# Face Reconstruction and Camera Pose Using Multi-dimensional Descent 

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#### Abstract

This paper aims to propose a novel, robust, and simple method for obtaining a human 3D face model and camera pose (position and orientation) from a video sequence. Given a video sequence of a face recorded from an off-the-shelf digital camera, feature points used to define facial parts are tracked using the ActiveAppearance Model (AAM). Then, the face's 3D structure and camera pose of each video frame can be simultaneously calculated from the obtained point correspondences. This proposed method is primarily based on the combined approaches of Gradient Descent and Powell's Multidimensional Minimization. Using this proposed method, temporarily occluded point including the case of self-occlusion does not pose a problem. As long as the point correspondences displayed in the video sequence have enough parallax, these missing points can still be reconstructed.


Keywords-Camera Pose, Face Reconstruction, Gradient Descent, Powell's Multidimensional Minimization.

## I. INTRODUCTION

ESTIMATION of point reconstruction and camera pose from multiple video frames can be done by many techniques. Some techniques for estimating camera pose require the placement of specific markers with known 3D world positions [3-7]. For certain environments, it may not be suitable or even possible to add markers to the scene. Even in cases where markers can be added to the scene, measuring their precise positions may still pose a problem. So, instead of using artificial markers in a scene, this paper uses natural feature-points that can be tracked by using the Active Appearance Model (AAM) [19], [20].

In order not to use any special markers of known world coordinates, auto- or self- calibration is proposed [8]. Autocalibration methods estimates structure from motion based on a given number of images. Examples of methodologies include fundamental matrix for two views [8], trifocal tensor for three [9], quadrifocal tensor for four [10], and factorization for multiple views [11]. In the auto-calibration, there are three levels of reconstruction including projective, affine, and metric reconstruction. Considering the many steps involved

[^0]and the details needed for reconstruction, noisy results from a lower level of reconstruction can propagate the error to a higher level. Thus, an optimization process is needed after each level of reconstruction. Furthermore, if points are missing because of temporary occlusions, special strategies must be applied to find correspondences prior to the 3D reconstruction process. In the case of self-occlusion, searching for correspondences is even harder. This proposed method is a simpler and more practical method where self-occlusions can be ignored while the robustness is still retained.

Gradient descent is a robust minimization method which has been used for various tasks such as training radial basis function (RBF) neural networks [12], predicting or searching motion of point in images [13], [15], and estimating 3D camera motion [14], [16]. Unfortunately, one main disadvantage of the original gradient descent is that the derivative of system of equations or the conjugate direction [13] must be calculated. This derivative can then be used to define the step size for parameter updating. An incorrect derivative-formula may cause the system to be prone to errors. With the method presented here, the derivative need not be calculated.

Another approach for finding the minimization and maximization of the system functions is Powell's lineminimization method [1]. Powell's minimization for multidimensional system can be seen in [1], [17], [18]. Line minimization used in Powell's multi-dimensional search can be illustratively explained as climbing down the valley shaped like a 3D-parabola graph. Although many variables make the multi-dimensional direction, only one variable or one line needs to be considered at a time. While considering a variable/line, the deepest point along that line may not be the correct result to obtain a global solution. Thus, the method proposed in this paper chooses not to climb down until the line minimization but, instead, climb with only one small step at a time in the currently correct direction.

For face reconstruction, Y. Zheng, J. Chang, Z. Zheng and Z. Wang [22] proposed 3D face reconstruction in 2007 using stereo images together with a reference 3D face model. Since they found that the traditional stereo methods based on intensity failed to provide good results in 3D face reconstruction, a reference 3 D model of the human face is used to help in correspondence calculation. Although a reference 3D model is used, non-linear deformations and camera registration have to be solved in order to find correspondences in the stereo images. In 2008, S. W. Park, J.
face for 3D reconstruction. An average 3D face model is created from 3D training images. If the given image is not a frontal image of the face, pose correction of feature points must be done to re-create points for the frontal view before mapping the re-created points to the 3D average model via Delaunay triangulation. Reconstructing 3D structure from one single image may appear to provide a big advantage, but a 3D model of the underlying object needs to be prepared prior to using the algorithm. In other words, one must know what kind of object is being reconstructed. Also, the optimal solution of 3D structure for the unseen feature points may not be obtained because an assumption that the reconstructed object is symmetric is needed.
Y. Zheng and Z. Wang [23] reconstructed a 3D face model from a single frontal face image. They used a learning based approach for the reconstruction. That is, a database indicating the relationship between feature point and point depth must be prepared in advance. Since only one image is needed, pointdepth estimation must be solved accurately. To estimate the point depth from just one image, a learning step of mapping between texture on a face and depth determination cannot be avoided. In other words, a database with depth of facial features must be available. Furthermore, the learning process to convert from the pattern map to the depth map from the prepared database is a high dimensional, nonlinear problem.

Combination concepts of the line minimization used in Powell's multi-dimensional search and the simple gradient descent optimization are adapted in the work proposed in this paper. The main concept of this proposed method is based on the fact that a feature point's 2D image points seen in several images will be back-projected to the same point in three dimensions [10]. Given 2D correspondences of $N$ 3D-points seen in $M$ frames provides more equations than unknowns. This constraint is sufficient to solve for all unknowns of the system of equations. Using the assumption that each contiguous frame from a video sequence has a small difference in image motion, camera motion can be easily achieved. The case missing feature points due to temporary occlusion or self-occlusion does not pose a problem in our proposed method as long as the points are seen in a sufficient number of frames to provide the above mentioned constraint. If there is sufficient parallax among the points, each point's 3D structure can be obtained by this proposed method.

After obtaining the face's 3D structure and camera motion, one may use them directly for applications such as Augmented Reality (AR), security, person identification, etc., or even use its re-projection to improve on searching correspondences of the missing points in AAM or other tracking systems.

## II. Methodology

To reconstruct a human's face, 68 feature points are defined for the important parts of the face such as eyebrows, eyes, nose, mouth, and chin. The feature points defining facial parts on a human face are determined as shown in Fig. 1:


Fig. 1 Feature points used to define facial parts
Some video frames may have some invisible points due to self-occlusion of the face as shown in Fig. 2:


Fig. 2 An example of point missing due to self-occlusion
Let a transformation matrix composed of rotation and translation parameters be

$$
M_{4 \times 4}=\left[\begin{array}{cc}
R_{3 \times 3} & \boldsymbol{T}_{3 \times 1}  \tag{1}\\
0_{1 \times 3} & 1
\end{array}\right]
$$

The 3D world coordinates of the feature points in the realworld can be defined as $\boldsymbol{W}=\left[\begin{array}{llll}X_{w} & Y_{w} & Z_{w} & 1\end{array}\right]^{T}$. The 3D transformation between the world coordinate and the camera coordinates, $\boldsymbol{C}=\left[\begin{array}{llll}X_{c} & Y_{c} & Z_{c} & 1\end{array}\right]^{T}$, can be obtained by

$$
\begin{equation*}
\boldsymbol{C}=M^{w \rightarrow c} \boldsymbol{W} \tag{2}
\end{equation*}
$$

From (1) and (2), the transformation matrix can be rewritten as:

$$
\begin{equation*}
\boldsymbol{C} \quad=\left(R_{x}^{-1} R_{y}^{-1} R_{z}^{-1} T^{-1}\right) \boldsymbol{W} \tag{3}
\end{equation*}
$$

where the rotation and translation matrices in (3) are defined as:

$$
\begin{align*}
& R_{x}^{-1}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & \cos \theta_{x} & \sin \theta_{x} & 0 \\
0 & -\sin \theta_{x} & \cos \theta_{x} & 0 \\
0 & 0 & 0 & 1
\end{array}\right]  \tag{4}\\
& R_{y}^{-1}=\left[\begin{array}{cccc}
\cos \theta_{y} & 0 & -\sin \theta_{y} & 0 \\
0 & 1 & 0 & 0 \\
\sin \theta_{y} & 0 & \cos \theta_{y} & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \\
& R_{z}^{-1}=\left[\begin{array}{cccc}
\cos \theta_{z} & \sin \theta_{z} & 0 & 0 \\
-\sin \theta_{z} & \cos \theta_{z} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \\
& T^{-1}=\left[\begin{array}{llll}
1 & 0 & 0 & -t_{x} \\
0 & 1 & 0 & -t_{y} \\
0 & 0 & 1 & -t_{z} \\
0 & 0 & 0 & 1
\end{array}\right]
\end{align*}
$$

After transforming a 3D-point from its world coordinate to a camera one, the camera coordinate can then be projected into
the image captured from this camera by using equation $\mathfrak{V}(\mathbb{)}: 3, \mathrm{No}: 12$, 2009 positive and negative value of the step size must

The obtained 2D image coordinate point, $\boldsymbol{S}=\left[\begin{array}{lll}u & v & 1\end{array}\right]^{T}$, is the feature-point that serves as the input.

$$
\boldsymbol{S}=\left[\begin{array}{ccc}
f & 0 & 0  \tag{5}\\
0 & f & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
X_{c} / Z_{c} \\
Y_{c} / Z_{c} \\
1
\end{array}\right]
$$

Considering the system equations, the unknown variables include camera's position ( $t_{x}, t_{y}, t_{z}$ ) and orientation ( $\theta_{x}, \theta_{y}$, $\theta_{z}$ ) for each video frame, a single focal length ( $f$ ), and 3D reconstructed points $\left(X_{w}, Y_{w}, Z_{w}\right)$ for each feature point representing facial parts. So with $M$ frames and $N$ feature points, there will be $6 M+3 N+1$ variables to be estimated. As input of the system, each 2 D feature-point has two components $(u, v)$ so $N$ feature-points seen in $M$ frames will provide $2 M N$ system-equations in total. The number of equations must be more than the number of unknown variables.

The steps in the proposed method are briefly explained as follows:

1. Feature points are tracked with the precision of subpixels by the AAM method. Note that, in the method proposed here, feature points need not be visible in all the frames.
2. Initialize all variables and step sizes (each variable has its own step size) used for updating variables; $Z_{w}$ to a negative value such as $-1, f$ to a sensibly positive (one may choose the range of $300-1,000$ ), step-size to a positive small value such as 1 , and the other variables can be initialized as 0 's.
3. Repeat steps 4 and 5 for all variables until the convergence to a global energy solution calculated by average distance of point re-projection or until a maximum number of iterations.
4. Add the current variable with its step size. Note that
be tried.
5. Decrease step-size of the current variable by half or by a ratio smaller than one until local energy of the variable is reduced or until the step size is too small, e.g., $1 \times 10^{-8}$.

Since for video recordings consecutive frames generally have a small difference in camera motion, the updated camerapose of the current frame should not be too different from the previous frame. If a big difference is observed, the current camera's pose is reset to the previous one. Also, since all 3Dpoints should be in front of the camera only (i.e. all $Z_{w}$ 's should be negative), a swap in signs of $Z_{w}$ must be done whenever $Z_{w}$ becomes a positive number. Furthermore, the focal length must be positive and non-zero, so an absolute value is needed, to avoid negative focal length values. Also, if the focal length becomes zero, it is re-initialized to a positive value such as 1 instead.

## III. Experimental Results

## A. Synthetic Environment

Experimentation on synthetic data can be used to test the accuracy both in two and three dimensions because the exact 3D information is known. For two dimensions, pixel-errors can be calculated from the displacement of point's reprojection. The accuracy in three dimensions can be demonstrated from both the reconstructed 3D-structure of a synthetic face and the camera pose of all video frames.

A video sequence is generated from 3DS Max with 225 frames in length, frame rate is 15 frames per second, and image frame size is $640 \times 480$ pixels. Some of the video frames and pixel-errors calculated from point re-projection are shown in Fig. 3.

(a) Frame 1, 122, and 225 respectively of the video sequence

(b) Corresponding displacement from point re-projection

Fig. 3 Input video frames and their corresponding pixel-errors calculated from re-projection
From Fig. 3(b), the pixel-errors are very low so that the observed (input) points and the re-projected ones are seen as
overlaid in the same positions. The average pixel-error of this experiment is calculated from the Root Mean Square (RMS)

Vol:3, Nece2, qdoogamera motion are displayed in the OpenGL environment as shown in Fig. 4. Since, in this experiment, a camera pans around a synthetic face from one side to the other, the camera motion of all video frames represented by several colored cones looks like a long ribbon in Fig. 4.

| AVERAGE PIXEL-ERRORS FOR SYNTHETIC FACE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| \#Equations | \#Variables | Iterations | Time(sec.) | Error(pixels) |
| 30,600 | 1,549 | 3,500 | 407 | $2.54 \mathrm{e}-4$ |



Fig. 4 Example graphic-results showing the face model and camera motion in OpenGL's 3D environment

Numerical errors in three dimensions are shown in Table II. Note that, for calculating the 3D numerical-errors, the position and orientation of the first calculated camera must be transformed to be the same as that of the first input camera.

The real size of the face model can be recovered by re-scaling it with a ratio between sizes of the input face and the calculated face.

Table II
Average 3D-Errors for Synthetic Face

| Orientation (degrees) |  |  | Position (specified unit) |  |  | Face Model (specified unit) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r_{x}$ | $r_{y}$ | $r_{z}$ | $t_{x}$ | $t_{y}$ | $t_{z}$ | $X_{w}$ | $Y_{w}$ | $Z_{w}$ |
| 0.015105 | 0.039641 | 0.031228 | 0.030606 | 0.003449 | 0.010833 | 0.002064 | 0.002856 |  |

## B. Real-World Environment

Since the precise distance between any two contiguous feature-points on a real person's face cannot be measured, only 2D errors calculated from displacement of point's reprojection are shown. In this experiment, the camera motion can be explained as that the camera pans around a human face; starts from the front of the person, then moves to the right of starts from the front of the person, then moves to the right of the face, and finally moves back until to the left of the face. The video sequence used for this experiment is recorded by a
hand-held digital-camera with 128 frames in length, a frame rate of 15 frames per second, and a frame size of $320 \times 240$ pixels. Some examples of video frames and pixel-errors calculated from the intermediate and the final results are shown in Fig. 5. From Fig. 5, the two sets of colored points representing the observed and the re-projected points become closer after a greater numbers of iterations.


(c) Displacement from point re-projection after 5,000 iterations

Fig. 5 Input video frames and their corresponding pixel-errors calculated from re-projection
The average pixel-errors of this experiment are calculated after various numbers of iterations and shown in Table III. After the feature points of the facial parts are reconstructed, the Delaunay triangulation method introduced in [1], [2] is used to create a 3D mesh followed by texture-mapping and 3D-lighting using OpenGL libraries. The final 3D graphicsresult showing the reconstructed 3D-face and the camera

| AVERAGE PIXEL-ERRORS FOR REAL HUMAN's FACE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| \#Equations | \#Variables | Iterations | Time(sec.) | Error(pixels) |
| 14,836 | 967 | 500 | 32 | $3.59 \mathrm{e}-2$ |
| 14,836 | 967 | 1,000 | 64 | $3.48 \mathrm{e}-2$ |
| 14,836 | 967 | 3,500 | 221 | $2.60 \mathrm{e}-2$ |
| 14,836 | 967 | 4,500 | 284 | $2.43 \mathrm{e}-2$ |
| 14,836 | 967 | 5,000 | 315 | $2.41 \mathrm{e}-2$ | motion is displayed for different viewpoints in Fig. 6.


(b) Perspective view 1

(c) Perspective view 2

Fig. 4 Example graphic-results showing the face model and camera motion in OpenGL's 3D environment

## IV. CONCLUSION

From the experiments on the synthetic face and real human face, one may see that the missing image-points do not pose a problem in our proposed method for estimating 3D-face reconstruction and camera pose computation. The proposed approach is simple, practical, but yet robust.

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