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Efficient Design Optimization of Multi-State Flow Network for Multiple Commodities

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***Abstract*—**The network of delivering commodities has been an important design problem in our daily lives and many transportation applications. The delivery performance is evaluated based on the system reliability of delivering commodities from a source node to a sink node in the network. The system reliability is thus maximized to find the optimal routing. However, the design problem is not simple because (1) each path segment has randomly distributed attributes; (2) there are multiple commodities that consume various path capacities; (3) the optimal routing must successfully complete the delivery process within the allowable time constraints. In this paper, we want to focus on the design optimization of the Multi-State Flow Network (MSFN) for multiple commodities. We propose an efficient approach to evaluate the system reliability in the MSFN with respect to randomly distributed path attributes and find the optimal routing subject to the allowable time constraints. The delivery rates, also known as delivery currents, of the path segments are evaluated and the minimal-current arcs are eliminated to reduce the complexity of the MSFN. Accordingly, the correct optimal routing is found and the worst-case reliability is evaluated. It has been shown that the reliability of the optimal routing is at least higher than worst-case measure. Two benchmark examples are utilized to demonstrate the proposed method. The comparisons between the original and the reduced networks show that the proposed method is very efficient.

***Keywords*—**Multiple Commodities, Multi-State Flow Network (MSFN), Time Constraints, Worst-Case Reliability (WCR)

I. Introduction

THE network of commodity is a complex design optimization problem in the fields of data transmission, telecommunication, distribution of gas and oil, power transmission, transportation systems, and so on [1-7]. It is essential to analysis the reliability level of the network in order to evaluate the system performance. The most common quantification approach is to measure the probability of a successful path from a source node to a sink node in a commodity network. The estimation of network reliability has been shown NP-hard [8]; therefore, an efficient design optimization approach to evaluate the system reliability of the commodity network is desirable.

The most straightforward method is to determine the shortest path from the source node to the sink node in the commodity network; however, it may not be the quickest. The quickness of the path depends on the attributes of the path: the capacity and lead time of the segment in the path, also known as an edge (undirected) or an arc (directed) in a graph [9]. The quickest path (QP) problems [10-15] have then been formulated to find the optimal path from source to sink subject to the allowable time constraints. The modeling of the arcs therefore is critical for the reliability evaluation in the finding of optimal routing of the commodities.

The most common models have only binary states. For instance, the binary model considers that the transmission of oil through a specific segment of the pipeline system is either successful or failed. In reality, the delivery system is more complicated than the simple binary model and it has multiple states. Lin [16] utilized a probabilistic model to describe the multiple states of the arc capacities. In other words, the arc capacity is randomly distributed and the probabilities for each capacity state are determined. Moreover, The reliability analysis of the delivery network is not simple because there is usually more than one type of commodity in the delivery network [17]. Each type of commodity consumes different capacity in the delivery processes. In this paper, we want to focus on the design optimization of the Multi-State Flow Network (MSFN) [16, 18-20] for multiple commodities. We propose an efficient approach to evaluate the system reliability in the MSFN with respect to randomly distributed arc attributes and find the optimal routing subject to the allowable time constraints.

The section II first shows the method to evaluate the system reliability in the MSFN. The main idea of the evaluation approach is to send one commodity by one minimal path (MP) at a time and check if the time constraints are satisfied. The section III introduces the proposed methodology to evaluate the delivery rates, also known as delivery currents, of the path segments. The minimal-current arc is then eliminated to reduce the complexity of the MSFN while a worst-case reliability is evaluated accordingly. The section IV utilizes two benchmark network examples to demonstrate the proposed method. Furthermore, the comparisons between the original and reduced MSFN are used to show the effectiveness of the proposed method.

II. Reliability Evaluation of Multi-State Flow Network

A minimal path (MP) in MSFN is a direct walk from the source node to the sink node such that no available edge occurs more than once. Yeh et al. [21] utilized the depth-first search (DFS) to study the timely and capacitive conditions of the commodity deliveries in order to find the feasible commodity routings (FCR) under the time constraints. The network reliability [22] of the FCR is then maximized to find the optimal routing of the commodity deliveries.

The determination of the FCRs using the DFS-based approach by Yeh et al. [21] is summarized in the following steps:

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(i) Determine the combinations of the commodities and the used MPs. Calculate the consumed bandwidth for each commodity via each MP. Define two empty sets S and Ω. Begin the DFS from the first combination, i.e. first commodity via first MP.
(ii) Stack the studied combination above other elements in S. If the consumed bandwidth of the stacking combination does not exceed the allowable level, go to (iii); otherwise, go to (v).
(iii) Focus on the stored combinations in S. If both sum of the consumed bandwidths for each commonly used MP and the sum of the consumed bandwidths for each commonly used edges do not exceed the allowable levels, go to (iv); otherwise, go to (v).
(iv) If the last commodity is not reached, move to the next commodity and go to (ii);otherwise, S is a possible FCR. Keep S as the last element in Ω and go to (v).
(v) If the last MP is not reached, pop the first element in the S, study the next MP, and go to (iii); otherwise, pop the elements in S and go to (vi).
(vi) If S is not empty, go to (v); otherwise, go to (vii).
(vii) For the i-th FCR in Ω, evaluate the total consumed capacity x_j^i for the j-th edge.
(viii) If there exists $x_j^{i'} \le x_j^i$ for any j and $i' < i$, the i-th FCR is not a real FCR; otherwise, the i-th FCR is a real FCR.
(ix) Finally, the MSFN reliability is calculated by the inclusion-exclusion method [22].

Suppose the numbers of commodities and MPs are denoted by u and m respectively, the abovementioned algorithm search for all feasible solutions in an implicit graph with branching factor m searched to depth u. Therefore, the time complexity of the algorithm is in the order of $O(m^u)$. In the next section, we want to propose a novel methodology to decrease the number of m in order to efficiently evaluate the reliability and find the optimal routing for the commodity delivery.

III. Evaluation of Worst-Case Reliability

The Multi-Sate Flow Network (MSFN) is a *simple* graph $G(N,E)$ with a set $N = \{n_i \mid i = 1...v\}$ of nodes connected by a set $E = \{e_{i,j} \mid i, j = 1...v;\ j > i\}$ of edges where v is the number of nodes. In the graph theory [9], a *simple* graph is said to contain zero self-loops and multi-edges; therefore, the connection between n_i and n_j is unique and is denoted by $e_{i,j}$. In MSFN, each edge has two attributes to represent its availability: lead time $l_{i,j}$ and capacity $c_{i,j}$. In this paper, the lead times of the available edges are assumed constant and known. The capacities are randomly distributed and the corresponding probabilities are known. On the other hand, the unavailable edges have infinite lead times and their capacities are always zero. Fig. 1 shows a six-node MSFN with eight available edges, illustrated by solid lines. The edge $e_{1,6}$ is unavailable and illustrated by a dash line. For simplicity, the unavailable edges are omitted.

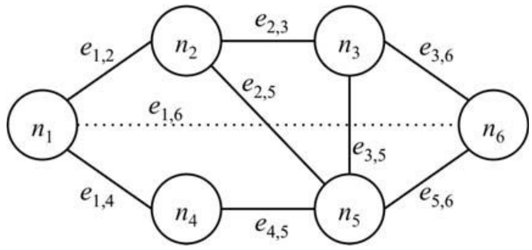

Fig. 1 Graph of MSFN [11]

We define the potential $\pi_{i,j}$ of a MSFN for moving a commodity from n_i to n_j in order to describe the delivery distributions at full capacities while the potentials follow a Delivery Potential Law (DPL). The DPL states that the potential sum around a closed loop in a MSFN is zero because a hypothetical test commodity moving around a complete loop does not receive *delivery energy* itself on the average. The potential is a delivery analog for the voltage in electric circuits as DPL is an analogy for Kirchhoff's Voltage Law [23]. Therefore, the potential $\pi_{i,j}$ can be calculated by

$$\pi_{i,j} = \pi_i - \pi_j \tag{1}$$

where π_i represents the nodal potential at n_i.

The consumed potential is inversely proportional to the edge capacity and proportional to the lead time; therefore, a delivery resistance $R_{i,j}$ is defined as the ratio of $l_{i,j}$ over $c_{i,j}$. Therefore, shorter $l_{i,j}$ or higher $c_{i,j}$ represents lower $R_{i,j}$. Referring to the Ohm's Law and Kirchhoff's Current Law [23], we define a delivery current as

$$\chi_{i,j} = \pi_{i,j} \big/ E\left[R_{i,j}\right] \tag{2}$$

where $E[R_{i,j}]$ is the expected value of the randomly distributed delivery resistance. A Delivery Current Law (DCL) then states that the sum of the delivery currents referenced into a node is equal at all times to the sum of the currents referenced out of the node, as in

$$\sum_{\substack{j=1...v \\ j\ne i}} \chi_{i,j} = \sum_{\substack{j=1...v \\ j\ne i}} \pi_{i,j} \big/ E\left[R_{i,j}\right] = \sum_{\substack{j=1...v \\ j\ne i}} \left(\pi_i - \pi_j\right) E\left[c_{i,j}\right] \big/ l_{i,j} = 0 \tag{3}$$

Given the potential $\pi_{1,v}$ between the source node n_1 and the sink node n_v, Eq. (3) is rewritten as

$$\Phi \cdot \Pi = \mathrm{X} \tag{4}$$

where Φ is a $(v-2)\times(v-2)$ matrix of edge attributes, as in (5); Π is a $(v-2)\times 1$ vector of unknown nodal potentials to be determined, as in (6); and X is a $(v-2)\times 1$ vector of source and sink currents, as in (7).

$$\Phi = \sum_{i=2}^{v-1}\sum_{j=1}^{v}\left(\mathrm{E}\left[c_{i,j}\right]/l_{i,j}\right)\mathbf{v}_i\mathbf{v}_i - \sum_{i=2}^{v-1}\sum_{j=2}^{v-1}\left(\mathrm{E}\left[c_{i,j}\right]/l_{i,j}\right)\mathbf{v}_i\mathbf{v}_j \quad (5)$$

$$\Pi = \sum_{i=2}^{v-1}\pi_i\mathbf{v}_i \quad (6)$$

$$\mathrm{X} = \sum_{i=2}^{v-1}\left(\pi_1\mathrm{E}\left[c_{i,1}\right]/l_{i,1} + \pi_v\mathrm{E}\left[c_{i,v}\right]/l_{i,v}\right)\mathbf{v}_i \quad (7)$$

In Eqs. (5) to (7), $\mathbf{v}_i$ represents i-th normal basis. The vector dyad $\mathbf{v}_i\mathbf{v}_j$ is for the quantity in the i-th row and j-th column of matrix. The quantities $c_{i,i}$ and $l_{i,i}$ are equal to zero because simple graph does not have any self-loops. The quantities $c_{i,j}/l_{i,j}$ for the unavailable edges are equal to zero since $c_{i,j} = 0$ and $l_{i,j} \to \infty$.

When the MSFN is not trivial and appropriately connected, Φ is symmetric and the unknowns can be determined by

$$\Pi = \Phi^{-1} \cdot \mathrm{X} \quad (8)$$

Accordingly, the delivery current for each edge can be calculated by Eq. (2). An edge with minimal delivery current has the lowest probability of contributing to deliver the commodity, that is, its existence has minimal significance in the evaluation of the reliability and optimal decision of the community routing. In this paper, we propose to evaluate the worst-case reliability of the reduced MSFN based on the eliminations of the minimal-current edges.

Based on the proposed framework, the delivery currents are first evaluated using Eq. (8) in the beginning of the design optimization process. The edges with the minimal delivery currents are then eliminated and the complexity of the reliability analysis is greatly reduced. The optimal routing is determined based the reliability analysis of the reduced MSFN and the optimal reliability is at least higher than the worst-case reliability.

IV. Numerical Examples

In this section, two benchmark networks are considered. The first network is the six-node MSFN shown in Fig. 1. The second problem focuses on a complex 26-node MSFN. The proposed method is utilized to evaluate the worst-case reliabilities and determine the optimal routings. The true reliabilities will be higher than or at least equal to the worst-case reliabilities. Meanwhile, the optimal routings obtained in the worst case will be identical to those obtained in the normal case.

A. Example 1: A Six-Node MSFN

The first example considers the six-node MSFN in Fig. 1. Based on DCL, the delivery current of the edges in a series remains constant. Therefore, the MSFN in Fig. 1 is simplified for the evaluation of the delivery current by merging the serial edges into a hyperedge, shown by the thick line in Fig. 2. It is noted that one hyperedge can only contains the adjacent edges in a simple series and the common incident nodes in the series. For instance, we use "+" to represent the operation of merging; therefore, the hyperedge $e_{1,5} = e_{1,4} + e_{4,5}$ contains two edges and one node: $\{e_{1,4}, e_{4,5}, n_4\}$. The proposed method is then utilized to evaluate the delivery currents in the remaining edges in the figure.

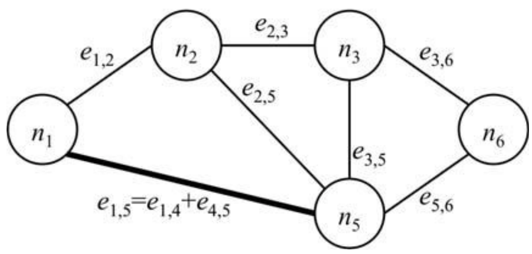

Fig. 2 Merged graph for the six-node MSFN

Suppose each available edge has the common attributes $c_{i,j} = c$ and $l_{i,j} = l$, we denote that $E[c] = C$. Given the reference potential $\pi_6 = 0$ and the source potential $\pi_1 = \pi$, the vector X is given by

$$\mathrm{X} = \begin{bmatrix} \pi_1\mathrm{E}\left[c_{2,1}\right]/l_{2,1} \\ \pi_1\mathrm{E}\left[c_{3,1}\right]/l_{3,1} \\ \pi_1\mathrm{E}\left[c_{5,1}\right]/l_{5,1} \end{bmatrix} = \begin{bmatrix} \pi C/l \\ 0 \\ \pi C/2l \end{bmatrix} \quad (9)$$

The diagonal terms in the matrix Φ are given by

$$\begin{aligned}\Phi_{11} &= \mathrm{E}\left[c_{2,1}\right]/l_{2,1} + \mathrm{E}\left[c_{2,3}\right]/l_{2,3} + \mathrm{E}\left[c_{2,5}\right]/l_{2,5} \\ &= C/l + C/l + C/l = 3C/l\end{aligned} \quad (10)$$

$$\begin{aligned}\Phi_{22} &= \mathrm{E}\left[c_{3,2}\right]/l_{3,2} + \mathrm{E}\left[c_{3,5}\right]/l_{3,5} + \mathrm{E}\left[c_{3,6}\right]/l_{3,6} \\ &= C/l + C/l + C/2l = 3C/l\end{aligned} \quad (11)$$

$$\begin{aligned}\Phi_{33} &= \mathrm{E}\left[c_{5,1}\right]/l_{5,1} + \mathrm{E}\left[c_{5,2}\right]/l_{5,2} + \mathrm{E}\left[c_{5,3}\right]/l_{5,3} + + \mathrm{E}\left[c_{5,6}\right]/l_{5,6} \\ &= C/2l + C/l + C/l + C/l = 3.5C/l\end{aligned} \quad (12)$$

The off-diagonal terms in Φ are shown as

$$\Phi_{12} = \Phi_{21} = -\mathrm{E}\left[c_{2,3}\right]/l_{2,3} = -C/l \quad (13)$$

$$\Phi_{13} = \Phi_{31} = -\mathrm{E}\left[c_{2,5}\right]/l_{2,5} = -C/l \quad (14)$$

$$\Phi_{23} = \Phi_{32} = -\mathrm{E}\left[c_{3,5}\right]/l_{3,5} = -C/l \quad (15)$$

The unknown potentials are determined using (8), as in

$$\left[\pi_2, \pi_3, \pi_5\right] = \left[0.575, 0.325, 0.400\right]\pi \quad (16)$$

The delivery currents are calculated using (2), as in

$$\begin{aligned}&\left[\chi_{1,2}, \chi_{1,5}, \chi_{2,3}, \chi_{2,5}, \chi_{3,5}, \chi_{3,6}, \chi_{5,6}\right] = \\ &\left[0.425, 0.200, 0.250, 0.175, -0.075, 0.325, 0.400\right]\pi C/l\end{aligned} \quad (17)$$

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Therefore, the edge $e_{3,5}$ has the smallest delivery current $\chi_{5,3} = -\chi_{3,5} = 0.075\pi C/l$ to move the commodity from n_5 to n_3 and can be eliminated for the worst-case reliability analysis without jeopardizing the main body of the network. Thus, the original six-node eight-edge MSFN is reduced to a six-node seven-edge MSFN, as shown in Fig. 3.

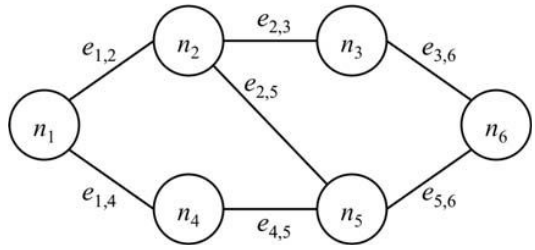

Fig. 3 A reduced six-node seven-edge MSFN

The original MSFN has a total of eight MPs as shown in Table I. Since edge $e_{3,5}$ is removed from the network, the reduced MSFN has a total of four MPs as shown in Table II.

Suppose there are three different types of commodities. Each type of commodity has the number of demand equal to 10. We assume each commodity consumes 1 unit of capacity during the delivery while the maximum capacity of each edge equals 7 units. The lead time of each edge is 2 units while the allowable delivery time is 12 units. Using the evaluation methodology in section II, the optimal routings of the commodity delivery for the original and reduced MSFN are shown in Table III respectively. The results are obtained from the same computer running a Windows operating system with a 3.07 GHz processor and 6 GB of RAM.

The original MSFN has the maximum network reliability when the first, second, and third commodities are delivered via the 1^{st}, 4^{th}, and 8^{th} MPs respectively, denoted by {1, 4, 8} in the Table III. On the other hand, the optimal routing solution is {1, 2, 4} for the reduced MSFN. According to Tables I and II, the optimal routing set for the reduced network is identical to the original solution: first commodity via the MP of $\{e_{1,2}, e_{2,3}, e_{3,6}\}$; second one via $\{e_{1,2}, e_{2,5}, e_{5,6}\}$; and the last one via $\{e_{1,4}, e_{4,5}, e_{5,6}\}$. Therefore, the proposed method is capable of finding the correct optimal routing solution under the appropriate amount of network reduction using the delivery laws.

Using the inclusion-exclusion method [22], the network reliability of the MSFN is higher than or equal to the worst-case measure: 0.985. We confirmed that the true reliability of the original MSFN is 0.986. It is expected that the worst-case reliability is slightly lower than the true measure because the delivery current of the removed edge $e_{3,5}$ is at most only 42% of the other edges, which contributes the least in the commodity delivery. Lastly, the time complexity is greatly reduced from $O(8^3)$ to $O(4^3)$; therefore, the computational efficiency is tremendously improved. The simulation turnaround time for the reduced MSFN is 1325.5 times faster than that for the original MSFN.

TABLE I
MINIMAL PATHS OF THE ORIGINAL SIX-NODE MSFN

MP	Edge Set
1	$\{e_{1,2}, e_{2,3}, e_{3,6}\}$
2	$\{e_{1,2}, e_{2,3}, e_{3,5}, e_{5,6}\}$
3	$\{e_{1,2}, e_{2,5}, e_{3,5}, e_{3,6}\}$
4	$\{e_{1,2}, e_{2,5}, e_{5,6}\}$
5	$\{e_{1,4}, e_{4,5}, e_{2,5}, e_{2,3}, e_{3,6}\}$
6	$\{e_{1,4}, e_{4,5}, e_{2,5}, e_{2,3}, e_{3,5}, e_{5,6}\}$
7	$\{e_{1,4}, e_{4,5}, e_{3,5}, e_{3,6}\}$
8	$\{e_{1,4}, e_{4,5}, e_{5,6}\}$

TABLE II
MINIMAL PATHS OF THE REDUCED SIX-NODE MSFN

MP	Edge Set
1	$\{e_{1,2}, e_{2,3}, e_{3,6}\}$
2	$\{e_{1,2}, e_{2,5}, e_{5,6}\}$
3	$\{e_{1,4}, e_{4,5}, e_{2,5}, e_{2,3}, e_{3,6}\}$
4	$\{e_{1,4}, e_{4,5}, e_{5,6}\}$

TABLE III
SIMULATED RESULTS FOR THE SIX-NODE MSFN

MSFN	Optimal Route Set	Network Reliability	Turnaround Time (millisec)
Original	{1, 4, 8}	0.986	2651
Reduced	{1, 2, 4}	0.985	2

B. Example 2: A 26-Node MSFN

The second example considers a complex MSFN that contains 26 nodes and 29 available edges, as in Fig. 4.

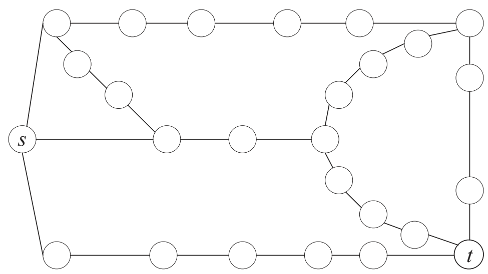

Fig. 4 A 26-Node MSFN [24]

The complex MSFN is first simplified to Fig. 5 based on the merging of serial edges. The proposed method is then utilized to evaluate the delivery currents at the edges $e_{1,2}$, $e_{1,5}$, $e_{1,26}$, $e_{2,5}$, $e_{2,15}$, $e_{5,11}$, $e_{11,15}$, $e_{11,26}$, and $e_{15,26}$.

Suppose each available edge has the common attributes $c_{ij} = c$ and $l_{ij} = l$, we denote that $\mathrm{E}[c] = C$. Given the reference potential $\pi_{26} = 0$ and the source potential $\pi_1 = \pi$, the vector X is calculated as

$$\mathrm{X} = \begin{bmatrix} \pi_1 \mathrm{E}[c_{2,1}]/l_{2,1} \\ \pi_1 \mathrm{E}[c_{5,1}]/l_{5,1} \\ \pi_1 \mathrm{E}[c_{11,1}]/l_{11,1} \\ \pi_1 \mathrm{E}[c_{15,1}]/l_{15,1} \end{bmatrix} = \begin{bmatrix} \pi C/l \\ \pi C/l \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

The diagonal terms in the matrix Φ are given by

$$\begin{aligned}\Phi_{11} &= \mathrm{E}[c_{2,1}]/l_{2,1} + \mathrm{E}[c_{2,5}]/l_{2,5} + \mathrm{E}[c_{2,15}]/l_{2,15} \\ &= C/l + C/3l + C/5l = 1.533C/l\end{aligned} \quad (19)$$

$$\begin{aligned}\Phi_{22} &= \mathrm{E}[c_{5,1}]/l_{5,1} + \mathrm{E}[c_{5,2}]/l_{5,2} + \mathrm{E}[c_{5,11}]/l_{5,11} \\ &= C/l + C/3l + C/2l = 1.833C/l\end{aligned} \quad (20)$$

$$\begin{aligned}\Phi_{33} &= \mathrm{E}[c_{11,5}]/l_{11,5} + \mathrm{E}[c_{11,15}]/l_{11,15} + \mathrm{E}[c_{11,26}]/l_{11,26} \\ &= C/2l + C/4l + C/4l = C/l\end{aligned} \quad (21)$$

$$\begin{aligned}\Phi_{44} &= \mathrm{E}[c_{15,2}]/l_{15,2} + \mathrm{E}[c_{15,11}]/l_{15,11} + \mathrm{E}[c_{15,26}]/l_{15,26} \\ &= C/5l + C/4l + C/3l = 0.783C/l\end{aligned} \quad (22)$$

The off-diagonal terms in Φ are shown as

$$\Phi_{12} = \Phi_{21} = -\mathrm{E}[c_{2,5}]/l_{2,5} = -C/3l \quad (23)$$

$$\Phi_{14} = \Phi_{41} = -\mathrm{E}[c_{2,15}]/l_{2,15} = -C/5l \quad (24)$$

$$\Phi_{23} = \Phi_{32} = -\mathrm{E}[c_{5,11}]/l_{5,11} = -C/2l \quad (25)$$

$$\Phi_{34} = \Phi_{43} = -\mathrm{E}[c_{15,11}]/l_{15,11} = -C/4l \quad (26)$$

$$\Phi_{13} = \Phi_{31} = \Phi_{24} = \Phi_{42} = 0 \quad (27)$$

The unknown potentials are determined using (8), as in

$$\begin{aligned}&[\pi_2, \pi_5, \pi_{11}, \pi_{15}] = \\ &[0.889, 0.850, 0.524, 0.394]\pi\end{aligned} \quad (28)$$

Therefore, the delivery currents are calculated using (2), as in

$$\begin{aligned}&[\chi_{1,2}, \chi_{1,5}, \chi_{1,26}, \chi_{2,5}, \chi_{2,15}, \chi_{5,11}, \chi_{11,15}, \chi_{11,26}, \chi_{15,26}] = \\ &[0.111, 0.150, 0.167, 0.013, 0.099, \\ &0.163, 0.033, 0.131, 0.131] \times \pi C/l\end{aligned} \quad (29)$$

The hyperedge $e_{2,5}$ is found to have the lowest delivery current and provides the smallest contributions for the commodity delivery. Therefore, the hyperedge $e_{2,5}$, equivalent to the combination of edges $e_{2,3}$, $e_{3,4}$, and $e_{4,5}$, is eliminated from the original 26-node 29-edge MSFN, which leads to a 24-node 26-edge MSFN. The original MSFN has nine paths as shown in Table IV, whereas the reduced MSFN has five paths as shown in Table V. The computational complexity is then reduced from $O(9^3)$ to $O(5^3)$.

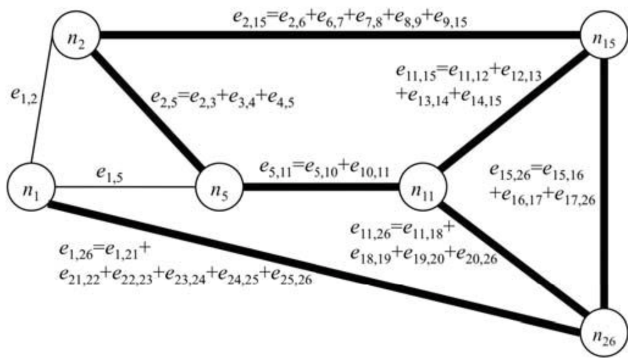

Fig. 5 Merged graph for the 26-node MSFN

TABLE IV
MINIMAL PATHS OF THE ORIGINAL 26-NODE MSFN

MP	Edge Set
1	$\{e_{1,2}, e_{2,15}, e_{15,26}\}$
2	$\{e_{1,2}, e_{2,15}, e_{11,15}, e_{11,26}\}$
3	$\{e_{1,2}, e_{2,5}, e_{5,11}, e_{11,15}, e_{15,26}\}$
4	$\{e_{1,2}, e_{2,5}, e_{5,11}, e_{11,26}\}$
5	$\{e_{1,5}, e_{2,5}, e_{2,15}, e_{11,15}, e_{11,26}\}$
6	$\{e_{1,5}, e_{2,5}, e_{2,15}, e_{15,26}\}$
7	$\{e_{1,5}, e_{5,11}, e_{11,15}, e_{15,26}\}$
8	$\{e_{1,5}, e_{5,11}, e_{11,26}\}$
9	$\{e_{1,26}\}$

TABLE V
MINIMAL PATHS OF THE REDUCED 24-NODE MSFN

MP	Edge Set
1	$\{e_{1,2}, e_{2,15}, e_{15,26}\}$
2	$\{e_{1,2}, e_{2,15}, e_{11,15}, e_{11,26}\}$
3	$\{e_{1,5}, e_{5,11}, e_{11,15}, e_{15,26}\}$
4	$\{e_{1,5}, e_{5,11}, e_{11,26}\}$
5	$\{e_{1,26}\}$

TABLE VI
SIMULATED RESULTS FOR THE 26-NODE MSFN

MSFN	Optimal Route Set	Network Reliability	Turnaround Time (millisec)
Original	{8, 9, 9}	0.979	18096
Reduced	{4, 5, 5}	0.977	47

Assuming the lead time of each edge is 1 unit and the time constraint is 14 units, the other simulation settings follow the Example 1. The simulated results are listed in Table VI. The optimal routing for the three commodities obtained from the reduced MSFN is {4, 5, 5}, which is identical to the true solution {8, 9, 9} in the original MSFN. The worst-case network reliability is evaluated as 0.977, which is just slightly lower than the true reliability: 0.979. Using the proposed method to reduce the network complexity, the simulation turnaround time is greatly improved to 385 times faster than original calculation as expected.

V. CONCLUSIONS

The design optimization and reliability analysis of the Multi-State Flow Network for multiple commodities are important but very complicated. We have reviewed Yeh's methods to search for the feasible commodity routings under the desirable time constraints and find the optimal network reliabilities. However, the computational requirement is in the order of number of minimal paths to the power of commodity number. In this paper, we have proposed a novel method to improve the computational efficiency. The Delivery Laws have been defined to evaluate the delivery rate for each edge. The minimal-current edge is eliminated to reduce the number of minimal paths since it has the lowest contribution of commodity delivery. Two benchmark examples have shown the proposed method is capable of efficiently determining the optimal routing and evaluating the worst-case reliability that is slightly lower than the true reliability level.

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