratic programming needs special and computationally costlier optimization tool-box. To obviate the need of quadratic programming solver, we propose UDV-Hop algorithm that provides similar localization accuracy as in QDV-Hop algorithm. In UDV-Hop algorithm, error terms from estimated distances between the unknown node and anchor nodes are separated in the same way as in QDV-Hop algorithm. Then, these error terms are minimized by using unconstrained optimization method. In both the schemes, the error terms are separated in the same way.

A. Separation of Error Terms

We first separate the error terms from estimated distances between the unknown node and anchor nodes and then convert the system of equations (made by measuring the distances between the unknown node and anchor nodes) into linear form in which error terms are in explicit form.

Let the true distance between unknown node $p(x, y)$ and anchor node $i(x_i, y_i)$ be $d_i + e_i$, where d_i is the estimated distance calculated by hop-size and number of hops between unknown node p and anchor node i , and e_i is correction a factor corresponding to the estimated distance d_i . This correction factor e_i is equivalent to error in estimated distance between unknown node p and anchor node i . Thus, the distance of the unknown node p from the anchor node i , $\forall i = 1, 2, \dots, n$ is described by (13).

$$\left. \begin{aligned} (x - x_1)^2 + (y - y_1)^2 &= (d_1 + e_1)^2 \\ (x - x_2)^2 + (y - y_2)^2 &= (d_2 + e_2)^2 \\ &\vdots \\ (x - x_n)^2 + (y - y_n)^2 &= (d_n + e_n)^2 \end{aligned} \right\} \quad (13)$$

Subtracting the last equation from first $n-1$ equations and dividing both side by d_n^2 , and with assumption $d_n \gg e_n$, we can neglect the terms $(e_n^2 - e_i^2)/d_n^2$, $\forall i = 1, 2, \dots, n-1$. Now simplifying them, the system of $n-1$ equations is obtained to be

$$\left. \begin{aligned} a_1x + b_1y + c_1e_1 + ce_n &= D_1 \\ a_2x + b_2y + c_2e_2 + ce_n &= D_2 \\ &\vdots \\ a_{n-1}x + b_{n-1}y + c_{n-1}e_{n-1} + ce_n &= D_{n-1} \end{aligned} \right\} \quad (14)$$

where, a_i, b_i, c_i, c and $D_i \forall i = 1, 2, \dots, n-1$ are constants, defined as

$$a_i = 2(x_n - x_i)/d_n^2, \quad b_i = 2(y_n - y_i)/d_n^2, \quad c_i = -2d_i/d_n^2, \\ c = 2/d_n, \quad D_i = ((x_n^2 + y_n^2) - (x_i^2 + y_i^2) + d_i^2)/d_n^2 - 1$$

The system of $n-1$ equations (14) has $n+2$ variables $x, y, e_1, e_2, \dots, e_n$. This underdetermined system of equations can be rewritten in matrix form as

$$GZ = D \quad (15)$$

$$\text{where } G = \begin{bmatrix} a_1 & b_1 & c_1 & 0 & \cdots & 0 & c \\ a_2 & b_2 & 0 & c_2 & \cdots & 0 & c \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ a_{n-1} & b_{n-1} & 0 & 0 & \cdots & c_{n-1} & c \end{bmatrix}$$

$D = [D_1 \ D_2 \ \cdots \ D_{n-1}]'$, and $Z = [x \ y \ e_1 \ \cdots \ e_n]'$. Here, $e_i \forall i = 1, 2, \dots, n$ are error terms.

Since the underdetermined system of equations usually has infinitely many solutions, it does not meet our requirement of

localizing the unknown node. On the other hand, our aim is to obtain better localization accuracy with less computational cost. In our problem, better localization accuracy can be obtained by minimizing the magnitude of error terms. We exploit the above factors by minimizing the magnitude of error terms e_1, e_2, \dots, e_n while using (15) as a set of constraints for the minimization problem.

B. QDV-Hop Algorithm

The first part of QDV-Hop algorithm (separation of error terms from distances estimated between the unknown node and anchor nodes) is described above in section 4.1. Now, norm-2 of error terms $e_1, e_2, e_3, \dots, e_n$ is minimized by using quadratic programming. The minimization problem of (15) in quadratic form can be obtained as

$$\min_{e_i} \sum_{i=1}^n e_i^2, \text{ subject to } GZ = D$$

where G , D , and Z are defined by equation (15).

The distance between the unknown node and the anchor node is estimated by using hop-size and number of hops. Hop-size for each pair of unknown node and anchor node is erroneous. The anchor nodes that are far are assumed to have more erroneous assessment of distance. Thus, an appropriate weight matrix can be used that prefers the distance provided by nearer anchor nodes compared with farther ones. Therefore, further improvement in accuracy of unknown node localization can be achieved by introducing weight matrix in the objective function of the above problem.

$$\min_e e'We, \text{ subject to } GZ = D \quad (16)$$

where W is the weight matrix defined as

$$W = \begin{bmatrix} w_{p,1} & 0 & \dots & 0 \\ 0 & w_{p,2} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & w_{p,n} \end{bmatrix},$$

where $w_{p,i}$ is the weight of the unknown node p for the i^{th} anchor node and is given as $w_{p,i} = (h_{p,i})^{-1}$, where $h_{p,i}$ is the minimum number of hops between the unknown node p and the anchor node i .

In solving (16), error terms $e_1, e_2, e_3, \dots, e_n$ are minimized, together with coordinate of unknown node (x, y) contained in the variable Z also obtained. Results obtained by QDV-Hop algorithm are superior to DV-Hop, IDV-Hop, and RNLEDV-Hop. But the use of quadratic programming to minimize error terms requires a special quadratic tool box that makes heavy computation and hence increases consumption of

time and energy. A WSN cannot allow high energy consumption and computation time to improve localization accuracy. Therefore, to overcome these drawbacks, we minimize error terms by using unconstrained optimization method that provides localization accuracy of the same order as that obtained by using quadratic programming tool.

C. UDV-Hop Algorithm

The first part of UDV-Hop algorithm (separation of error terms from distances estimated between the unknown node and anchor nodes) is depicted above in section 4.1. Here, we minimize the error terms by using unconstrained optimization method.

The quadratic form of the problem (15) minimizes the correction factor and is obtained as (16). In this section, we reformulate the problem (16) as an unconstrained minimization problem as:

$$\min_e (e'We + \nu \|GZ - D\|^2), \quad (17)$$

where $\nu > 0$ is a constant and W is the weight matrix defined in (16). Now in (17)

$$\text{Let } L = e'We + \nu \|GZ - D\|^2.$$

The optimization problem (17) will have optimal solution, where ∇L vanishes and Hessian matrix $\nabla^2 L$ of the optimization problem should be positive definite.

Below, we find ∇L and $\nabla^2 L$.

$$\text{Let } L_1 = e'We \text{ and } L_2 = \nu \|GZ - D\|^2$$

Converting $L_1 = e'We$ in terms of Z , we obtain

$$L_1 = Z'PZ$$

$$\text{where } P = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & & W \\ 0 & 0 & \dots & 0 \end{bmatrix}_{(n+2) \times (n+2)}.$$

Writing $L_1 = Z'PZ$ in summation form, we obtain:

$$L_1 = \sum_{i=1}^{n+2} \sum_{j=1}^{n+2} Z_i P_{ij} Z_j.$$

Differentiating it with respect to Z

$$\frac{\partial L_1}{\partial Z_\alpha} = \sum_{j=1}^{n+2} P_{\alpha j} Z_j + \sum_{i=1}^{n+2} Z_i P_{i\alpha} \quad \forall \alpha = 1, 2, \dots, n+2. \quad (18)$$

Matrix form of (18) be

$$\nabla L_1 = PZ + P'Z.$$

As P is symmetric matrix; therefore,

$$\nabla L_1 = 2PZ \quad (19)$$

Differentiating (18) with respect to Z

$$\frac{\partial^2 L_1}{\partial Z_\beta \partial Z_\alpha} = \frac{\partial}{\partial Z_\beta} \left(\sum_{j=1}^{n+2} P_{\alpha j} Z_j + \sum_{i=1}^{n+2} Z_i P_{i\alpha} \right) \\ \forall \alpha, \beta = 1, 2, \dots, n+2. \\ \frac{\partial^2 L_1}{\partial Z_\beta \partial Z_\alpha} = P_{\alpha\beta} + P_{\beta\alpha}.$$

Its matrix form be

$$\nabla^2 L_1 = 2P \quad (20)$$

Consider
$$L_2 = \nu \|GZ - D\|^2 \\ = \nu (GZ - D)'(GZ - D).$$

Writing L_2 in summation form,

$$L_2 = \nu \sum_{i=1}^{n-1} \left(\sum_{j=1}^{n+2} G_{ij} Z_j - D_i \right)^2.$$

Differentiating L_2 with respect to Z ,

$$\frac{\partial L_2}{\partial Z_\alpha} = 2\nu \sum_{i=1}^{n-1} \left(\sum_{j=1}^{n+2} G_{ij} Z_j - D_i \right) G_{i\alpha} \quad \forall \alpha = 1, 2, \dots, n+2. \quad (21)$$

Matrix form of (21) be

$$\nabla L_2 = 2\nu G'(GZ - D). \quad (22)$$

Differentiating (21) with respect to Z ,

$$\frac{\partial^2 L_2}{\partial Z_\beta \partial Z_\alpha} = \frac{\partial}{\partial Z_\beta} \left[2\nu \sum_{i=1}^{n-1} \left(\sum_{j=1}^{n+2} G_{ij} Z_j - D_i \right) G_{i\alpha} \right] \\ \forall \alpha, \beta = 1, 2, \dots, n+2. \\ \frac{\partial^2 L_2}{\partial Z_\beta \partial Z_\alpha} = 2\nu \sum_{i=1}^{n-1} G_{i\alpha} G_{i\beta}.$$

Its matrix form be

$$\nabla^2 L_2 = 2\nu G'G. \quad (23)$$

Now ∇L is calculates by (19) and (22),

$$\nabla L = 2PZ + 2\nu G'(GZ - D). \quad (24)$$

For stationary points, equating ∇L to 0, we get

$$2PZ + 2\nu G'(GZ - D) = 0 \\ 2(P + \nu G'G)Z - 2\nu G'D = 0 \\ Z = (P + \nu G'G)^{-1} \nu G'D. \quad (25)$$

Find $\nabla^2 L$ by (20) and (23),

$$\nabla^2 L = 2(P + \nu G'G). \quad (26)$$

Now, optimization problem (17) has an optimal solution, if the Hessian defined in (26) is a positive definite. In Theorem 1 below, we prove that the Hessian is a positive semi-definite.

Theorem.1 The Hessian matrix defined in (26) is a positive semi-definite.

Proof: For any positive semi-definite matrix [25] $S_{m \times m}$, $\lambda' S \lambda \geq 0, \forall \lambda \in R^m$.

Considering
$$\lambda' (\nabla^2 L) \lambda = \lambda' [2(P + \nu G'G)] \lambda \\ = 2\lambda' P \lambda + 2\nu \lambda' G'G \lambda.$$

Expanding the first term

$$2\lambda' P \lambda = 2 \sum_{j=1}^{n+2} P_{jj} \lambda_j^2.$$

Since P is a diagonal matrix with first two diagonal elements as zero and rest as strictly positive.

Thus,
$$2 \sum_{j=1}^{n+2} P_{jj} \lambda_j^2 \geq 0.$$

Considering the second term

$$2\nu \lambda' G'G \lambda = 2\nu \|G\lambda\|^2 \geq 0.$$

Therefore, $\lambda' (\nabla^2 L) \lambda \geq 0 \quad \forall \lambda \in R^{n+2}$ that proves Hessian defined in (26) is a positive semi-definite. But Hessian defined in (26) should be a positive definite to ensure unique solution of equation (25). Thus, Tikhnov's regularization term δI (where $0 < \delta \in R$ and I is the identity matrix of order $n+2$), is added to Hessian defined in (26) to make it a positive definite. Hence, using the Tikhnov's regularization term δI in (25), unique solution of our problem will be given by (27)

$$Z = (P + \nu G'G + \delta I)^{-1} \nu G'D. \quad (27)$$

Equation (27) obtains the coordinate (x, y) of unknown node included in Z while minimizing error in distance. The solution provided by UDV-Hop scheme reduces computational effort significantly and thus improves energy

and time efficiency of the system to estimate the coordinate of unknown nodes.

IV. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

To demonstrate the effectiveness of the proposed algorithms, simulation experiments are performed on randomly generated scenarios. The results are compared with other algorithms viz. DV-Hop, IDV-Hop, and RNLEDV-Hop. All simulations are conducted on MATLAB 2008b. The simulations of all the algorithms including our proposed algorithm run on 100 times randomly generated sensor node deployment scenarios, and the average values are used for comparison.

The experimental region is taken to be a square of fixed area $100\text{ m} \times 100\text{ m}$ in all the experiments. The sensor nodes are randomly distributed in the region. Each node (unknown or anchor) has the same communication radius.

In the simulation, the localization error is defined as the average error function illustrated in (28).

$$\text{Localization Error (LE)} = \frac{\sum_{i=n+1}^N \sqrt{(x_i^e - x_i^a)^2 + (y_i^e - y_i^a)^2}}{R \times (N - n)}, \quad (28)$$

where (x_i^a, y_i^a) is the true coordinate of the unknown node i , the estimated coordinate of the unknown node i is (x_i^e, y_i^e) , R is the communication radius of the sensor nodes, N is the total number of the nodes in the sensor field, and n is the number of anchor nodes. Lower localization error of the algorithm shows better performance. Since localization error calculated from (28) depends on number of unknown nodes, number of anchor nodes and communication radius of sensor nodes, following simulation experiments are conducted to analyze the behavior of localization error:

- Experiment 1: On changing the total number of nodes.
- Experiment 2: On changing the percentage of anchor nodes to the total number of nodes.
- Experiment 3: On changing the communication radius of sensor nodes.

In the real-time scenario, radio signals are affected by the environment. Therefore, the communicating radius of the sensor nodes does not make standard circle but an anomalous polygon. Each experiment considers three different scenarios of communication ranging error 0–10%, 0–20% and 0–30% to evaluate and compare the performance of our proposed methods (QDV-Hop and UDV-Hop) with DV-Hop, IDV-Hop, and RNLEDV-Hop.

Experiment 1. On changing the total number of nodes

In this simulation experiment, the total number of nodes varies from 200 to 500 for three different scenarios of communication ranging error 0–10%, 0–20%, and 0–30%. The number of anchor nodes is 10% of the total nodes, and the communication radius is assumed to be 15 m.

In Fig. 2(a), Fig. 2(b), and Fig. 2(c), variations in

localization error are recorded by increasing the total number of nodes. From these figures, we observe that as the number of total nodes increases in the region, localization error of DV-Hop, IDV-Hop, RNLEDV-Hop, and proposed methods (QDV-Hop and UDV-Hop) decreases. With increasing total number of nodes in the network, the average number of neighbors of each node increases. Hence, the network becomes well connected. Also, the shortest hop path has lesser deviation from the line joining the anchor pairs. Thus, the average hop-size of the anchor node will be more accurate, and the estimated distance between the anchor node and the unknown node will be closer to its true distance. Therefore, localization error of the algorithm decreases with an increase in the number of unknown nodes.

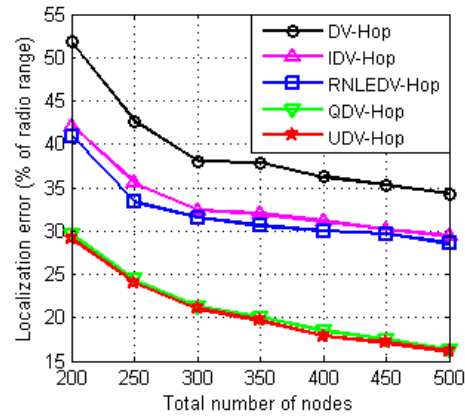


Fig. 2(a) Total number of nodes vs. localization error with 0–10% communication ranging error

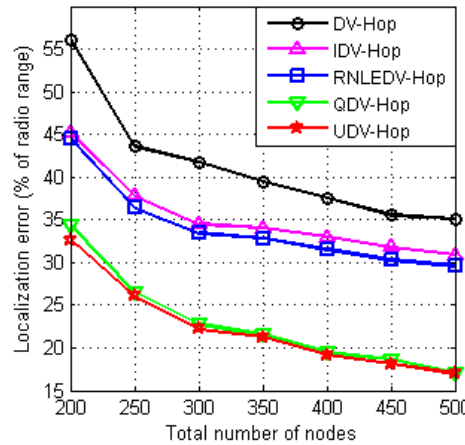


Fig. 2(b) Total number of nodes vs. localization error with 0–20% communication ranging error

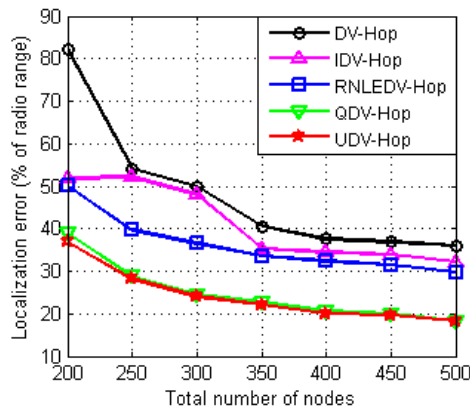


Fig. 2(c) Total number of nodes vs. localization error with 0–30% communication ranging error

In Fig. 2(a), Fig. 2(b) and Fig. 2(c), we observe that the proposed algorithms (QDV-Hop and UDV-Hop) have about 18%, 19%, and 23% lesser localization error on average compared with DV-Hop for communication ranging error 0–10%, 0–20%, and 0–30%, respectively. On the other hand, the average localization error of proposed algorithms (QDV-Hop and UDV-Hop) is about 11%, 13%, and 15% lesser compared with IDV-Hop and RNLEDV-Hop. As the communication ranging error increases, the localization error of all algorithms also increases, but this increment is less for the proposed approaches compared with other algorithms.

Experiment 2. On changing the percentage of anchor nodes to the total number of nodes

This experiment is conducted by changing the percentage of anchor nodes from 5% to 30% of total number of nodes in three different scenarios of communication ranging error 0–10%, 0–20%, and 0–30%. The total number of nodes is kept fixed at 300, and communication radius is kept constant at 15 m.

In Fig. 3(a), Fig. 3(b), and Fig. 3(c), variation in localization error with change in percentage of anchor nodes to the total number of nodes is observed. From these figures, we examine that as the number of anchor nodes increases in the region, localization error of DV-Hop, IDV-Hop, RNLEDV-Hop, and proposed schemes (QDV-Hop and UDV-Hop) decreases. When the number of anchor nodes increases in the network for a fixed number of total nodes, the number of hops between the anchor and unknown nodes decrease. Due to lesser number of hops, distance between the unknown node and the anchor node is less erroneous. Therefore, the estimated distance between the unknown node and the anchor node is closer to the true distance. Hence, localization error of the algorithm decreases.

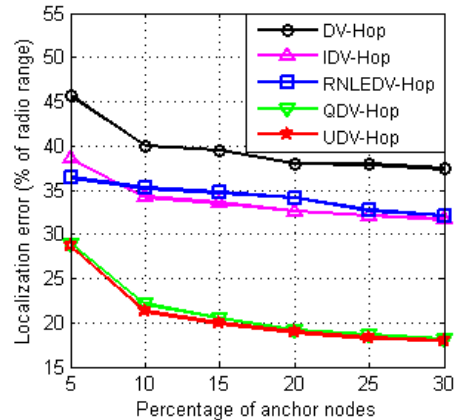


Fig. 3(a) Percentage of anchor nodes vs. localization error with 0–10% communication ranging error

In Fig. 3(a), Fig. 3(b), and Fig. 3(c), it can be observed that localization error is reduced by about 17%, 18%, and 19% when results of proposed algorithms (QDV-Hop and UDV-Hop) are compared with DV-Hop for communication ranging error 0–10%, 0–20%, and 0–30%, respectively. The average localization errors of proposed methods (QDV-Hop and UDV-Hop) are about 12%, 12%, and 13% lesser, respectively, compared with IDV-Hop and RNLEDV-Hop. It is also observed that as the communication ranging error increases, the localization error of all the algorithms increase, but this increment is lesser for the proposed approaches than for other algorithms.

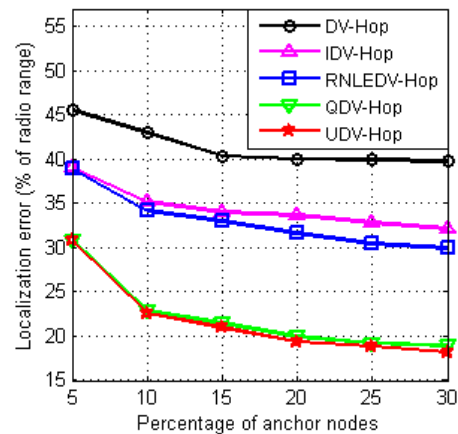


Fig. 3(b) Percentage of anchor nodes vs. localization error with 0–20% communication ranging error

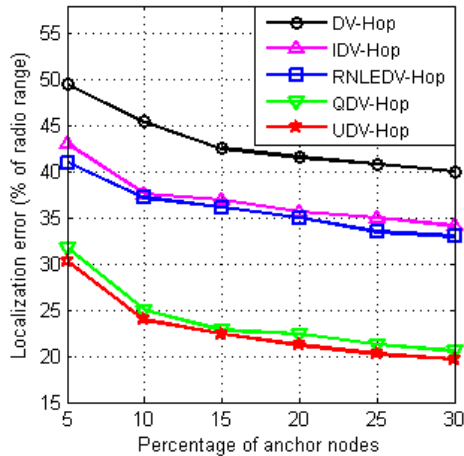


Fig. 3(c) Percentage of anchor nodes vs. localization error with 0–30% communication ranging error

Experiment 3. On changing the communication radius of sensor nodes

To understand variation in performance of all the algorithms while varying communication radius of sensor nodes, we conducted experiments with the total number of nodes fixed at 300 and anchor nodes at 10% of the total nodes in all considered scenarios of ranging error 0–10%, 0–20%, and 0–30%. From Fig. 4(a), Fig. 4(b), and Fig. 4(c), it is observed that as the communication radius of sensor nodes increases, localization error of DV-Hop, IDV-Hop, RNLEDV-Hop, and proposed algorithms (QDV-Hop and UDV-Hop) decreases. Also, with increase in communication radius for a fixed number of nodes, the network connectivity gets increased. Consequently, the number of neighboring anchor nodes per unknown node will increase. Therefore, localization error of the algorithm decreases.

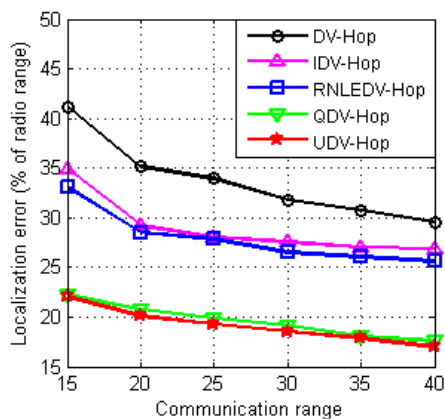


Fig. 4(a) Communication range vs. localization error with 0–10% communication ranging error

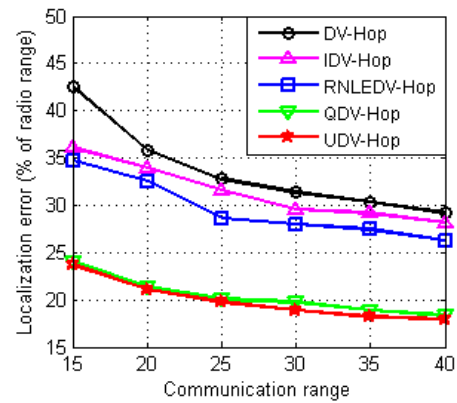


Fig. 4(b) Communication range vs. localization error with 0–20% communication ranging error

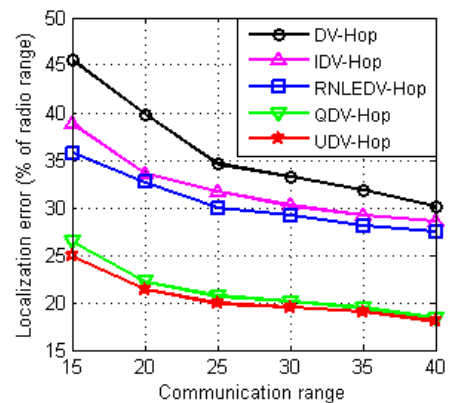


Fig. 4(c) Communication range vs. localization error with 0–30% communication ranging error

Fig. 4(a), Fig. 4(b), and Fig. 4(c), show that proposed algorithms (QDV-Hop and UDV-Hop) have about 13%, 13%, and 15% lesser error on average than DV-Hop for communication ranging error 0–10%, 0–20%, and 0–30%, respectively. On the other hand, average localization error is reduced by about 9%, 9%, and 12%, respectively, when results of proposed schemes (QDV-Hop and UDV-Hop) are compared with IDV-Hop and RNLEDV-Hop algorithms. It is thus observed that as the communication ranging error increases, the localization error of all algorithms increases, but for the proposed algorithm, this increase is lesser than for other algorithms.

V. COMMUNICATION COST AND COMPUTATIONAL EFFICIENCY

The cost-effectiveness is a significant issue with improvement in the localization accuracy of the algorithm. Therefore, in this section we discuss the communication cost and computational complexity of the algorithms.

A. Communication Cost

The communication cost of the algorithms is represented by the number of transmitting and receiving packets by the nodes in the localization process. All the localization algorithms

(DV-Hop, IDV-Hop, RNLEDV-Hop, QDV-Hop, and UDV-Hop) have three steps. Communications between nodes take place only in the first two steps. Communications in the first step of DV-Hop, IDV-Hop, RNLEDV-Hop, QDV-Hop, and UDV-Hop are same. In the second step of DV-Hop, RNLEDV-Hop, QDV-Hop, and UDV-Hop algorithms, unknown nodes receive only first-arrived hop-size message and forward it to their neighbor nodes. In IDV-Hop algorithm, unknown nodes receive hop-size messages from all anchor nodes and forward to their neighbor nodes. On receiving and forwarding all hop-size messages to their neighbor nodes in IDV-Hop, unknown nodes in IDV-Hop need more communicational work than those in DV-Hop, RNLEDV-Hop, QDV-Hop, and UDV-Hop algorithms. Extra communicational work increases communication cost of IDV-Hop. As discussed earlier that first two steps of QDV-Hop and UDV-Hop are same as in DV-Hop, the communication cost of QDV-Hop and UDV-Hop algorithms are same as the communication costs of DV-Hop and RNLEDV-Hop algorithm but lower than that of IDV-Hop algorithm.

B. Computational Efficiency

Computational efficiency is used to explain properties of an algorithm relating to how much various types of resources it consumes. By the simulation experiments we discuss localization time of the nodes for two cases.

Case 1: When the total number of nodes varies and the percentage of anchor nodes is fixed.

Case 2: When the percentage of anchor nodes varies and the total number of nodes is fixed.

Case 1. When the total number of nodes varies and the percentage of anchor nodes is fixed

In the simulation experiment, 10% of total nodes are taken as anchor nodes and communication radius is 15 m. The total number of nodes varies from 200 to 500. Table 1 shows the comparison of localization time for the algorithms. It is observed that localization time of QDV-Hop and UDV-Hop algorithms are more than localization time of DV-Hop, IDV-Hop, and RNLEDV-Hop, but localization time of UDV-Hop is comparable with the localization time of DV-Hop, IDV-Hop, and RNLEDV-Hop. On an average, the UDV-Hop algorithm is 10^{-4} seconds slower than DV-Hop, IDV-Hop, and RNLEDV-Hop algorithms. Meanwhile, localization error is reduced significantly. Therefore, we need to balance the localization accuracy and the calculation in the practical application.

TABLE I
COMPARISON OF LOCALIZATION TIME (IN SECOND) TO CHANGE IN THE TOTAL NUMBER OF NODES

Total nodes	Localization algorithms				
	DV-Hop	IDV-Hop	RNLEDV-Hop	QDV-Hop	UDV-Hop
200	0.0243	0.0112	0.0226	1.4368	0.0460
250	0.0374	0.0203	0.0341	1.7993	0.0737
300	0.0539	0.0286	0.0488	2.1775	0.1005
350	0.0734	0.0398	0.0674	2.5556	0.1496
400	0.0981	0.0517	0.0894	3.0640	0.1968
450	0.1238	0.0631	0.1138	3.4247	0.2853
500	0.1547	0.0753	0.1414	3.7695	0.3433

Case 2. When the percentage of anchor nodes varies and the total number of nodes is fixed

In this case, for simulation experiment, the total number of nodes is 300 and communication radius is 15 m. Anchor node varies from 5% to 30%. Table 2 shows localization time of the algorithms. Localization time of QDV-Hop algorithm is higher than those of other algorithms, but localization time of UDV-Hop algorithm is comparable with those of DV-Hop, IDV-Hop, and RNLEDV-Hop algorithms. On an average, UDV-Hop algorithm takes 10^{-4} more seconds than DV-Hop, IDV-Hop, and RNLEDV-Hop algorithms to localize an unknown node. Though the proposed algorithm is little slow yet it proves its effectiveness by achieving better localization accuracy.

TABLE II
COMPARISON OF LOCALIZATION TIME (IN SECOND) TO CHANGE IN THE % OF ANCHOR NODES

% of anchor nodes	Localization algorithms				
	DV-Hop	IDV-Hop	RNLEDV-Hop	QDV-Hop	UDV-Hop
5	0.0290	0.0091	0.0259	2.2489	0.0576
10	0.0523	0.0103	0.0494	2.1820	0.1002
15	0.0801	0.0150	0.0794	2.1158	0.1685
20	0.0988	0.0332	0.1145	2.3016	0.2317
25	0.1186	0.0647	0.1575	3.5419	0.3154
30	0.1389	0.0731	0.2078	3.4157	0.3534

VI. CONCLUSION

In this paper, we improve the localization accuracy of DV-Hop algorithm without increasing computational complexity and requiring any other tool. We proposed two improved schemes, i.e., QDV-Hop and UDV-Hop. In both schemes, error terms are separated from the estimated distances between unknown node and anchor nodes in the same way. QDV-Hop algorithm minimizes the error terms by using quadratic programming that requires optimization tool box. The requirement of the computationally costlier quadratic tool box can be obviated by introducing the UDV-Hop algorithm. In UDV-Hop algorithm, error terms are minimized by using unconstrained optimization problem that results into solving a system of linear equations without involving iterative procedure. The experiments verify better performance

obtained by QDV-Hop and UDV-Hop against DV-Hop, IDV-Hop and RNLEDV-Hop algorithms, in all considered scenarios.

The improved localization accuracy in the proposed work has proved its application. Future directions for this work include reducing computational complexity while further improving accuracy of localization using some other methods.

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REFERENCES

- [1] I.F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: a survey," *Computer Networks*, vol. 38, issue 4, pp. 393-422, 2002.
- [2] S.-M. Lee, H. Cha, and R. Ha, "Energy aware location Error handling for object tracking applications in wireless sensor networks," *Computer Communications*, vol. 30, issue 7, pp. 1443-1450, 2007.
- [3] Q. Xiao-gang and Q. Chen-xi, "An Improvement of GAF for Lifetime Elongation in Wireless Sensor Networks," *Journal of Convergence Information Technology*, vol. 5, no. 7, pp. 112-119, 2010.
- [4] M. Broxton, J. Lifton, and J. Paradiso, "Localizing a Sensor Network via Collaborative Processing of Global Stimuli," In *Proc. of the 2nd European Workshop on Wireless Sensor Networks*, pp. 321-332, 2005.
- [5] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins, *Global Positioning System: Theory and Practice*. 2nd edition, Springer-Verlag, New York, 1993.
- [6] J. Lee, W. Chung, and E. Kim, "A new range-free localization method using quadratic programming," *Computer Communications*. vol. 34, issue 8, pp. 998-1010, 2011.
- [7] H. Lee, M. Wicke, B. Kusy, and L. Guibas, "Localization of mobile users using trajectory matching. In MELT '08," *Proceedings of the first ACM International Workshop on Mobile Entity Localization and Tracking in GPS-less Environments*, New York, USA, pp. 123-128, 2008.
- [8] T. He, C. Huang, B.M. Blum, J.A. Stankovic, and T. Abdelzaher, "Range-Free Localization Schemes for Large Scale Sensor Networks," In *Proceedings of the 9th annual international conference on Mobile computing and networking*, pp. 81-95, 2003.
- [9] G. Q. Gao, and L. Lei, "An Improved node Localization Algorithm based on DV-Hop in WSN," In *2nd International Conference on Advanced Computer Control (ICACC)*, vol. 4, pp. 321-324, 2010.
- [10] A. Savvides, C. Han, and M.B. Strivastava, "Dynamic fine grained localization in ad-hoc networks of sensors," *Proceedings of the International Conference on Mobile Computing and Networking*, pp.166-179, 2001.
- [11] S. Yun, J. Lee, W. Chung, E. Kim, and S. Kim, "A soft computing approach to localization in wireless sensor networks," *Expert Systems with Applications*, vol. 36, issue 4, pp. 552-7561, 2009.
- [12] C.-R. Li, "The Self-Location Technology Research for Wireless Sensor Network," *SOUTHWEST JIAOTONG UNIVERSITY*, Master's Paper, 2006.
- [13] S. Capkun, M. Hamdi, and J.-P. Hubaux, "Gps-free positioning in mobile Ad Hoc networks," In *Proceedings of Hawaii International Conference System Sciences*, pp. 3481-3490, 2001.
- [14] R. Nagpal, "Organizing a global coordinate system from local information on an amorphous computer," *A.I. Memo 1666*, MIT A.I. Laboratory, 1999.
- [15] D. Niculescu, and B. Nath, "Ad Hoc Positioning System (APS)," In *Proceeding of IEEE Global Telecommunications Conference*. San Antonio: IEEE Communication Society, vol. 5, pp. 2926-2931, 2001.
- [16] H. Chen, K. Sezaki, P. Deng, and H. CheungSo, "An Improved DV-Hop Localization Algorithm for Wireless Sensor Networks," In *3rd IEEE International Conference on Industrial Electronics and Application (ICIEA)*, pp. 1557-1561, 2008.
- [17] H. Chen, K. Sezaki, P. Deng, and H. CheungSo, "An Improved DV-Hop Localization Algorithm with Reduced Node Location Error for WSNs. *IEICE TRANS. FUNDAMENTALS*, E 91 A(8), pp. 2232-2236, 2008.
- [18] S. Hou, X. Zhou, and X. Liu, "A Novel DV-Hop Localization Algorithm for Asymmetry Distributed WSNs," In *3rd IEEE International Conference on Computer Science and Information Technology (ICCSIT)*, vol. 4, pp. 243-248, 2010.
- [19] Y.Y. Li, "Improved DV-Hop Localization Algorithm Based on Local Estimating and Dynamic Correction in Location for WSNs," *International Journal of Digital Content Technology and its Applications*, vol. 5, issue 8, pp. 196-202, 2011.
- [20] S. Tian, X. Zhang, P. Liu, P. Sun, and X. Wang, "A RSSI-based DV-hop algorithm for wireless sensor networks," *Wireless Communications, Networking and Mobile Computin*, pp. 2555 - 2558, 2007.
- [21] K. Benkic, M. Malajner, P. Planinsic, and Z.K. Cucej, "Using RSSI value for distance estimation in wireless sensor networks based on ZigBee," In *Proc. of 15th International Conference on Systems, Signals and Image Processing*, pp. 303-306, 2008.
- [22] L. Gui, and T. Val, "A Novel Two-Class Localization Algorithm in Wireless Sensor Networks," *Laboratory CNRS-IRIT-UT2, University of Toulouse*, vol. 3, issue 3, 2011.
- [23] Q. Qian, X. Shen, and H. Chen, "An Improved Node Localization Algorithm Based on DV-Hop for Wireless Sensor Networks," *Computer Science and Information Systems*, vol. 8, issue 4, pp. 953-972, 2011.
- [24] Y.T. Chan, and K.C. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Transactions on Signal Processing*. vol. 42, issue 8, pp. 1905-1915, 1994.
- [25] D.G. Luenberger, *Linear and Non-Linear Programming*, 2nd Edition, Kluwer Academic Publishers, London, 2003.