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Do Students Really Understand Topology in the Lesson? A Case Study

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***Abstract*—**This study aims to specify to what extent students understand topology during the lesson and to determine possible misconceptions. 14 teacher trainees registered at Secondary School Mathematics education department were observed in the topology lessons throughout a semester and data collected at the first topology lesson is presented here. Students' knowledge was evaluated using a written test right before and after the topology lesson. Thus, what the students learnt in terms of the definition and examples of topologic space were specified as well as possible misconceptions. The findings indicated that students did not fully comprehend the topic and misunderstandings were due to insufficient pre-requisite knowledge of abstract mathematical topics and mathematical notation.

***Keywords*—**Mathematics Education, Teacher Education, Topology.

I. Introduction

STUDIES on conceptual understanding in science and mathematics, which started in late 1970's internationally and peaked in the beginnings of 1980's, have been widely documented in literature [1-5]. Leading researchers – conceptualists report that teaching in the area at various educational levels around the world is generally finalised without conceptual understanding. These studies, which base their work on constructivism, report that in order to finalise teaching with conceptual understanding students' comprehension levels of the relevant topics should be tested before or after the lesson using methods such as tests/interviews/observations.

One of the disciplines in which conceptual understanding is crucial is topology. Topology was recognized as part of mathematics following a real life problem in geometry. Euler's study in 1736 on "Seven Bridges in Königsberg" is accepted to be one of the first topological findings. Subsequently, topology saw a rapid development and today it has become indispensible with its wide range of applications from digital medicine and artificial intelligence to language studies. Topology allows mathematics to be generalized to all sets and provides an overview of mathematical topics. In order to investigate mathematical topics from a wider perspective, students' understanding of topology is essential. Given that the history of mathematics dates back to BCE, topology, a comparatively recent area in the history of science, is a course that might cause misconceptions among university students due to various abstract concepts it involves.

"I had heard from my maths teacher and the trainee teacher in my secondary school. They said it was a very difficult lesson…"

"When I was in my first year, my friends in senior years told me. They told me that it was a very difficult lesson, they memorised to pass, only a few students in the class could understand it and they couldn't explain what they understood, and I thought I would possibly fail..."

Student views given above about the topology course belong to students registered at secondary school mathematics department in the 2008-2009 academic year. Similar views are believed to be echoed by most of the mathematics students. These students stated that their views remained the same after the completion of the course. Success levels in the topology course are considerably low. This might suggest inefficient topology instruction.

Although plenty of studies exist on topology as pure mathematics, there is relatively less on only topology instruction. Some studies conducted in 70's stand out in the literature [6-11]. Afterwards, fewer studies are found in the field [12-14]. Moreover, most of these studies may not be completely considered as educational. The effects of student centred instruction on topology education were investigated in a thesis conducted in 2001 [15]. Some studies also exist on the use of Moore method in topology instruction [16-19].

Few studies investigate students' understanding and misconceptions in the topology course. With an aim to fill this gap, students were observed during an academic term in the topology course. In order to determine how and to what extent students understood the course topics, data collected in the first topology lesson of the course were analysed. This preliminary study is expected to be a stepping stone for the investigation of the topology course in which many conceptual errors could occur.

II. Methods

Research design is the organisation of required conditions for data collection and analysis economically in line with the research questions [20]. Two main approaches that meet these requirements are survey and experimental designs.

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Survey models are research designs that describe a past or a present situation as it was / is. General survey models, on the other hand, are survey models conducted on the entire population or a group, example or sample from the population in order to draw conclusions on a population which consists of many elements [20].

Thus, the current study adopted a general survey design. A group of students were observed during the topology course for an academic term. Experts were also consulted and the method was subsequently chosen. Student views on the topology course were investigated at the beginning and end of the term using open-ended questionnaires. Semi-structured interviews were also conducted at the beginning and end of the term with five students each. Moreover, at the beginning and end of each lesson, a written test of a few questions was conducted about the teaching point of the relevant lesson. Mid-term and final examinations aimed to test conceptual understanding were administered and five students were interviewed about the questions in the final examination. Several lessons were audio recorded. Student responses in a final examination conducted in the previous years as part of the topology course were analysed and main student errors were identified.

The scope of the current paper includes student errors identified in the written test mentioned above and data collected in the first topology lesson. In the first lesson, set families were roughly covered and the concept of constructing topologic structures was then introduced via open axioms and various examples of topologic structures were discussed. A written test was administered right before and after this lesson in order to describe students' comprehension.

A. Participants

The sample of this study consisted of 17 students registered at the Secondary School Mathematics Education Department. The sample included students who were taking the topology course for the first time and students who failed the course before.

B. Data Collection Tools

Data was collected from a topology exam conducted in the previous years and pre and post written tests (WT) administered before and after the first lesson. Expert reviews of the written test were consulted and a two-question test was found appropriate for research. The test questions were as follows:

1. What are topologic structure and topologic space? Please discuss.

2. Given set of real numbers R and interval A= [3, 7). Is the family $\Im=\{T : A\subset T \text{ or } T=\emptyset\}$ a topologic structure on R? Please show.

C. Validity and Reliability

Content validity of data collection tools was ensured via a detailed consideration of the scope of research with two lecturers at the Department of Mathematics Education. For the reliability study, all qualitative data was categorised and coded separately by each researcher. Following this coding stage, consistency of the categories coded by each researcher was found to be %88.

III. Results

This section initially describes student errors observed in previous examination papers. Some of these errors are presented in the table below, unchanged.

TABLE I
SOME STUDENTS ERRORS OBSERVED IN A WRITTEN TOPOLOGY EXAM

Student	Error
Gamze	*If f is continuous* $f(x)\in\tau_2\subset\tau_1$ ... *For* $f(x)\in\tau_2(x\in f^1(\tau_2))$ $\quad x\in\tau_1(x\in f^1(\tau_2))$, $A(X,\delta)\in\ \tau_1$ *f is continuous*
Mehtap	*f is continuous* $f(U)\subset T\ f^1(U)\ \in\tau_1$
Serap	*Let's take axiom* $T\in\tau_1$. $T^c\in\tau_1$ *is closed* $\Rightarrow T^c\in\tau_2$ *in other words each open set according to* τ_1 *are open according to* τ_2 *thus* $\tau_2\subset\tau_1$
Özlem	*If* $F\in\tau_1$ *is closed then* $F^c\in\tau_1$ *is open...*
Kadir	$A^c\notin\tau_2\Rightarrow A\in\tau_2$ *For a function to be open each of its subsets should be open, in other words the image of each element in f is open...*
Dilek	*As more closed wouldn't require more open, we can't compare them.*
Hasan	*Given* $\forall T\in\tau_1\Rightarrow T^c$ *is closed. Thus* f^1 *is open* $\Rightarrow f^1(V)$ *For* $\in\tau_1\Rightarrow\ \forall x\in U\, f^1$*is* $(f(x))\ \in\tau_1$
Harun	*Given* $B\subset\tau_1$ *is closed for the topology* $B\ \tau_1$. *If it is also closed for* $B\subset\tau_2$, *then it is also closed in* $B\ \tau_2$. *One open in* $B^c\subset\tau_1$ *and one open in* $B^c\subset\tau_2$, *thus* $\tau_2\subset\tau_1$.
Burak	*For* $T\in\tau_1$, T^c *is closed for* τ_1 *and closed for* τ_2, *T is open for* τ_2, $T\in\tau_2$ *thus* $\tau_2\subset\tau_1$.
Filiz	*Each set closed for the topology* $(X,\ \tau_1)\ (X,\ \tau_2)$, τ_1 *is closed for* τ_2; *according to this statement topology* τ_1 *is a subset of* τ_2 *in other words is more closed than* τ_2. *They can be compared and* τ_1 *is more closed than* τ_2
Yasin	*Given* $\forall T$ *is closed for* τ_1 *and* τ_2. $T\in\tau_1\Rightarrow T^c$ *is closed for* τ_1; $T\in\tau_2$ *is open and* $\tau_1\subset\tau_2$.
Bekir	*Given A is closed for* τ_1. *A is closed for* τ_2. A^c *is open As* $f^1(A^c)=\{\ f^1(A)\}^c$, *A is open for* τ_2, *As it is closed for* τ_1 *and closed for* τ_2, *then* $\tau_1\subset\tau_2$.
Coşkun	*If* $T\in\tau_1$, *then* $T^c\notin\tau_1$ *and* $T^c\notin\tau_2$ *and* $T\in\tau_2$
İlyas	$\tau_1^c\subset\tau_2$ *and* $\tau_2^c\subset\tau_1$ *and* $\tau_1\subset\tau_2$.
Mehtap	$X\in\tau_1\ \Rightarrow X^c\in\tau_1$, *then* $\tau_1\subset\tau_2\ \Rightarrow X^c\in\tau_2$. *Thus* $X\in\tau_2$ *in other words* $\tau_1\subset\tau_2$

14 students were present in the first lesson. Nine of these

TABLE I
CONTINUITED…

Student	Error
Gonca	*Function can be open if* $\in\tau$ *...*
Turan	*Topologic structures* $\tau_1=\{\varnothing, X, [1,2]\}$ *and* $\tau_2=\{\varnothing, X, [1,2], [3,5)\}$ *cannot be compared*
Nurşah	$A\subset X$ *is closed, for* $A^c\in\tau_1$ $\forall A\in\tau_1$ *, there is a closed set A,* $A\subset\tau_1$ *in other words* $\tau_1\subset\tau_2$

students had failed the course before and were re-taking it. The answer to the first question in the test was: "Given the power set of set A, not an empty set, is $\wp(A)$ and $\tau\subset\wp(A)$, if it satisfies three axioms called τ open axioms, then family τ is called the topologic structure in set A and the pair (A,τ) is called topologic space". In the answer, the open axioms O1 ($\varnothing$,R$\in\tau$), O2 (family τ should be closed according to finite or infinite union operation) and O3 (family τ should be closed according to finite intersection operation) should, of course, be stated. In the second question, family τ should be shown to satisfy open axioms O1, O2 and O3. According to the results of the WT before the lesson, students who had failed the course before were observed to provide really insufficient or wrong answers. These answers can be summarised as follows:

TABLE II
ANSWERS TO THE WRITTEN TEST BEFORE THE LESSON

Student	Error
Gülşah	*I remember titles such as open set, accumulation point…, τ, set families. I didn't understand it when studying for the exam.*
Çetin	$A=[3,7)$*,* $A^{-1}=(-\infty, 3)\cup[7,+\infty)$ *Is a topologic structure because it is the reverse!*
Mustafa	τ*:* $x\in X$ *and* $x\in\tau$
Yakup	*Topologic structure: Are structures composed of topologic families.*
Evrim	*I think there were 3 properties we looked to decide whether it is a topologic structure. The given set was not an empty set. We looked at intersection and combination. And they should belong to τ. But I can't remember completely.*
Seyfettin	*I took the course but I don't remember anything. It was supposed to be a topologic structure if it satisfies two properties but I don't remember the properties.*
Gonca	*3 conditions should be met in order to talk about the existence of a topologic structure. I don't remember. I assume they were things like open and closed.*
İrfan	*Like the subsets composed of the elements of the power set of set A made a topologic structure in space… There were three rules to be met for a topologic structure but I don't remember.*

Only one of the students who took the course before, and not included in Table 2, was able to explain open axioms correctly. However, he was not able to answer the second question. Students who were taking the course for the first time, naturally, were unable to answer the questions.

Some wrong answers were also observed in the written test administered after the lesson. These answers could be summarised as follows:

- Students generally provided correct answers to the first question except two students who did not state open axioms precisely. Notational errors were observed when explaining open axioms.
- For the second question, answers of four students were wrong for axiom O1; the rest of the answers were correct for O1.
- Only three students could provide correct answers to axiom O2. Some of the rest of the students printed the axiom as it was without applying it to the example. There were also students who understood the concept of infinite union in this axiom as the union of all sets that belong to family τ.
- Only two students correctly answered axiom O3. Some of the rest of the students only explained the axiom generally and some others stated that the intersection of sets that belong to family τ would always b equal to set A=[3,7).
- Almost all students made mistakes of notation.
- Students were observed to have difficulties in concepts such as sets, set families, union in set families and intersection.

IV. CONCLUSION

As presented in Table1, students misunderstand topologic concepts as well as abstract mathematical concepts such as sets, set families and combination, intersection in families which provide the foundations of topology. Students were observed to be insufficient in the use of notation. Similar observations were made in the test results conducted before and after the presentation of the lesson. Students were not even aware that topological structure is a set family.

It is significant that students who had taken the course before could not remember anything even in relation to basic concepts. Following the presentation of the lesson, these students could define topological structure, however, struggled to apply the definition to the example. This might suggest that students' mathematical expression skills were weak or they did not fully comprehend the definitions. For example, infinite combination in topologic structures was misunderstood.

According to the data collected this study it is possible to claim that these concepts were not fully understood during the lesson. Major source of the problem could be deficiencies in abstract mathematics and the use of notation which provide the basis of the topology course. Abstract mathematics concepts are prerequisites of the topology course. Therefore, concepts such as sets, sets families, etc. could be introduced at the beginning of the topology course.

Observations during the lesson indicated that students perceived the lesson to be abstract and tended to drift away.

Student-centred approaches could be used to keep students alert. Likewise, studies have evidenced that the method known as the Moore Method increased success in topology instruction.

However, research is scarce on topology instruction in which various problems are encountered. Further research in the area is required. Students seemed to have negative attitudes towards topology. Hence, research on attitude towards the topology course would also be beneficial. Studies could focus on conceptual learning in the topology course or on the effects of the modified moore method in topology instruction. Future studies of the author will touch on these problems.

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