

# DMC with Adaptive Weighted Output

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**Abstract**—This paper presents a new adaptive DMC controller that improves the controller performance in case of plant-model mismatch. The new controller monitors the plant measured output, compares it with the model output and calculates weights applied to the controller move. Simulations show that the new controller can help improve control performance and avoid instability in case of severe model mismatches.

**Keywords**—Adaptive control, dynamic matrix control, DMC, model predictive control

## I. INTRODUCTION

IN many industrial plants, distributed control systems (DCS), mainly composed of PID controllers, have been used to control the process. The continuous need to increase productivity, improve efficiency, and the challenges caused by process disturbances, process nonlinearity, variance in raw material quality, have motivated the use of advanced process control (APC). Among different APC schemes, model predictive control (MPC), has received the most attention especially in refining, petrochemical and chemical industries [1]. Dynamic models play a central role in the MPC technology. Imprecise model can significantly degrade control performance and may lead to plant instability. The most difficult and time consuming work during an industrial MPC project is modeling and identification. It is estimated that up to 80% of time and expense in the design and installation of MPC is attributed to modeling and system identification [1]. Model is usually identified by applying a step change on each manipulated variable (MV) and record the change in all controlled variables (CV). This process should be repeated several times at all operating ranges to reach a consistent dynamic model. The accuracy of the model highly depends on the number of step tests done, the magnitude of the step change, and the lack of external disturbances or process instabilities. Process control engineers are usually challenged by the restrictions imposed by plant operators on the number of step tests and the allowed changes in controlled variables making a precise model hard to achieve. Imprecise model identification, in addition to plant nonlinearities have motivated the research in adaptive MPC. In adaptive MPC, a supervisory module is continuously collecting measurements, estimating the process model and updating the MPC controller. Although the adaptive MPC described looks simple and reasonable, the difficulty is how to construct the adaptive MPC while maintain closed loop stability [7]. Different techniques for adaptive MPC are well summarized in [7].

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Lots of research efforts were focused on using artificial neural networks in online model identification in adaptive MPC, especially for nonlinear applications [2-4]. In [5], Xue M., et al. described an adaptive MPC based on fuzzy compensation mechanism. In [6], a different technique was adopted by measuring time domain modeling error indicators, and applying updates to a parametric DMC through linear regression equations and a fuzzy system. In [10] Authors proposed using a sliding mode controller in parallel with MPC. The role of the additional controller is to produce a control action that compensate the process nonlinearities and hence improve robustness.

This paper presents a new adaptive MPC scheme where a supervisory module inspect the plant measured output after each change in controller set point and then apply weights on controller output to avoid the effects of plant model mismatch. The paper is organized as follows: Notation used in the paper is presented in section II. Section III is dedicated for a quick overview on the theory behind a famous type of MPC which is the dynamic matrix control (DMC), while section V explains the new adaptive DMC proposed. Simulation results and the conclusion are presented in sections IV and V respectively.

## II. NOTATION

Bold lower case letters are used for vectors while bold upper case letters are used for matrices. The hat accent is used to indicate that the variable is an estimated one. All notation used in this paper are given in Table I.

TABLE I  
NOTATION

Symbol	Description
$z^{-n}$	Delay operator
$h$	Impulse response coefficients
$y$	Plant measured output
$u$	Controller output
$k$	Process gain
$\theta$	Process dead time
$\tau$	Process time constant
$d$	Measured disturbances
$r$	Reference trajectory or set point
$q$	Output weighting factor
$\lambda$	Move suppression factor

## III. MODEL PREDICTIVE CONTROL

### A. Basic concepts and equations

MPC refers to a family of controllers that uses a discrete form of the process model to predict future values of a process variable based on past values of controller output. The main idea behind MPC-type controllers is illustrated in Fig. 1 for a SISO system[12]. At sampling time  $k$ , a set of  $m$

future manipulated variable moves (control horizon) are selected, so that the predicted response over a finite horizon  $p$  (prediction horizon) has certain desirable characteristics. This is achieved by minimizing an objective function based on the deviation of the future controlled variables from a desired trajectory over the prediction horizon  $p$  and the control energy over the control horizon  $m$ . The MPC optimization is performed for a sequence of hypothetical future control moves over the control horizon and only the first move is implemented [13]. The problem is solved again at time  $k + 1$  with the measured output  $y(k + 1)$  as the new starting point. Model uncertainty and unmeasured process disturbances are handled by calculating an additive disturbance as the difference between the process measurement and the model prediction at the current time step.

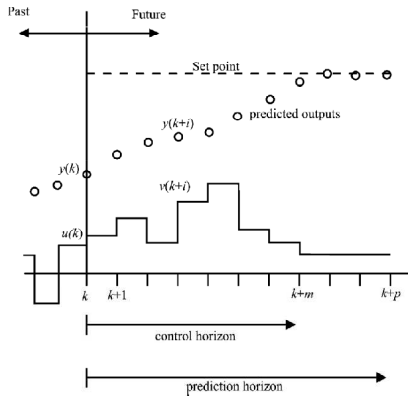


Fig. 1 Graphical representation for MPC

MPC algorithm can be easily extended to control MIMO processes, subject to numerous disturbances and dynamically varying constraints. Based on the model used, different MPC algorithms are described in literatures [11]. Dynamic matrix control DMC is a widely used algorithm developed by Cutler and Ramaker in the seventies. The DMC uses a step response model which consists of values representing the step response of the model

$$\mathbf{h} = [h_1 \ h_2 \ h_3 \ \dots \ h_p] \quad (1)$$

where  $p$  is the prediction horizon.

The future process values can be predicted by:

$$\mathbf{y} = \mathbf{H}\Delta\mathbf{u} + \mathbf{d} \quad (2)$$

Where  $\mathbf{u}$  is the future controller moves,  $\mathbf{d}$  is the unmeasured disturbances and  $\mathbf{H}$  is the dynamic matrix. If the control horizon equals to  $m$ ,  $\mathbf{H}$  can be constructed as:

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & 0 & \dots & 0 \\ h_2 & h_1 & 0 & \dots & 0 \\ h_3 & h_2 & h_1 & \dots & \cdot \\ h_4 & h_3 & h_2 & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ h_p & h_{p-1} & h_{p-2} & \dots & h_{p-m+1} \end{bmatrix} \quad (3)$$

Assuming that the future set points are known and  $\mathbf{d}=0$ , the controller can estimate the optimum future moves, by minimizing a cost function defined by:

$$\begin{aligned} J &= \sum_{i=1}^p (r_i - y_i)_Q^2 + \sum_{i=1}^m \lambda \Delta u^2 \\ &= (\mathbf{r} - \mathbf{H}\Delta\mathbf{u})^T \mathbf{Q} (\mathbf{r} - \mathbf{H}\Delta\mathbf{u}) + \Delta\mathbf{u}^T \mathbf{\Lambda} \Delta\mathbf{u} \end{aligned} \quad (4)$$

Where  $\Delta\mathbf{u}$  is a control output vector of size  $m$ ,  $\mathbf{r}$  is the set point vector,  $\mathbf{Q}$  and  $\mathbf{\Lambda}$  are diagonal matrices for the input weighting and move suppression respectively and are considered as tuning parameters. Differentiating and equating to zero:

$$\begin{aligned} \Delta\mathbf{u} &= [\mathbf{H}^T \mathbf{H}]^{-1} \mathbf{H}^T (\mathbf{r}_o - y_o) \\ &= \mathbf{W}_1 \cdot (\mathbf{r}_o - y_o) \end{aligned} \quad (5)$$

Only the first calculated move is applied to the plant and can be calculated using the first row of the matrix  $\mathbf{W}$ . Hence,

$$\Delta m_o = \mathbf{W}_1 \cdot (\mathbf{r}_o - y_o) \quad (6)$$

where  $\mathbf{W}_1$  is a vector representing the first row of matrix  $\mathbf{W}$ . Since the current measured value  $y_o$  is used to estimate the future move, the controller is able to account for model mismatch and the offset error will finally reduce to zero.

#### B. Effect of model-plant mismatch:

Although the DMC can generate offset free response even in the presence of model mismatch, the latter can degrade the overall controller performance. Many researchers studied the effect of model plant mismatch (MPM) on MPC and how to develop performance assessment indicators [8,9]. To demonstrate the impact of MPM, assume that a plant is represented in s-domain by a first order plus dead time (FOPDT) model expressed as:

$$y = k \cdot \frac{e^{-\theta s}}{(1 + \tau s)} \quad (7)$$

Where  $k$  is the process gain,  $\theta$  is the dead time and  $\tau$  is the time constant.

Fig.2 shows the plant measured value when the DMC has a perfect model compared to DMC with MPM (+10% in gain and -10% in time constant).

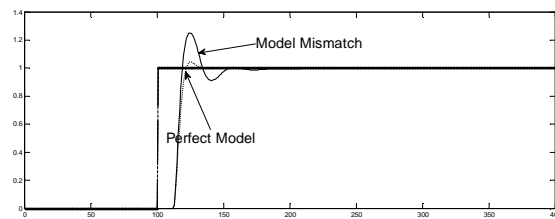


Fig. 2 Effect of model mismatch on control performance

#### IV. MPC WITH ADAPTIVE WEIGHTED OUTPUT

Now, assume that the controller output is applied to the plant model used in the design phase as well as the real plant as illustrated in fig. 3

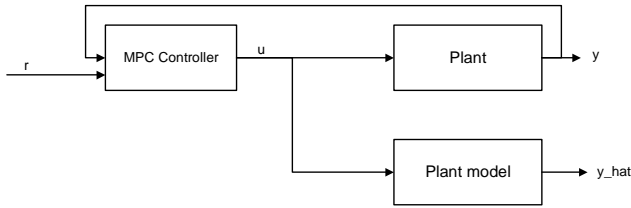


Fig. 3 MPC controller output applied to plant and plant model

$y$  and  $\hat{y}$  can be represented by:

$$y_k = \sum_{i=1}^N h_i \cdot u_{k-i} \quad (8)$$

$$\hat{y}_k = \sum_{i=1}^N \hat{h}_i \cdot u_{k-i} \quad (9)$$

First consider the case where the process gain and time constant are matched and the dead time in the model is underestimated. Then:

$$\mathbf{h} = z^{-n} \cdot \hat{\mathbf{h}} \quad (10)$$

It can be easily shown that if the controller output is delayed by  $n$ ,  $y$  and  $\hat{y}$  will be identical.

If the adaptive controller detects a delay  $n$  between  $y$  and  $\hat{y}$ , it should respond by delaying controller output by  $n$  samples. The time gap created in controller output should be filled by repeating the last output before detecting the mismatch in dead time.

Now consider the case where the model has a mismatch in both gain and time constant and assume that the output to the plant is subjected to a nonlinear transformation such that:

$$u_p(t) = (a \cdot u(t - n))^b \quad (11)$$

The adaptive problem can be defined as finding  $a$ ,  $b$ ,  $n$  such that:

$$y \approx \hat{y} + c \quad (12)$$

where  $c$  is the offset error between  $y$  and  $\hat{y}$  due to model mismatch.

Equation (12) can be represented in the following alternative form:

$$\frac{dy}{dt} \approx \frac{d\hat{y}}{dt} \quad (13)$$

If  $b=1$ , it can be easily shown that for unit step change the optimum value of  $a$  can be calculated as:

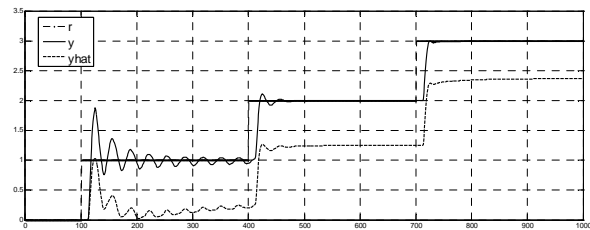
$$a = \frac{\hat{y}'|_{\text{at first peak}}}{y'|_{\text{at first peak}}} \quad (14)$$

It is worthy to note that the weight “ $a$ ” is enough to avoid overshoots and the final controller response will be satisfactory. Parameter  $b$  can be selected to improve controller response in case of severe time constant mismatch. Through experimental tests done on first order plants,  $b$  can be estimated using the following equations:

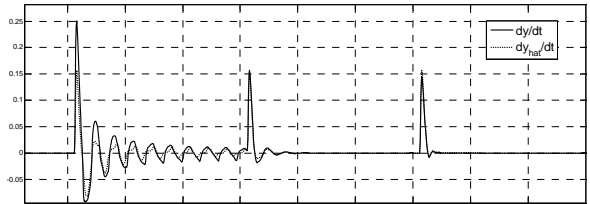
$$b = \frac{y'|_{\text{at second peak}}}{\hat{y}'|_{\text{at second peak}}}_{u_p=a.u} \quad (15)$$

Once a change in set point is detected, weights are recalculated and the result is multiplied by the stored values for  $a$  and  $b$ . Weights updating can be stopped once a certain criteria on overshoot is achieved.

To illustrate the idea of output weighting, consider an FOPDT system defined by  $k = 6.5$ ,  $\theta = 10s$  and  $\tau = 30s$ , controlled by a DMC modeled by  $\hat{k} = 5$ ,  $\hat{\theta} = 10s$ , and  $\hat{\tau} = 42s$ . Fig. 4 shows the trend for  $y$ ,  $\hat{y}$ ,  $y'$  and  $\hat{y}'$  for  $a=0.626$  and  $b=0.635$ . Note that at the third set point change,  $y'$  and  $\hat{y}'$  become superimposed and thus  $a$  and  $b$  stop updating. Note also that the first overshoot cannot be avoided for fast processes or in case of large dead time. A good approach is to apply small steps to controller set point until the control performance become satisfactory.



(a)



(b)

 Fig. 4 (a) Plot for  $y$  and  $\hat{y}$  (b) Plot for  $y'$  and  $\hat{y}'$ 

Fig. 5 illustrates the proposed adaptive MPC.

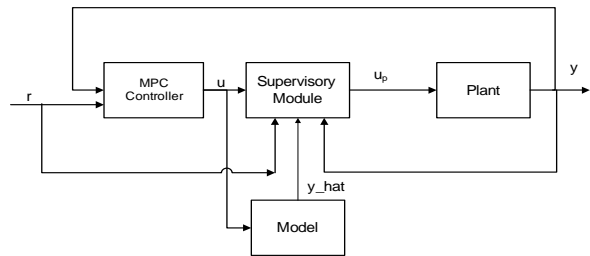


Fig. 5 Proposed adaptive DMC with weighted output

Following each change in the set point applied to the controller, the supervisory module compares  $y$  and  $\hat{y}$  samples and decide the required changes in  $a$  and  $b$  weighting parameters using the approach described earlier.

## V. SIMULATION RESULTS

All simulations shown in this section are performed using Matlab MPC toolbox and the dmc function developed in [14].

### A. First order plant, unstable Controller

Consider a first order plant modeled by  $\hat{k} = 2$ ,  $\hat{\theta} = 10$  s and  $\hat{\tau} = 60$  s and sampling time  $T_s=1$ s. If the real plant has  $k = 2.8$ ,  $\theta = 10$  s and  $\tau = 36$  s, the DMC will be unstable. Fig.6 shows the simulation results for Adaptive weighted output DMC compared to the conventional one. Noise is added to controller output and plant measured value. Unmeasured disturbance is applied to the plant output at  $t=600$ s. The calculated weights are  $a=0.31$ , and  $b=0.56$ . This example shows how the proposed controller can improve stability in case of severe model mismatch.

### B. Second order plant

Consider a second order plant modeled as:

$$\hat{y}(s) = \frac{3.4}{(1 + 40s)^2}$$

If the real plant is defined by

$$y(s) = \frac{4}{(1 + 32s)^2}$$

Fig. 7 shows the simulation results for AWO-DMC with a condition added to stop weight updates once overshoot is within accepted limits.

### C. Overestimated Gain

In some cases the process gain is overestimated or the time constant is underestimated in the model leading to a sluggish response. Applying AWO-DMC can help improving the response. Note that in this case the weights are higher than one. Care must be taken to set upper limits and upper rate of change for weights to avoid instability. Fig. 8 shows the simulation results for model described in simulation A when the actual process gain is equal to 1.4,  $a=1.43$  and  $b=1$ .

### D. Mismatch in dead Time

Consider a stirring process modeled as FOPDT using  $\hat{k} = 0.88$ ,  $\hat{\theta} = 40$  s and  $\hat{\tau} = 57$  s. If the real plant has  $k = 1.056$ ,  $\theta = 30$  s and  $\tau = 51$  s. Fig.9 shows the simulation results for AWO-DMC with  $\hat{y}$  plotted to illustrate how the mismatch in dead time was detected and compensated. The final weights calculated are  $a=0.7396$ ,  $b=1$  and  $n=9$ .

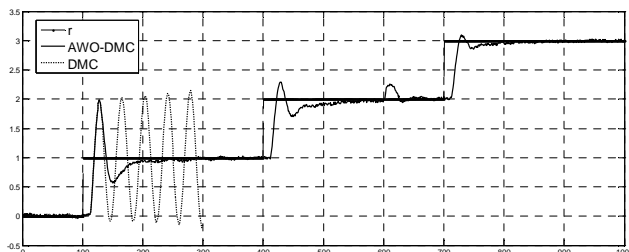


Fig. 6 AWO-DMC with severe model mismatch

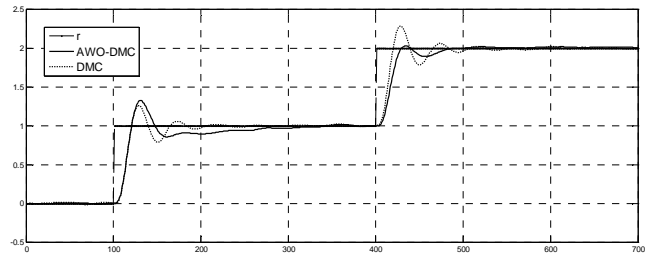


Fig. 7 AWO-DMC applied on second order plant

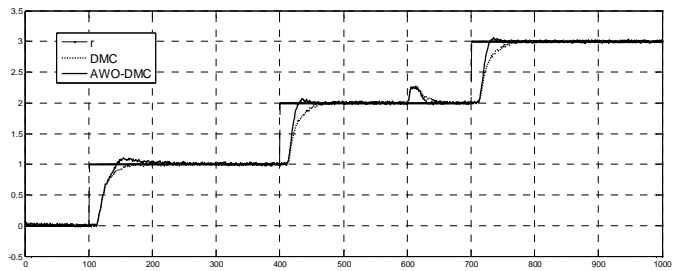


Fig. 8 AWO-DMC for overestimated gain

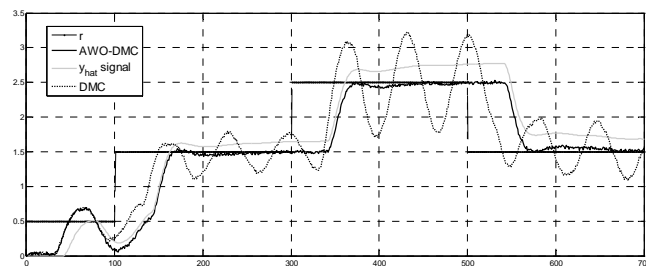


Fig. 9 AWO-DMC for mismatches in Gain, time constant and dead time

## VI. CONCLUSION

This paper presents a new approach in the implementation of SISO MPC controller. In this approach, approximate models collected from physical equations, dynamic simulations packages can be used directly to the plant. Poor performance and instability are avoided by using a simple adaptation to controller output. The simplicity of the calculations used allows the implementation of this controller on DCS currently used in industry. Adaptation can usually be accomplished in one or two step changes. Calculated weights should be continuously monitored and can be helpful in model fine tuning. This approach can considerably save the large cost spent on online identification packages and on expert process control engineers.

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