# Constraint Active Contour Model with Application to Automated Three-Dimensional Airway Wall Segmentation

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Abstract—For evaluating the severity of Chronic Obstructive Pulmonary Disease (COPD), one is interested in inspecting the airway wall thickening due to inflammation. Although airway segmentations have being well developed to reconstruct in high order, airway wall segmentation remains a challenge task. While tackling such problem as a multi-surface segmentation, the interrelation within surfaces needs to be considered. We propose a new method for three-dimensional airway wall segmentation using spring structural active contour model. The method incorporates the gravitational field of the image and repelling force field of the inner lumen as the soft constraint and the geometric spring structure of active contour as the hard constraint to approximate a three-dimensional coupled surface readily for thickness measurements. The results show the preservation of topology constraints of coupled surfaces. In conclusion, our springy, soft-tissue-like structure ensures the globally optimal solution and waives the shortness following by the inevitable improper inner surface constraint.

*Keywords*—active contour model, airway wall, COPD, geometric spring structure

## I. INTRODUCTION

DIAGNOSIS of Chronic Obstructive Pulmonary Disease (COPD), such as emphysema, chronic bronchitis and bronchiectasis utilizes CT imaging to study the anatomic al structure correlating with physiologic symptoms. Accurately measuring the airway wall thickness in three-dimensional is important for the longitudinal studies of airways as the chronic airflow limitation can be caused by a combination of airway remodeling and parenchymal destruction [1]. The interrelation within surfaces needs to be considered to ensure the preservation of their topology constraints, especially for complex shapes with high curvatures, like airway and cerebral cortex. Hence, the challenge remains when attempting to obtain the global optimality that takes the contextual properties into account with little or no human intervention.

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In medical images, the techniques employed to perform multi-surface segmentation can be generally modeled in graph space and image space. Recently, the optimal graph based surface segmentation even though guarantees a globally optimal solution by introducing the graph columns and extended arcs as geometric constraints defining the interrelations and surface smoothness respectively, is limited to cylindrical shape or terrain [2]. To overcome the bifurcation problem when segmenting the airway wall, in [3] the flow lines given by the convolution kernel along the surface are utilized as guidance to construct graph columns. Although claiming to be globally optimal solution as the method also allows columns in varying lengths, the graph based method requires the valid representation of graph columns fully across the surface in order to differentiate the outer surface from inner surface. However, the weak edges, taking place in the distal airways, may lead the inner lumen being over-segmented as the result of attempting to reconstruct high-order bronchi, in which the inner graph column points may not cross the inner sought surface properly. The proposed method, on the other hand, while maintaining the smoothness properties of simple active contour model, also has the similar geometric constraint as in the works of graph-based optimal surface segmentation, but does not use voxel-based information such as intensity from the initial segmentation as in graph-based methods. Instead, we only apply the shape constraint from the inner lumen as the physical property of the tube-like structure.

Several methods have being proposed to solve this in image space [4, 5, 6], but only a few is able to pursue the global optimality as the local boundary can easily trapped in local minima, and none of them can preserve the interrelation of coupled surfaces in three dimensions. The closest work related to solve multi-surface segmentation in image space is coupled active contour model [5]. The method uses two nodes, the inner and outer nodes along the coupled contours, to profile the intensity in between. Although pursuing to preserve the topology by regulating the radial lines as the thickness constraint and contour smoothness as shape constraint, the two dimensions coupled active contour approach requires not only a precise initial segmentation as it would affect the center of the gravity, but also a manually adjusted distance between the two nodes in order to have them allocated closely to the edge. The independency of radial lines also prevents it from obtaining globally optimal solution. In our work, we develop a new active contour model finding the globally optimal coupled surfaces in

two dimensions and three dimensions by constructing a double layered mesh where the static interior vertices represent the inner surface and the dynamic exterior vertices represent the outer surface. The mesh is then iteratively shaped by the external energy from both the image and the initial segmentation, and minimized by the internal energy constraining by the geometrical spring structures representing the edges within.

Mass-spring method is a well-established deformable model simulating the soft tissue in the area of medical imaging and computer graphic. The formulation of our method is derived from the classical approach of gradient vector flow snake (GVF Snake) [6], but incorporates the new form of reconstructing the active contour model by spring structures in both two dimensions and three dimensions. Consequently, the resulting coupled surface can be automatically extracted following by the contributions of our work. In particular, we aim to solve the common leakage problem in most active contour models while preserving the topology of surfaces by energy minimization as the geometric spring structure, the internal force formulation in elastically deformable model [7], reaches its equilibrium for the new constraint model.

This paper is organized as follows. After the brief introduction of the original active contour model following by its variation of modified external force in the gradient vector flow snake, and we whereas present the geometric spring structure as the internal force to solve the coupled surfaces segmentation by adding constraint energy which is seamlessly adapted to the energy functional of GVF snake. We then define the new optimal surface segmentation method, namely the constraint active contour model. We also apply the method to a suitable application problem segmenting the airway wall in both two dimensions and three dimensions.

# II. METHOD

The physical interpretation of the proposed constraint active contour model with spring structure is originated from the work in [6, 7]. The sheet-like structure such as airway wall can be defined as a thin layer described in elastically deformable model. To simulate the deformable surface as a physical object interacting with the surroundings in three dimensions space, it involves numerically solving the partial differential equations governs the non-rigid shape evolving in motion through time and space, based on the principles of elasticity theory. We incorporate the geometric springy representation of snake in active contour model to take gradient vector flow of the image as the gravitational field into account for the surfaces to fall on the objects.

#### A. Active Contour Model

Snake [8] is a class of active contour models that falls into the desired solution nearby the local edges while keeping its shape as a spline. Snake can be seen as a special case of deformable model presented in [9, 10]. The flexibility of its energy functional is written as:

$$E_{snake} = \int_{0}^{1} E_{snake} (S(s)) ds$$

$$\text{minmize} = \int_{0}^{1} (E_{int} (S(s)) + E_{image} (S(s)) + E_{con} (S(s))) ds$$

$$(1)$$

The snake S(s) = [x(s), y(s), z(s)],  $s \in [0,1]$  is an arbitrary surface in  $\mathbb{R}^3$ .  $E_{image}$  gives rise to the image energy comprising three different energy functionals which attract the spline to lines, edges and terminations. Since we only consider the edges of image in our application, edge detection can be obtained by taking the gradient of image:

$$E_{image}(x, y, z) = \left|\nabla I(x, y, z)\right|^{2}$$
 (2)

Adding the Gaussian  $G_{\sigma}$  smoothing term allows the spline to be drawn from further distance:

$$E_{image}(x,y) = \left| \nabla \left( G_{\sigma}(x,y,z) * I(x,y,z) \right) \right|^{2}$$
(3)

The energy functional is then followed by the application specific constraint. An example of utilizing such external constraint energy ( $E_{con}$ ) is in the work of balloon model [4]. In order to push the initial contour outward from the steady state, the method introduces an internal pressure as pumping the air to the inflated balloon. However, the tradeoff of this tactic is the risk of having weak edge with respect to the air pressure, causing the contours to cross the desired edges. Our approach prevents this by introducing the hard constraint which is introduced in section 2.4 and 2.5.  $E_{int}$  represents the internal energy of the spline due to stretching and bending:

$$E_{int}(x, y, z) = \left(\alpha \left| S'(s) \right|^2 - \beta \left| S''(s) \right|^2\right) / 2 \tag{4}$$

B. Gradient Vector Flow Snake

Without loss of generality to snake's variational approach, the minimization of energy functional (1) can be seen as the fundamental force balance equations:

$$F_{int} + F_{ext} = 0 ag{5}$$

$$F_{ext} = -\nabla E_{ext}(x, y, z) \tag{6}$$

where the image energy (3) is written as external constraint to address the dependency of gradient factor:

$$\nabla E_{ext}(x, y, z) = \nabla \left( G_{\sigma}(x, y, z) * I(x, y, z) \right) \tag{7}$$

Note that  $E_{ext}$  can now be seen as the edge map f(x,y) of the image, such that  $V(x,y,z) = \nabla f$  generates a vector field, namely the *gradient vector flow field*. Using the *calculus of variations*, we can solve V(x,y,z) = (u(x,y,z), v(x,y,z), w(x,y,z)) where u, v and w are the vector map of the edge map in time domain. We also treat this *generalized diffusion equation* as the gravitational field in our proposed method. Once the vector field is computed, we can replace the  $-\nabla E_{ext}(x,y,z)$  with V(x,y,z). If S reaches the local minima of the energy functional, then solving the internal energy (4) of the surface becomes the Euler-Lagrange equation [1, 10]:

$$F_{int} = \alpha \nabla^2 S - \beta \nabla^2 S \left( \nabla^2 S \right) \tag{8}$$

$$F_{int} + F_{ext} = \alpha S''(s,t) - \beta S''''(s,t) + V(x,y,z)$$
 (9)

Where  $\nabla''$  is the *Laplacian* operator  $\frac{\partial^2}{(\partial u)^2} + \frac{\partial^2}{(\partial v)^2} + \frac{\partial^2}{(\partial w)^2}$  and S is the deformable surface in s-space at certain t-time. Equation (9) derives the formulation of GVF snake.

# C. Spring Structural Active Contour Model

The total exerting force of a single spring segment is calculated by Hooke's law:  $F_{spring} = -kS'$ , where  $F_{spring}$  is the force measured by the product of spring constant k and displacement S'. The potential energy of the spring is  $E_{spring} = \frac{1}{2}k(S')^2$ . Another damping force  $F_{damping} = -bS''$  is applied to calculate the resistance, which is the force proportional to the velocity S'' of the displacement with friction b. The summation of the force of spring in motion becomes:

$$F_{spring} + F_{damping} + F_{gravity} = F_{total}$$
 (10)

 $F_{gravity} = mV$ , where V is gravitational field from GVF, and m is the mass of the node. Replacing the internal force  $F_{int}$  in (5) with  $F_{spring} + F_{damping}$ . Resolving this equation with Newton's law of motion,  $F_{int} = ma$ , and the definition of acceleration can be defined as the second derivative of displacement: ma = mS''', then the differential equation of internal force becomes:

$$mS^{"} = kS' + bS^{"} \tag{11}$$

In order to minimize the energy functional, the spring force is balanced with damping force and gravitational force at equilibrium as stated:  $F_{int} = F_{ext}$ , such that

$$kS' + bS'' = mV(S(s)) \tag{12}$$

Note that the mass can be application specific. Spring constant and friction becomes smoothness term specifying the stretching and bending coefficient of the springy structure. The surface that is constructed by nodes attaching to the end-points of the springs with spring constant and friction utilizing the gradient vector flow forms the constraint active contour model.

# D.Two-dimension Geometric Spring Structure

The mostly related to the two dimensions version of this work is the "coupled active contour model" [5] using two or more nodes to profile the intensity in between. Although preserving the topology, the coupled active contour approach requires not only a precise initial segmentation as it would affect the center of the gravity, but also a manually adjusted distance between the two nodes in order to have them allocated closely to the edge. The main difference is also the hard constraint of the two dimensions geometric spring structure being integrated in our model.

#### E. Three-dimension Geometric Spring Structure

The mostly related to the three dimensions version of this work is the "weighted balloon model" which as mentioned above suffers from the lack of constraint as its regulation term. The work of "constraints on deformable models" introduces the geometric constraint-based modeling. We call this the hard constraint of the proposed constraint active contour model. As illustrated in Figure 1(a), the inner points are connected to the outer points to apply the inherent geometric constraint (red dashes) prescribing the involuntary muscle contraction in our application. The edges (black dashes) connecting the outer points to form an elastic membrane are the geometric constraint regularizing the smoothness. Note that this geometric spring structure does not have bending spring as those in the following two dimensions version (see Figure 1(b)). This is similar to the minimization of balloon forces by maximizing the area of the outer surface. The outer points move outward in the direction same as the gradient vector flow is in distant.

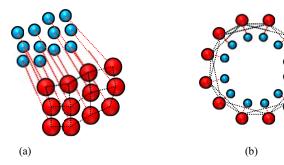


Fig. 1 Geometric spring structure illustrates each outer point (blue) being pushed and pulled by the springs (red dashed lines) perpendicular to the surface representing the inner points (red). (a) The forces acting on the outer points of the three dimensions surfaces are balanced by the springs (black dashed lines) connected to their neighbor points. (b) The two dimensions geometric spring structure requires extra cross linking on the outer surface to possess bending energy.

#### III. APPLICATION AND RESULTS

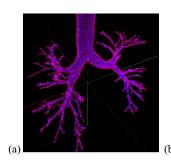
#### A. Data

To study the applicability of the proposed method, we conduct an experiment in segmenting the human pulmonary airway wall. The human pulmonary CT images used have a nearly isotropic resolution of 0.78 mm x 0.78 mm x 0.65 mm consisting 512 slices of 512 x 512 pixels. The subject shows the physiological phenotype of emphysema which can be a challenge for airway lumen segmentation as well as for the proposed method in airway wall segmentation.

# B. Airway Wall Segmentation

The intra-thoracic airway is extracted in three dimensional using our previously developed algorithm and the voxels of the segmented surface are denoted with sets of vertices as mentioned in the method above. To study the applicability in image sources with different dimensionalities, we propagate

the outer surface through slices with multiple initial contours, as well as through the volumetric data with outer surface as a whole mesh. The preliminary result in Fig. 2(b) shows a segmentation result in two dimensions obtained from the cross-sections. The outer contour stays closely to the edge of the airway wall while preserving the interrelation with the inner surface. Other interesting phenomena during the process of minimization can be observed here. Consequently, the repelling force from the initial segmentation may push some of the outer points travelling beyond the desired edge and trap them in the local minima. Fortunately, the hard constraint not only keeps the outer contour intact by length and curvature, but also applies the contraction to those drawing away spuriously. This characteristic gives better performance comparing to the other geometric active contour models [4, 5]. Fig 2(a) shows a segmentation result in three dimensions. The blue points are the initial vertices and the red points are the vertices of the outer mesh. The result shows that the thickness of the wall is correlated to the diameter of the segment in the sense that it gets thinner for smaller diameters.



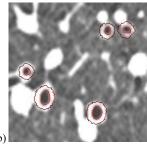


Fig. 2 The three dimensions result (a) and the two dimensions result (b) of airway wall segmentation using constraint active contour model.

## IV. DISCUSSION AND CONCLUSION

The new formulation of active contour model is presented. For the simplicity of the proposed method, the singular surface version of the model is not thoroughly covered in this paper, but rather to emphasize on the strength in coupled springy structure as it can achieve the globally optimal solution for outer surface segmentation. The well-defined physics-based interpretation in image space, allows it being easily extended to handle more than two surfaces by attaching another layer of spring structures to the outer surface. The spring constant and damping friction of the hard constraints, as well as the compositions of gravitational field all provide the flexibility for further investigation in this physical-based segmentation. Although some global parameter tunings and application-specific design of springy structure are required to achieve better segmentation, the proposed method shows prominent result with consistency. The result shows the robustness and global optimality respect to high noise resistant in full three dimensions. The preliminary result even though shows promising human pulmonary airway wall segmentation, requires further quantitative analysis for clinical validation in the further work.

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